LECTURE 6

19-JAN-2016

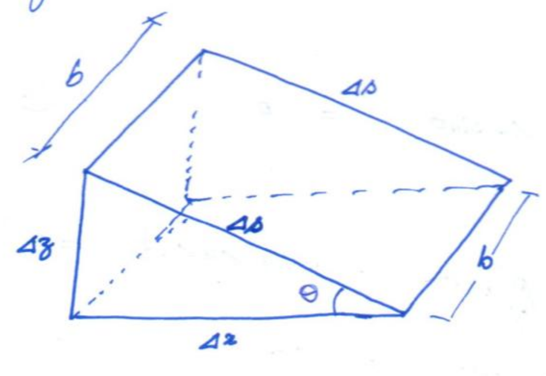
PRESSURE DISTRIBUTION IN A FLUID

→ When the fluid velocity is zero, we call it as hydrostatic conditions.

→ The pressure of the liquid varies with respect to the weight of fluid.

→ As discussed earlier, a fluid at rest cannot support shear stress

→ Let us describe the equilibrium of forces (static liquid) of a small wedge of fluid at rest.



→ Recall of your engineering mechanics of study about free body diagrams. As this is static liquid all the forces need to be balanced.

Obvious, from the figure, the forces in y-direction balance. Still we can work it out.

$\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma F_z = 0$.

As it is a static liquid, let us suggest that forces act into the plane in respective directions.

→ Let p_x be force per unit area acting on the plane $b\Delta z$.

→ Let p_z be the force per unit area acting on the plane $b\Delta x$

→ p_y be the force per unit area acting on the planes $\Delta x\Delta z$ in front side and $\Delta x\Delta z$ in back side.

→ p_s be the force per unit area on the plane $b\Delta s$.

∴ For $\Sigma F_y = 0$

$$p_y \Delta x\Delta z - p_y \Delta x\Delta z = 0 \rightarrow (1)$$

(Both forces are same)

For $\Sigma F_x = 0$;

$$p_x b\Delta z - p_s b\Delta s \sin\theta = 0 \rightarrow (2)$$

For $\Sigma F_z = 0$;

$$-p_s b\Delta s\cos\theta + p_z b\Delta x - \frac{1}{2}\rho g \Delta x\Delta z b = 0 \rightarrow (3)$$

From geometry, $\Delta s\cos\theta = \Delta x$
$\Delta s\sin\theta = \Delta z$

∴ In (2); $p_x b\Delta z - p_s b\Delta z = 0$; or $p_x = p_s$

In (3); $-p_s b\Delta x + p_z b\Delta x - \frac{1}{2}\rho g b\Delta x\Delta z = 0$

or $p_z = p_s + \frac{1}{2}\rho g\Delta z$

In $\lim_{\substack{\Delta z\to 0\\ \Delta x\to 0}}$; $p_z = p_s = p_x = p_y = p$ = pressure

That is at a static point, pressure is a ~~static~~ scalar property without any orientation.

At any mathematical point in space and time, the pressure is a <u>scalar value</u>. $p(x, y, z, t)$

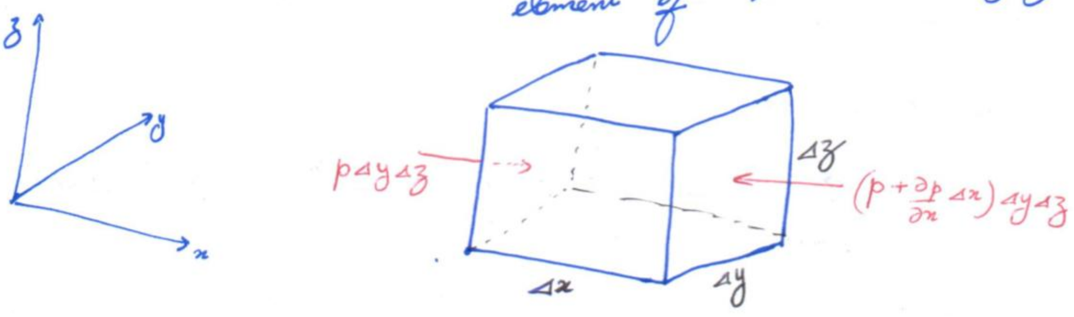
Pressure Force

Due to pressure, forces act on respective planes of interest. Because, you know pressure is having dimensions of force per unit area - i.e. $[M L^{-1} T^{-2}]$.

→ So, pressure × Area = Force.

→ The force due to pressure is called <u>pressure force</u>.

In our spatial co-ordinate system, consider ~~an~~ fluid element of volume $\Delta x \Delta y \Delta z$



→ There are two planes of area = $\Delta y \Delta z$ that are perpendicular to x-axis.

Let us say that as pressure is scalar, it is given as $p(x, y, z, t)$

Let pressure 'p' act on left side plane.

Let pressure $p + \frac{\partial p}{\partial x} \Delta x$ act on right side plane.

The <u>net force</u> acting ~~on~~ in the <u>x- direction</u> will be:

$$\Delta F_x = p\,\Delta y \Delta z - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \Delta z$$

$$= -\frac{\partial p}{\partial x}\Delta x \Delta y \Delta z$$

Similarly, the net force in the y- direction (only due to pressure) is:

$$\Delta F_y = -\frac{\partial p}{\partial y}\Delta x \Delta y \Delta z$$

Net force in z- direction is:

$$\Delta F_z = -\frac{\partial p}{\partial z}\Delta x \Delta y \Delta z$$

As force is vector, the net pressure force can be given as

$$\Delta \vec{F}_{pressure} = \left(-\frac{\partial p}{\partial x}\hat{i} - \frac{\partial p}{\partial y}\hat{j} - \frac{\partial p}{\partial z}\hat{k}\right)\Delta x \Delta y \Delta z$$

Since the volume $\Delta x \Delta y \Delta z$ is arbitrary and chosen by us:, we can define net pressure force ~~due~~ per unit volume $\vec{f}_{pres}$

$$\vec{f}_{pressure} = -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)$$

$$= -\nabla p \qquad \text{(gradient of pressure)}$$

That is, it is the pressure gradient that causes <u>pressure force</u>.

Gage pressure, vacuum pressure

You have seen pressure expressed as:

→ Absolute pressure (The actual magnitude of pressure)

→ Gage pressure

If the ~~atmospheric~~ actual pressure is greater than atmospheric pressure, people also use gage pressure. If $p > p_a \rightarrow p_{gage} = p - p_a$

→ Vacuum pressure

If $p < p_a \rightarrow p_{vacuum} = p_a - p$

→ You also know:

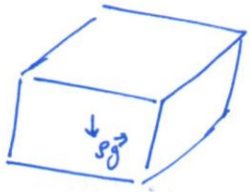
$$1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa}$$

Hydrostatic Pressure Conditions

Consider the earlier drawn fluid element of volume $\Delta x \Delta y \Delta z$ to be in static condition.

→ That is that fluid is in rest with no acceleration and no velocity.

→ If the velocity is absent or velocity is constant, then there won't be any viscous forces (Viscous Stress * Area) acting on the fluid element.



→ In static conditions

$$\Sigma \vec{F} = 0$$

→ The forces acting on this element will be - pressure forces and gravity forces.

i.e.

$$\Sigma F_x = 0 \; ; \qquad \Sigma F_y = 0 \; ; \qquad \Sigma F_z = 0$$

For $\Sigma \vec{F} = 0$; $\quad -\vec{\nabla} p \, \Delta x \Delta y \Delta z + \rho \vec{g} \, \Delta x \Delta y \Delta z = 0$

i.e. $\boxed{\vec{\nabla} p = \rho \vec{g}}$

→ This particular distribution of pressure is called hydrostatic pressure distribution.

→ You should note that this hydrostatic distribution is true for all fluids at rest, irrespective of their viscosities.

$$\vec{\nabla} p = \frac{\partial p}{\partial x} \hat{i} + \frac{\partial p}{\partial y} \hat{j} + \frac{\partial p}{\partial z} \hat{k}$$

$$\vec{g} = 0 \hat{i} + 0 \hat{j} - g \hat{k}$$

As $\Sigma F_x = 0$, this implies $\frac{\partial p}{\partial x} = 0$

III^ly $\Sigma F_y = 0$; implies $\frac{\partial p}{\partial y} = 0$

and $\Sigma F_z = 0$; implies $\underline{\underline{\frac{\partial p}{\partial z} = -\rho g}}$

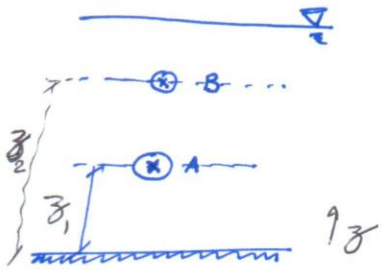
So it is clear to you that in hydrostatic conditions,

$$\frac{\partial p}{\partial x} = 0 \quad \text{and} \quad \frac{\partial p}{\partial y} = 0$$

i.e. 'p' is independent of x and y

Then we can write $\frac{\partial p}{\partial z}$ as $\frac{dp}{dz}$

$$\frac{dp}{dz} = -\rho g$$



This equation can be solved in respective domains

$$dp = -\rho g \, dz$$

$$\int_A^B dp = -\int_A^B \rho g \, dz$$

i.e. $$p_B - p_A = -\rho g (z_2 - z_1)$$

or $$p_A = p_B + \rho g (z_2 - z_1)$$

If B was water surface having p_B as atmospheric pressure. Usually in gage pressure,

$p_B = 0$ and $z_2 - z_1 = h$ (height of water)

Then $$p_A = \rho g h$$