LECTURE 39
19 - APRIL - 2016

MODEL SIMILARITY

→ We discussed yesterday that dimensional analysis is used in fluid mechanics, especially in modeling studies.

→ It was stated that the model and prototype have to be similar, if you want to design appropriate prototype.

→ ~~There are~~ Usually the experimenter perform dimensional analysis to achieve similarity between the model and prototype. For a phenomena, they may come up with dimensionless relations

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4, \ldots, \Pi_k)$$

** The flow conditions of a model test will be completely similar to the prototype if all the dimensionless parameters of model is same as prototype.

i.e. You require $\Pi_{2_m} = \Pi_{2_p}$; $\Pi_{3_m} = \Pi_{3_p}$; ...

→ To achieve complete similarity is very difficult. Instead, by comprising various features you can have

* geometric similarity
* kinematic similarity
* dynamic similarity
* thermal similarity

Geometeric Similarity

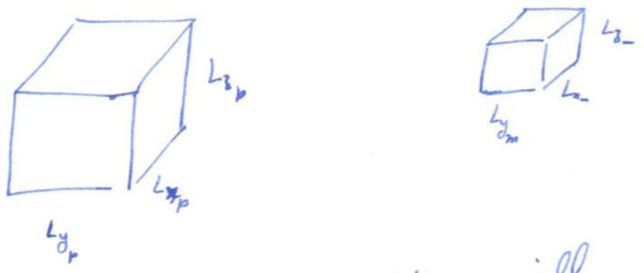
If all the length dimensions [L] of the model and the prototype are having same linear scale ratio, then the model and prototype is said to be geometrically similar.

i.e. For a body of length in x, y, z directions are L_{x_p}, L_{y_p}, L_{z_p}

If you make a model of that body with lengths L_{x_m}, L_{y_m}, L_{z_m}

And if $\dfrac{L_{x_m}}{L_{x_p}} = \dfrac{L_{y_m}}{L_{y_p}} = \dfrac{L_{z_m}}{L_{z_p}} = \text{constant}.$

Then they are geometrically similars.



Geometerially similar items will contain homologous points.

Kinematic Similarity

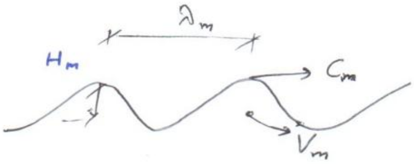
If the model and prototype have same length scale ratio and same time scale ratio, then they are kinematically similar.

For time scale equivalence, you now require equivalence of Reynolds number and Mach number, etc.

e.g.

→ Frictionless flow like free surface flow can be made kinematically similar, if their Froude numbers are equal.

i.e. In a Wave Motion Modeling Studies:



→ For kinematic similarity, the Froude numbers have to be equal.

$$Fr_m = \frac{V_m^2}{g L_m} = \frac{V_p^2}{g L_p} = Fr_p$$

→ Froude number consist of only length and time dimensions and hence suitable for adopting kinematic similarity.

★ From $\dfrac{V_m^2}{g L_m} = \dfrac{V_p^2}{g L_p}$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \quad ; \quad \text{If } \frac{L_m}{L_p} = \alpha$$

Then $\dfrac{V_m}{V_p} = \sqrt{\alpha}$, Also $\dfrac{T_m}{T_p} = \dfrac{L_m/V_m}{L_p/V_p} = \sqrt{\alpha}$

If viscosity, surface tension, etc. play important role then you may have to consider dynamic similarity.

Dynamic Similarity

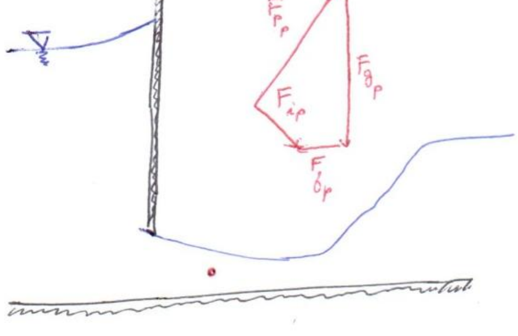
The model and the prototype are dynamically similar, if they have same length scale ratio, same time scale ratio, and same force scale ratio.

From Newton's law: for any fluid particle - the sum of pressure force, gravity force, and friction force is equal to the net force or inertial force.

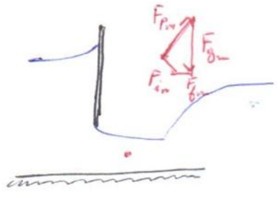
i.e. $$\vec{F}_p + \vec{F}_g + \vec{F}_f = \vec{F}_i$$

The dynamic similarity ~~laws~~ considers that

$$\frac{\vec{F}_{p_m}}{\vec{F}_{p_p}} = \frac{\vec{F}_{g_m}}{\vec{F}_{g_p}} = \frac{\vec{F}_{f_m}}{\vec{F}_{f_p}} .$$



PROTOTYPE



MODEL

→ In this flow through sluice gate experiment, force polygons at homologous points will have exactly the same shape if $Re_p = Re_m$ and $Fr_m = Fr_p$.

Example (As adapted from FM White's Fluid Mechanics)

The pressure drop due to friction in a long circular pipe is function of average flow velocity, density, viscosity, and pipe length and diameter.

i.e. $\Delta p = f(V, \rho, \mu, L, D)$.

We wish to know how Δp varies with V. Use Pi-theorem to rewrite this function in dimensionless form.

D (cm)	L (m)	Q (m^3/h)	Δp (Pa)	ρ (kg/m^3)	μ (kg/ms)	V (m/s)
1.0	5.0	0.3	4680	680	2.92×10^{-4}	1.06
1.0	7.0	0.6	22300	680	2.92×10^{-4}	2.12
1.0	9.0	1.0	70,800	680	2.92×10^{-4}	3.54
2.0	4.0	1.0	2080	998	0.0010	0.88
2.0	6.0	2.0	10500	998	0.0010	1.77
2.0	8.0	3.1	30400	998	0.0010	2.74
3.0	3.0	0.5	540	13550	1.56×10^{-3}	0.20
3.0	4.0	1.0	2480	13550	1.56×10^{-3}	0.39
3.0	5.0	1.7	9600	13550	1.56×10^{-3}	0.67

Note here, $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2}$

$$\therefore \Delta p = f(V, \rho, \mu, L, D)$$

There are six variables $\Delta p, V, \rho, \mu, L, D$ ($n = 6$)

i.e. The basic dimensions are M, L, T ($k = 3$)

$\therefore$ We expect $j = n - k = 6 - 3 = 3$ Π groups.

In this case we want to plot Δp versus V.
Therefore both of them will be ~~repeating~~ not the repeating variables.

We select ρ, μ, D as the repeating variables.

$\therefore \Pi_1 = \rho^a \mu^b D^c \Delta p$ → should have $M^0T^0L^0$

$\Pi_2 = \rho^d \mu^e D^f V$

$\Pi_3 = \rho^g \mu^h D^i L$

We will get

$$\Pi_1 = \frac{\rho D^2 \Delta p}{\mu^2}, \quad \Pi_2 = \frac{\rho V D}{\mu}, \quad \Pi_3 = \frac{L}{D}$$

$$\therefore \frac{\rho D^2 \Delta p}{\mu^2} = f\left(\frac{\rho V D}{\mu}, \frac{L}{D}\right)$$

We can now plot Π_1 versus Π_2 with Π_3 as a parameter

→ If you work-out the given data, you can see that there will be nine data points.

→ We can see Δp increases linearly with L.

So $\Pi_1 = f(\Pi_2, \Pi_3)$

and $\Pi_3 = \frac{L}{D}$, $\therefore \Pi_1 = \frac{L}{D}\, g(\Pi_2)$

or $\Pi_1 = h(\Pi_2)$

i.e. $\frac{\rho D^3 \Delta p}{\mu^2} = \text{function}\left(\frac{\rho V D}{\mu}\right)$