Lecture 7
20-Jan-17

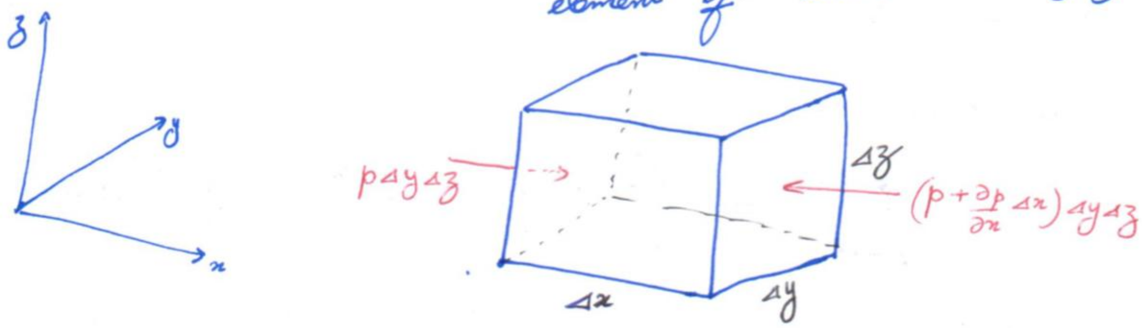
Pressure Force

Due to pressure, forces act on respective planes of interest. Because, you know pressure is having dimensions of force per unit area - i.e. $[M L^{-1} T^{-2}]$.

→ So, pressure × Area = Force.

→ The force due to pressure is called pressure force.

In our spatial co-ordinate system, consider ~~an~~ fluid element of volume $\Delta x \Delta y \Delta z$



→ There are two planes of area = $\Delta y \Delta z$ that are perpendicular to x - axis.

Let us say that as pressure is scalar, it is given as $p(x, y, z, t)$

Let pressure 'p' act on left side plane.

Let pressure $p + \frac{\partial p}{\partial x}\Delta x$ act on right side plane.

The net force acting ~~on~~ in the x- direction will be:

$$\Delta F_x = p\,\Delta y \Delta z - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \Delta z$$

$$= -\frac{\partial p}{\partial x}\Delta x \Delta y \Delta z$$

Similarly, the net force in the y- direction (only due to pressure) is:

$$\Delta F_y = -\frac{\partial p}{\partial y}\Delta x \Delta y \Delta z$$

Net force in z- direction is:

$$\Delta F_z = -\frac{\partial p}{\partial z}\Delta x \Delta y \Delta z$$

As force is vector, the net pressure force can be given as

$$\Delta \vec{F}_{pressure} = \left(-\frac{\partial p}{\partial x}\hat{i} - \frac{\partial p}{\partial y}\hat{j} - \frac{\partial p}{\partial z}\hat{k}\right)\Delta x \Delta y \Delta z$$

Since the volume $\Delta x \Delta y \Delta z$ is arbitrary and chosen by us:, we can define net pressure force ~~stre~~ per unit volume $\vec{f}_{pres}$

$$\vec{f}_{pressure} = -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)$$

$$= -\nabla p \qquad \text{(gradient of pressure)}$$

That is, it is the pressure gradient that causes pressure force.

Gage pressure, vacuum pressure

You have seen pressure expressed as:

→ Absolute pressure (The actual magnitude of pressure)

→ Gage pressure

If the ~~atmospheric~~ actual pressure is greater than atmospheric pressure, people also use gage pressure if $p > p_a \rightarrow p_{gage} = p - p_a$

→ Vacuum pressure

If $p < p_a \rightarrow p_{vacuum} = p_a - p$

→ You also know:

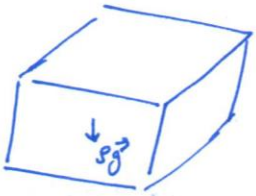
$$1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa}$$

Hydrostatic Pressure Conditions

Consider the earlier drawn fluid element of volume $\Delta x \Delta y \Delta z$ to be in static condition.

→ That is that fluid is in rest with no acceleration and no velocity.

→ If the velocity is absent or velocity is constant, then there won't be any viscous forces (Viscous Stress * Area) acting on the fluid element.



→ In static conditions

$$\sum \vec{F} = 0$$

→ The forces acting on this element will be - pressure forces and gravity forces.

i.e. $\sum F_x = 0$; $\sum F_y = 0$; $\sum F_z = 0$

For $\sum \vec{F} = 0$; $-\vec{\nabla} p \, \Delta x \Delta y \Delta z + \rho \vec{g} \, \Delta x \Delta y \Delta z = 0$

i.e. $$\boxed{\vec{\nabla} p = \rho \vec{g}}$$

→ This particular distribution of pressure is called <u>hydrostatic pressure</u> distribution.

→ You should note that this hydrostatic distribution is true for all fluids at rest, irrespective of their viscosities.

$$\vec{\nabla} p = \frac{\partial p}{\partial x} \hat{i} + \frac{\partial p}{\partial y} \hat{j} + \frac{\partial p}{\partial z} \hat{k}$$

$$\vec{g} = 0 \, \hat{i} + 0 \, \hat{j} - g \, \hat{k}$$

As $\sum F_x = 0$, this implies $\frac{\partial p}{\partial x} = 0$

III^ly $\sum F_y = 0$; implies $\frac{\partial p}{\partial y} = 0$

and $\sum F_z = 0$; implies $\underline{\underline{\frac{\partial p}{\partial z} = -\rho g}}$

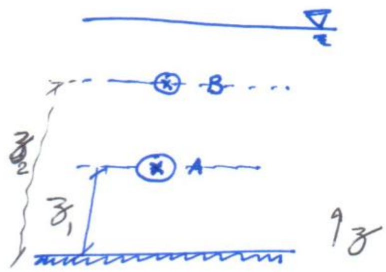
So it is clear to you that in hydrostatic conditions,

$$\frac{\partial p}{\partial x} = 0 \qquad \text{and} \qquad \frac{\partial p}{\partial y} = 0$$

i.e. 'p' is independent of x and y

Then we can write $\frac{\partial p}{\partial z}$ as $\frac{dp}{dz}$

$$\frac{dp}{dz} = -\rho g$$



This equation can be solved in respective domains

$$dp = -\rho g \, dz$$

$$\int_A^B dp = -\int_A^B \rho g \, dz$$

i.e. $$p_B - p_A = -\rho g (z_2 - z_1)$$

or $$p_A = p_B + \rho g (z_2 - z_1)$$

If B was water surface having p_B as atmospheric pressure. Usually in gage pressure,

$p_B = 0$ and $z_2 - z_1 = h$ (height of water)

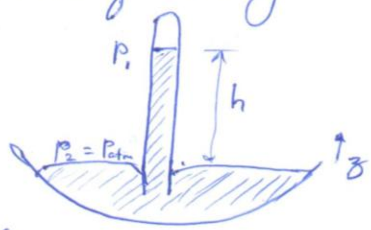
Then $$p_A = \rho g h$$

The simple example of application of hydrostatic formula is

Barometer

$\longrightarrow$ **Barometer** measures atmospheric pressure

* A sufficiently long tube closed at one end is filled with mercury.
* This tube is inverted into a reservoir of mercury.
* In static conditions,
  the mercury dips little bit at closed end at the top.



The vapor pressure of mercury is very small and approximately vacuum is assumed at the top.

$\therefore\ p_1 = 0$ (absolute value).

p_2 = atmospheric pressure.

$$p_2 - 0 = -\rho_{Hg}\, g\, (0 - h)$$

or $\underline{p_2 = \rho_{Hg}\, g\, h}$

As $p_2 = 101.35$ kPa

$\rho_{Hg} = 13.3 \times 1000$ kg/m^3

We have $\underline{h \approx 0.761 \text{ m}}$

So if you'll say the pressure is 0.761 m or 761 mm of Hg, it suggests about magnitude of atmospheric pressure.

Manometer Application

As you know that change in elevation creates pressure difference, the principle is applied in manometry.

i.e. $p_2 - p_1 = \Delta p = -\rho g (z_2 - z_1) = -\rho g \Delta z$