LECTURE 36

12- APRIL -2016

DIMENSIONAL ANAYSIS (PART-II)

Yesterday we started discussing on dimensional analysis.

We said that dimensional analysis can:

* Reduce complexity of experimental data
* Help plan the experiment or theory
* Helps in providing scaling laws that can convert cheap data of model to design information on full-scale prototype.

For example, in the case of body subjected to stream velocity V,

$$F = f(\rho, V, L, \mu) \rightarrow (1)$$

you came up with the non-dimensional relation

$$\frac{F}{\rho V^2 L^2} = g(Re)$$

$\Rightarrow$ Now if we keep the Reynolds number same for model and prototype.

i.e. $Re_m = Re_p$

Then you have $C_{F_m} = C_{F_p}$

That means that:

$$\frac{F_p}{F_m} = \frac{\rho_p}{\rho_m}\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 \rightarrow (2)$$

This equation is scaling law.

Principle of Dimensional Homogeneity

If you have an equation that describes a physical process, then this equation will be dimensionally homogeneous if each of its additive terms have same dimensions.

e.g. From your physics class, you had studied the displacement of a falling body can be given as:

$$S = S_0 + V_0 t + \frac{1}{2} g t^2 \rightarrow (3)$$

Looking into this:

$$[L] \quad [L] \quad [L T^{-1} T] \quad [L T^{-2} T^2]$$

$$\downarrow$$

$$[L] \quad [L] \quad [L] \quad [L]$$

All the terms are having the dimension $[L]$

Again for inviscid, incompressible fluid flow, you have the Bernoulli's equation in steady state:

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant} \rightarrow (4)$$

$$\frac{M L^{-1} T^{-2}}{M L^{-3}} \quad L^2 T^{-2} \quad L T^{-2} L$$

$$[L^2 T^{-2}] \quad [L^2 T^{-2}] \quad [L^2 T^{-2}] \quad - \quad \underset{[L^2 T^{-2}]}{\overset{\downarrow}{\text{constant}}}$$

Following quantities need to be expressed:

(i) Dimensional Variables : That vary in the experiments
e.g. S and t ; p, V, z, etc.

(ii) Dimensional constants : Constant that are having dimensions, S_0, V_0, g, constant ρ, etc.

iii) Pure constants : The coefft $\frac{1}{2}$ in the equation (3) & (4).

(iv) Angles and revolutions are dimensionless

Choice of variables and Scaling parameters

The variables are things, we would like to plot and also get information. 'S' and 't'

Parameters are those that effect on the variables

S_0, V_0, g, etc.

While non-dimensionalising, we need to check basic dimensions.

$$S = S_0 + V_0 t + \frac{1}{2} g t^2$$

the basic dimensions are [L] and [T]

Non-dimensional equations:

e.g.: 1) If we take scaling parameters S_0, V_0; effect of gravity g.

i.e. Define $S^* = \frac{S}{S_0}$, $t^* = \frac{V_0 t}{S_0}$

$$\therefore \; S^* = 1 + t^* + \frac{1}{2}\left(\frac{g S_0}{V_0^2}\right) t^{*2}$$

2) If we take scaling parameter V_0, g

$$S^{**} = \frac{S g}{V_0^2}, \quad t^{**} = \frac{g}{V_0}$$

$$S^{**} = \frac{g S_0}{V_0^2} + t^{**} + \frac{1}{2} t^{**2}$$

3) If we take S_0 and g as parameters

$$S^{***} = \frac{S}{S_0}; \quad t^{***} = t\left(\frac{g}{S_0}\right)^{1/2}$$

$$S^{***} = 1 + \frac{V_0}{\sqrt{g S_0}} t^{***} + \frac{1}{2} t^{***2}.$$

Buckhinghom Pi Theorem

Several methods to reduce number of dimensional variables into a smaller number of dimensionless group.

$\Pi \rightarrow$ in mathematics used for products.

We can define dimensionless groups $\Pi_1, \Pi_2, \Pi_3, \ldots$

→ A physical process satisfying PDH having n-dimensional variables. Then it can be reduced to a relation between only k-dimensionless variables or Π's.

The reduction is $k = n - j$, where $j \rightarrow$ basic dimensions.

e.g. in $F = f(\rho, L, U, \mu)$

we have basic dimensions $[M\ L\ T] = 3$

Total ~~dimens~~ variables $F, \rho, L, U, \mu = 5$

$\therefore k = 5 - 3 = 2$

$\therefore$ There will be two Π's for this.

Example:

$$F = f(L, U, \rho, \mu) \quad , \quad \therefore n = 5$$

F	L	U	ρ	μ
MLT^{-2}	L	LT^{-1}	ML^{-3}	$ML^{-1}T^{-1}$

The Π theorem suggest two independent Π's.

~~Π_1~~ L, U, and ρ cannot form a non-dimensional Π group.