LECTURE 4

13-JAN-2016

Secondary Properties of Fluid

Yesterday, we discussed about the

* Eulerian and Lagrangian approach of analysing a motion.
* We also saw that pressure, temperature, and density are considered as primary thermo-dynamic variables.
* Among the secondary properties, we discussed about viscosity.

→ We saw that

shear stress $\boxed{\tau = \mu \frac{du}{dy}}$ → ①

where μ → coefficient of viscosity

Looking into eqn ①, we can see that shear stress is linearly related to gradient of velocity.

→ Therefore, all fluids that ~~behave linearly~~ follows the above linear relation are called Newtonian fluids.

Reynolds Number

$$Re = \frac{\rho V L}{\mu}$$

where $V \rightarrow$ characteristic velocity of fluid

$L \rightarrow$ characteristic length

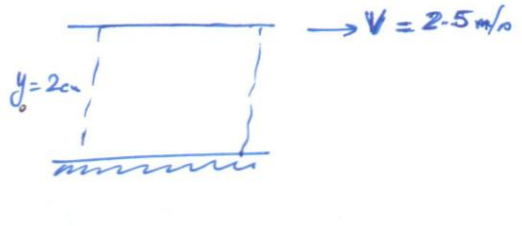
Reynolds number will suggest whether a flow is laminar or turbulent.

Example

The fluid SAE 30 oil at $20^\circ C$ is sheared by ~~seen~~ pulling the top portion ~~with~~ in x-direction with a velocity $V = 2.5$ m/s. The depth of water is $y_0 = 2$ cm. $\mu_{SAE} = 0.29$ kg/(m·s).

Solution Steps

(i) Sketch the problem



~~$\tau = \mu \frac{du}{dy}$~~

At $y = 0$, $u = 0$

$y_0 = 2$, $u = V = 2.5$ m/s

Now $\mu \frac{du}{dy} = \tau$,

i.e. $\frac{du}{dy} = \frac{\tau}{\mu} = \text{constant}$ $\left[\frac{\tau}{\mu} \rightarrow \text{constant for a given shear stress} \right]$.

Solving, $u = a + by$

Applying the boundary conditions given, we get

$$a = 0 \quad \text{and} \quad b = \frac{V}{y_0} = \frac{2.5}{2 \times 10^{-2}}$$

$$\therefore \; u = \frac{V}{y_0} \cdot y = \frac{2.5}{2 \times 10^{-2}} * y$$

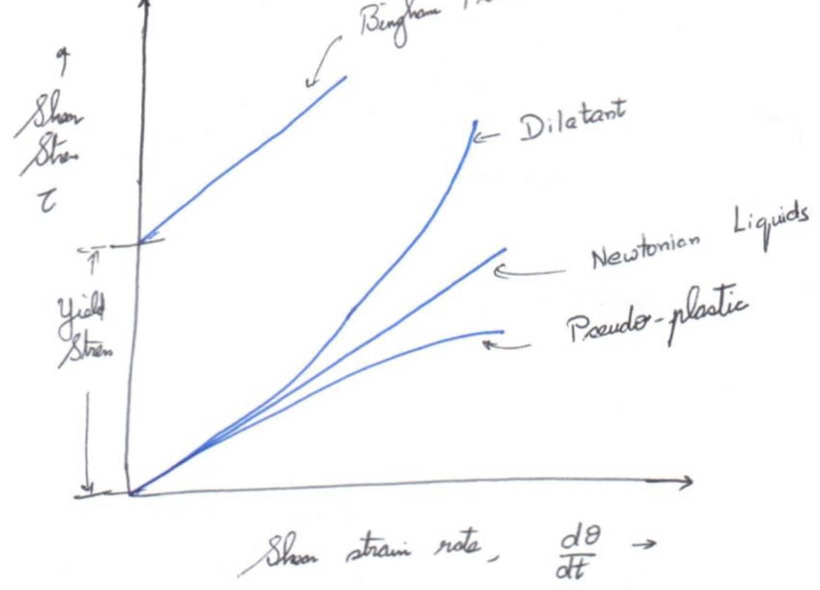
$$\therefore \; \frac{du}{dy} = 1.25 \times 10^{2} = \text{constant}$$

$$\therefore \; \tau = \mu \frac{V}{y_0} = 0.29 * 1.25 \times 10^{2} \; N/m^2$$

$$=$$

Non-Newtonian Fluids

There are many fluids that do not obey linear law of viscosity. They are Non-Newtonian fluids. If we plot Shear strain rate versus Shear stress τ



Dilatants are shear thickening liquids. That is such liquids increases the resistance with increasing strain rate. (e.g. Quick sand).

Pseudo-plastic are shear thinning fluid, which is less resistant at higher strain rates.
(e.g. Blood plasma, paint, colloidal suspensions etc.)

Bingham plastics are fluids that require some finite yield stress before it starts flow.
(e.g. mayyonise, toothpaste, etc.)

Vapor Pressure

The pressure at which a liquid boils and is in equilibrium with its own vapor.

→ If the liquid pressure is greater than the vapor pressure, the only exchange between liquid and vapor is evaporation at interface.

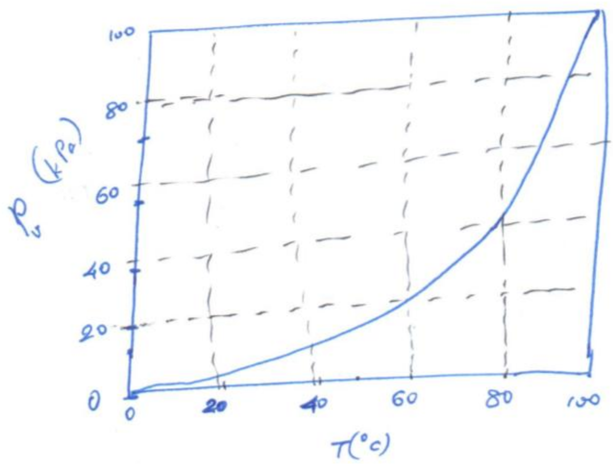
→ If liquid pressure falls below the vapor pressure, vapor bubbles begin to appear in liquid.

→ When the liquid pressure drops below vapor pressure due to flow phenomenon, the process is called **CAVITATION**.

→ If water is accelerated from rest to approximately 15 m/s, the pressure of it drops by nearly 1 atm.

You know how much is 1 atm $= 1.01325 \times 10^5$ N/m² (Pa)

$= 101.325$ kPa



→ If water is heated to 100°c, its vapor pressure rises to 101.325 kPa.

→ The dimensionless parameter that describe the flow induced boiling is called Cavitation Number

$$Ca = \frac{P_a - P_v}{\frac{1}{2} \rho V^2}$$

P_a → ambient pressure
P_v → Vapor pressure
V → characteristic flow velocity
ρ → density of fluid.

Example. (Adapted from FM White's "Fluid Mechanics")

A torpedo moving in fresh water at 15°C has minimum-pressure point given by $p_{min} = p_0 - 0.35\,\rho V^2$, where p_0 is a known value = ~~110~~ 120 kPa, ρ is water density, and V is torpedo velocity. Estimate the velocity at which cavitation bubbles will form on torpedo.

Note that at 15°C, density of water ~~can~~ need to be interpolated from the data

T °c	ρ (kg/m³)
0	1000
10	1000
20	998
30	996

→ The data for vapor pressure is as follows:

T °c	p_v (kPa)
0	0.611
10	1.227
20	2.337
30	4.242
40	7.375
⋮	⋮

Solution

→ Cavitation bubbles start to form when liquid pressure equals the vapor pressure at that temperature.

→ So let us take the minimum pressure point value to be same as vapor pressure, so that cavitation bubbles form.

$$p_{min} = p_v$$

$$p_{min} = 120 \times 10^3 - 0.35 * 999 * V^2$$

$$p_v \text{ at } 15°c \approx 1.80 \text{ kPa} = 1800 \text{ Pa}$$

$$\therefore \quad 1800 = 120000 - 0.35 * 999 * V^2$$

$$\therefore \quad V^2 = \qquad \text{or} \qquad V \approx$$