LECTURE 43

27-APRIL-2016

TURBULENT FLOWS

Yesterday, we discussed about laminar flow in pipes.

→ We discussed about head loss and for circular (or round pipes), if the flow is laminar, then

$$h_f = \frac{32 \mu L V}{\rho g D^2} = \frac{128 \mu L Q}{\pi \rho g D^4} \rightarrow (1)$$

As the general form of head loss is:

$$h_f = f \frac{L}{D} \frac{V^2}{2g}, \rightarrow (2)$$

we can deduce the Poiseuille laminar flow friction factor, $f_{laminar}$ by comparing (1) and (2).

You see, that this friction factor, $f = \frac{64}{Re}$.

⇒ In turbulent flows, the fluctuation of velocities within a cross section itself is huge.

That is, you may find it difficult to use the differential equations for flow for turbulent condition

For turbulent pipe flow, you need ~~to~~ not solve the differential equations at present.

Many scientists have worked on pipe flow and have deduced relations to find the friction factor.

e.g: In 1935, Prandtl formed the following empirical relation between friction factor and Reynolds number in turbulent pipe flow

$$\frac{1}{\sqrt{f}} = 2.0 \log\left[Re\sqrt{f}\right] - 0.8$$

From this, you can deduce the head loss,

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

Note: → If the pipe is horizontal, you may then see that $h_f = \frac{\Delta p}{\rho g}$

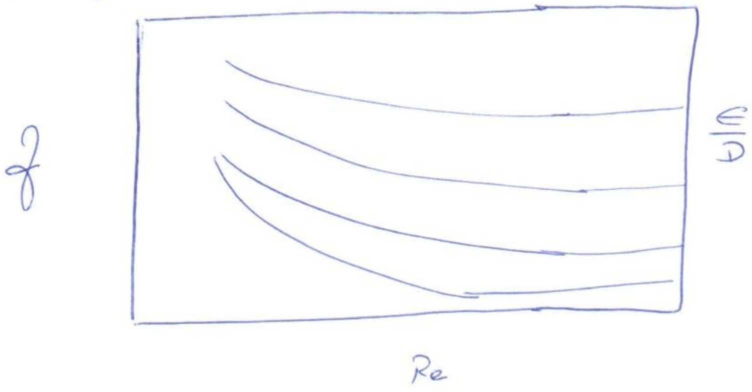
$$\text{or } \Delta p = \rho g\, h_f$$

where h_f is already evaluated using friction factor.

⇒ The Moody's chart for pipe friction is a well-established chart for friction factor f versus Reynolds number. It uses the formula:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left[\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right]$$

This Moody's chart is used in most of the standard-design of pipe flow problems.



Recall
D → diameter of pipe

Here the ϵ is the roughness thickness.

Some of the recommended roughness values are:

Material	Condition	ϵ mm
Steel	Sheet Metal, New	0.05
	Stainless, New	0.002
	Commercial, New	0.046
	Riveted	3.0
	Rusted	2.0
Iron	Cast, New	0.26
	Wrought, New	0.046
	Galvanised, New	0.15
	Asphalted, Cast	0.12
Brass		0.002
Plastic	Tubes.	0.0015
Glass	—	—
Rubber	Smoothed	0.01
Wood		0.5
Concrete	Rough	2.0
	Smooth	0.04

⇒ The scientist Haaland has given an explicit relation to Moody's chart with less than 2% variation.

$$\frac{1}{\sqrt{f}} \approx -1.8 \log_{10}\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$$

You may utilise this relation in turbulent flow computations.

Example: (As adopted from FM White's Fluid Mechanics)
Oil with $\rho = 900\ kg/m^3$ and kinematic viscosity, $\nu = 0.00001\ m^2/s$ flows at $0.2\ m^3/s$ through 500 m of 200 mm-diameter cast iron pipe. Determine a) the head loss, b) the pressure drop if the pipe slopes down at 10° in the flow direction.

Solution.

Given $\rho = 900\ kg/m^3$, $\nu = 0.00001\ m^2/s$.

$$\therefore \mu = \rho\nu = 900 \times 0.00001 = 0.009\ \frac{kg}{m\,s}$$

$Q = 0.2\ m^3/s$, $L = 500\ m$, $D = 200\ mm = 0.2\ m$

We are ~~considering~~ assuming steady and incompressible flow for this case:

$$\therefore V = \frac{Q}{A} = \frac{0.2}{\frac{\pi}{4}\times(0.2)^2} = 6.4\ m/s$$

Reynolds Number, $$Re = \frac{\rho V D}{\mu} = \frac{900 \times 6.4 \times 0.2}{0.009}$$

$$= \underline{\underline{128000}} \gg 2300$$

Flow is turbulent.

⇒ We have to compute head loss:

→ The pipe material is cast iron

$$\therefore \quad \epsilon = 0.26 \text{ mm}$$

$$\therefore \quad \frac{\epsilon}{D} = \frac{0.26}{200} = 0.0013$$

From the explicit formula

$$\frac{1}{\sqrt{f}} \approx -1.8 \log_{10}\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$$

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10}\left[\frac{6.9}{128000} + \left(\frac{0.0013}{3.7}\right)^{1.11}\right]$$

$$= -1.8 \times (-3.698) = 6.6566$$

$$\text{ie } \sqrt{f} = 0.1502 \; ; \quad f = 0.0226$$

$$\therefore \; h_f \approx f \frac{L}{D} \frac{V^2}{2g} = 0.0226 \times \frac{500}{0.2} \frac{(6.4)^2}{2 \times 9.81}$$

$$\approx \underline{118 \text{ m}}$$

$$\therefore \quad h_f = \underline{118 \text{ m}}$$

Pressure drop

$$h_f = \Delta z + \frac{\Delta p}{\rho g}$$

$$\text{i.e.} \quad 118 = L \sin(10) + \frac{\Delta p}{\rho g}$$

$$\text{ie} \quad \Delta p = [118 - 500 \sin(10)] \times 900 \times 9.81$$

$$\approx 273699 \approx \underline{273700 \text{ Pa}}$$

⇒ You can now also think, what does a <u>pump</u> do?

→ If the hydraulic head is lower at a portion and you want to take water from that location, then a pump will provide additional head, so that overall head at that location increases and water will flow to the location of lower head.

⇒ The following types of design problems can be seen in <u>Pipe Flows</u>.

(i) Given D, L, V (or Q), ρ, μ, g, → compute head loss h_f.

(ii) Given D, L, h_f, ρ, μ, g, → compute velocity V or discharge Q.

(iii) Given Q, L, h_f, ρ, μ, g → compute diameter of the pipe

(iv) Given Q, D, h_f, ρ, μ, g → compute the pipe length L