LECTURE 40

20 - APRIL - 2016

# MODEL SIMILARITY (PART-II)

Yesterday, we discussed about geometric, kinematic and dynamic similarities that can exist between a model and a prototype.

Today, we will quickly work on an example problem.

## Example (As adopted from FM White's Fluid Mechanics)

The pressure drop due to friction in a long circular pipe is a function of average flow velocity, density, viscosity, and pipe length and diameter

i.e. $\Delta p = f(V, \rho, \mu, L, D)$

We wish to know how $\Delta p$ varies with $V$. Use Pi-theorem to rewrite this function in dimensionless form.

| D (cm) | L (m) | Q ($m^3/h$) | $\Delta p$ (Pa) | $\rho$ ($kg/m^3$) | $\mu$ (kg/ms) | V (m/s) |
|---|---|---|---|---|---|---|
| 1.0 | 5.0 | 0.30 | 4680 | 680 | $2.92 \times 10^{-4}$ | 1.06 |
| 1.0 | 7.0 | 0.60 | 22300 | 680 | $2.92 \times 10^{-4}$ | 2.12 |
| 1.0 | 9.0 | 1.00 | 70800 | 680 | $2.92 \times 10^{-4}$ | 3.54 |
| 2.0 | 4.0 | 1.00 | 2080 | 998 | 0.0010 | 0.88 |
| 2.0 | 6.0 | 2.00 | 10500 | 998 | 0.0010 | 1.77 |
| 2.0 | 8.0 | 3.10 | 30400 | 998 | 0.0010 | 2.74 |
| 3.0 | 3.0 | 0.50 | 540 | 13550 | $1.56 \times 10^{-3}$ | 0.20 |
| 3.0 | 4.0 | 1.00 | 2480 | 13550 | $1.56 \times 10^{-3}$ | 0.39 |
| 3.0 | 6.0 | 1.70 | 9600 | 13550 | $1.56 \times 10^{-3}$ | 0.67 |

Note here the velocity, $V = \dfrac{Q}{A} = \dfrac{Q}{\frac{\pi}{4} D^2}$

You are provided the information

$$\Delta p = f(V, \rho, \mu, L, D)$$

There are six variables $\Delta p, V, \rho, \mu, L, D$ (i.e. $n = 6$)

The basic dimensions in the expressions are $M, L, T$ ($k = 3$)

∴ We expect $j = n - k = 6 - 3 = 3$ $\pi$ groups.

In this case, as we want to see how $\Delta p$ varies with $V$, we <u>would not</u> keep them as repeating variables.

∴ Therefore, as basic dimensions are $M, L, T$ here, we will keep $\rho, \mu, D$ as repeating variables

$$\Pi_1 = \rho^a \mu^b D^c \Delta p \rightarrow M^0 L^0 T^0$$

$$\Pi_2 = \rho^d \mu^e D^f V \rightarrow M^0 L^0 T^0$$

$$\Pi_3 = \rho^g \mu^h D^i L \rightarrow M^0 L^0 T^0$$

Solving for exponents, you get

$$\Pi_1 = \frac{\rho D^2 \Delta p}{\mu^2} \; ; \quad \Pi_2 = \frac{\rho V D}{\mu} \; , \quad \Pi_3 = \frac{L}{D}$$

∴ We will get a dimensionless expression

$$\frac{\rho D^2 \Delta p}{\mu^2} = f\left(\frac{\rho V D}{\mu}, \frac{L}{D}\right)$$

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

⇒ We can now plot $\Pi_1$ versus $\Pi_2$ with $\Pi_3$ as a parameter or $\Pi_1$ versus $\Pi_3$ with $\Pi_2$ as a parameter. As our objective was to find $\Delta p$ versus $V$ then we have to go with $\Pi_1$ versus $\Pi_2$ with $\Pi_3$ as a parameter.

→ From the first row of the data,

$\frac{L}{D} = 500$, evaluate $\frac{\rho D^2 \Delta p}{\mu^2} = 3.73 \times 10^9$

$$\frac{\rho V D}{\mu} = 24700$$

From second row of the data

$\frac{L}{D} = 700$, and $\frac{\rho D^2 \Delta p}{\mu^2} \approx 1.78 \times 10^{10}$

$$\frac{\rho V D}{\mu} \approx 49370$$

→ Like this you evaluate for each row.
You will get nine data points.

→ There are definitely length effects.

For same value of $\frac{L}{D}$ say $\frac{L}{D} = 200$, we get, for $D = 3.0$ cm, $\frac{\rho D^2 \Delta p}{\mu^2}$ ~~= 5.34 × 10⁹~~

$D = 2.0$ cm, $\frac{\rho D^2 \Delta p}{\mu^2} =$

~~Δp is now~~ For same $\frac{L}{D}$ ratio, we can ~~see~~ that $\Delta p$ is increasing linearly with $L$.

As $L$ is present only in $\Pi_3 = \frac{L}{D}$,

the function $\Pi_1 = f(\Pi_2, \Pi_3) = \frac{L}{D}\, g(\Pi_2)$

i.e. $$\frac{\rho D^2 \Delta p}{\mu^2} = \frac{L}{D}\, g\left(\frac{\rho V D}{\mu}\right)$$

or $$\frac{\rho D^3 \Delta p}{L \mu^2} = g\left(\frac{\rho V D}{\mu}\right)$$

By going through the given data, you can now see or plot the above relation and see that

$$\boxed{\frac{\rho D^3 \Delta p}{L \mu^2} \approx 0.155\left(\frac{\rho V D}{\mu}\right)^{1.75}}$$

All Newtonian smooth pipe flows correlate in this manner.

This will be the principle in many pipe flows.

# Viscous Flows or Pipe Flows

You have already been told about the basic flow equations
(i.e. Momentum equation and continuity equation).

→ These equations can be applied for many practical cases.

→ However, you should note that there is no general fluid motion analysis possible.

* You can have only particular solutions for particular problems.
* Sometimes for certain problems you may have to neglect viscosity.
* Sometimes you may have to neglect compressibility

As a civil engineer, you may have to consider mostly incompressible liquids.

→ For a fluid mechanics specialist, one of the most utilised application will be in Pipe Flow Analysis.

Any common man may first ask you ~~to~~ – how will you detect the type of flow or amount of flow in a pipe.

→ Pipe flows are generally viscous.

So Reynolds number play a prominent part in understanding the pipe flow.
You can have based on Reynolds number

→ Laminar flows
→ Turbulent flows

Example (Adapted from FM WHITE's Fluid Mechanics)

From Reynold's experiments on pipe flows of various diameters, it was found that there is a critical Reynolds number $Re_{critical} \approx 2300$. Below this value, pipe flow is laminar.
For flow through a 5-cm diameter pipe, at what velocity will this critical Reynolds number appear at 20°C.
a) For airflow, b) for water flow.

Solution

In pipe flow, the Reynolds number is defined as

$$Re = \frac{\rho V D}{\mu}$$

$$\mu_{air} = 1.80 \times 10^{-5} \ \frac{kg}{ms}, \quad \rho_{air} = 1.205 \ kg/m^3$$

$$\mu_{water} = 0.001 \ \frac{kg}{ms}, \quad \rho_{water} = 998 \ kg/m^3$$

$$\therefore Re_{critical}\Big|_{air} = 2300 = \frac{\rho V D}{\mu} = \frac{1.205 \times V \times 0.05}{1.80 \times 10^{-5}}$$

$$\text{or} \quad V_{air} = 0.687 \ m/s$$

$$Re_{critical}\Big|_{water} = 2300 = \frac{\rho V D}{\mu} = \frac{998 \times V \times 0.05}{0.001}; \quad V_w = 0.0461 \ m/s$$