LECTURE 42
26-APRIL-2016

FULLY DEVELOPED PIPE FLOW

Yesterday, we were discussing about fully developed pipe flow.

We were also going through Poiseuille's solution of Navier-Stokes equation for incompressible steady flow in a round pipe of diameter D (or radius R)

Recall the solution $u = u_{max}\left(1 - \frac{r^2}{R^2}\right)$

where $u_{max} = \left(-\frac{dp}{dx}\right)\frac{R^2}{4\mu}$

$$\left(-\frac{dp}{dx}\right) = \frac{\Delta p + \rho g \Delta z}{L} \quad ; \quad V = \frac{u_{max}}{2}$$

$\Rightarrow$ We have seen discharge, $Q = \int u \, dA$

$$= \frac{\pi R^4}{8\mu}\left(\frac{\Delta p + \rho g \Delta z}{L}\right)$$

Now the shear stress (wall shear stress)

$$\tau_w = \left|\mu \frac{du}{dr}\right|_{r=R} = \frac{4\mu V}{R} = \frac{R}{2}\left(\frac{\Delta p + \rho g \Delta z}{L}\right)$$

Recall, $h_f = \Delta z + \frac{\Delta p}{\rho g}$

$\therefore$ Here for Poiseuille flow, $h_f = \frac{4\mu V L}{R^2/2 \; \rho g}$

or $$h_f = \frac{8\mu V L}{\rho g R^2}$$

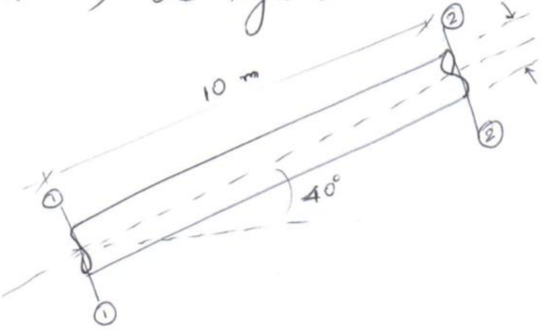
So, if you know, wall shear stress τ_w

Then $h_f = \dfrac{8 \mu V L}{\rho g R^2} = f_{laminar} \dfrac{2L}{R} \dfrac{V^2}{2g}$

or $f_{laminar} \rightarrow \text{function}\left(\dfrac{1}{Re}\right)$

Example (Adopted from FM White's Fluid Mechanics)

An oil with $\rho = 900$ kg/m³ and $\mu = 0.18 \frac{kg}{m s}$ flows through an inclined pipe. Two sections, section ① and section ② are 10 m apart. Assume steady laminar flow. a) Check whether flow is up or down. b) Compute h_f between ① and ②, c) compute the discharge Q, d) velocity V, and e) The Reynolds number.



Given $p_1 = 350000$ Pa

$z_1 = 0.0$

$p_2 = 250000$ Pa

$z_2 = 10 \sin 40$

$= 6.43$ m

$D = 6$ cm

→ The flow of oil will be in the direction of falling hydraulic gradient

This is because $V_1 = V_2$ and therefore $\dfrac{V_1^2}{2g}$ and $\dfrac{V_2^2}{2g}$ cancels off.

HGL = Datum Head + Pressure Head

$(HGL)_1 = 0.0 + \dfrac{350000}{900 \times 9.81} = 39.64$ m

$$(HGL)_2 = z_2 + \frac{p_2}{\rho g} = 6.43 + \frac{250000}{900 \times 9.81}$$

$$= \underline{\underline{34.74 \text{ m}}}$$

a) ∴ As the hydraulic gradient is higher at section 1, flow occurs from bottom to top.

b) Head loss between 1 and 2 $= 39.64 - 34.74$

$$h_f = \underline{\underline{4.9 \text{ m}}}$$

c) Discharge Q for circular pipe, $Q = \frac{\pi R^4}{8\mu}\left(\frac{\Delta p + \rho g \Delta z}{L}\right)$

$$\text{or } Q = \frac{\pi R^4 \rho g}{8 \mu L} h_f$$

$$= \frac{\pi \times 900 \times 9.81 \times (0.03)^4}{8 \times 0.18 \times 10} \times 4.9$$

$$= \underline{\underline{0.0076 \text{ m}^3/\text{s}}}$$

d) Velocity, $V = \frac{Q}{A} = \frac{0.0076}{\pi \times (0.03)^2} = 2.7 \text{ m/s}$

e) Reynolds number, $Re = \frac{\rho V D}{\mu} = \frac{900 \times 2.7 \times 0.06}{0.18}$

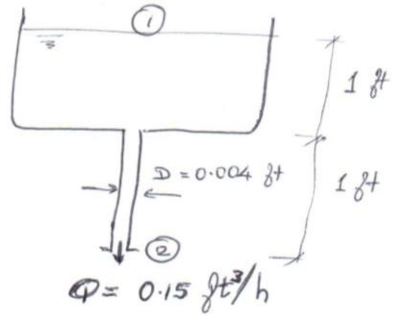
$$= \underline{\underline{810}}$$

$Re = 800 < 2300$.

(∴ Flow is LAMINAR).

Example

A liquid of specific weight $\rho g = 58 \text{ lbf/ft}^3$ flows by gravity through a 1-ft tank and 1-ft capillary tube at a rate of 0.15 ft³/h. Sections (1) and (2) are at atmospheric pressure. Neglecting entrance effects, compute viscosity of the liquid.



Answer:

For the capillary pipe,

length $L = 1$ ft,

diameter, $D = 0.004$ ft.

→ The flow is steady and incompressible.

Section (1) is free surface, ∴ We can approximate

$V_1 \approx 0.0$

→ As $\rho g = 58$ lbf/ft³,

we have $\rho = \frac{58}{32.2} = 1.801$ slug/ft³

We have $Q = 0.15$ ft³/h

$$\therefore \quad V_2 = \frac{Q}{A_2} = \frac{0.15/3600}{\frac{\pi}{4} \times 0.004^2}$$

$$= \underline{\underline{3.32 \text{ ft/s}}}$$

Writing the energy equation between (1) and (2)

$$\frac{p_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f$$

$$\text{or} \quad h_f = (z_1 - z_2) + \frac{1}{2g}\left[\alpha_1 V_1^2 - \alpha_2 V_2^2\right] + \frac{(p_1 - p_2)}{\rho g}$$

As both sections open to atmosphere, $p_1 = p_2$ and also $V_1 = 0$

$$\therefore \; h_f = (z_1 - z_2) - \alpha_2 \frac{V_2^2}{2g}$$

From laminar pipe flow, it is seen that $\alpha = 2.0$

$$= (2 - 0) - 2.0 \times \frac{3.32^2}{2 \times 32.2}$$

$$= 2 - 0.342$$

$$h_f = \underline{\underline{1.658 \text{ ft}}}$$

Suppose, if we take $\alpha_2 = 1.0$ itself, then the head loss $h_f = 1.83$ ft

→ To compute viscosity,

$$h_f = \frac{32 \mu L V}{\rho g D^2} \quad \text{for laminar circular pipe flow.}$$

$$\text{i.e.} \quad 1.658 = \frac{32 \times \mu \times 1.0 \times 3.32}{58 \times (0.004)^2}$$

$$\therefore \; \mu = 1.45 \times 10^{-5} \; \frac{\text{slug}}{\text{ft-s}}$$

→ The Reynolds number for pipe flow:

$$Re = \frac{\rho V D}{\mu} = \frac{1.80 \times 3.32 \times 0.004}{1.45 \times 10^{-5}}$$

$$\simeq 1650$$

(The flow is laminar).