LECTURE 35

11 - APRIL - 2016

DIMENSIONAL ANALYSIS

Recall, how we stated that there are three broad ways of analysing fluid motion.

→ The large scale control volume approach

→ The infinitesimal differential approach

→ The experimental or dimensional analysis.

⇒ In the differential approach, you had already developed partial differential equations for fluid motion, irrespective of volume. That is, they were mathematically applicable ~~to~~ in any spatial and temporal point.

→ ~~The~~ Although, these partial differential equations are applicable for various types of fluid flows, to solve them analytically is not that easy. This is because, according to the geometric and physical conditions, to incorporate these conditions mathematically is quite difficult.

→ In addition, to solve these PDE's with appropriate boundary conditions is also quite difficult, except in simplified cases.

Therefore, one require experimental studies to validate the relations between various fluid and flowing properties.

→ ~~@~~ By experimentations, there will be large number of data and dimensional analysis helps in presenting these data appropriately.

* Dimensional analysis helps in reducing the complexity of experimental variables.

* In fluid mechanics, we use four basic dimensions — Mass [M], Length [L], Time [T], and temperature [θ]

i.e. MLTθ dimensional system.

Most of the fluid properties can be described through MLTθ system.

* If a phenomenon depends on n - dimensional variables, then the dimensional analysis will reduce this problem to $j = n - k$ where $k = 1, 2, 3$, or 4.

*. ~~@~~ Dimensional analysis reduces the variables and group them in dimensionless form.

This way, we can save time and money.

e.g. We want to analyse force F on a complex body immersed in a stream of fluid that depends on the body length L, stream velocity V, fluid density ρ, fluid viscosity μ

i.e. $$F = f(L, V, \rho, \mu)$$

The geometry and flow conditions, let us suggest now that they are highly complicated and we are unable to solve them using Navier-Stokes and momentum equations.

$\Rightarrow$ So we try to develop the relation of F w.r.t. the L, V, ρ and μ experimentally or numerically.

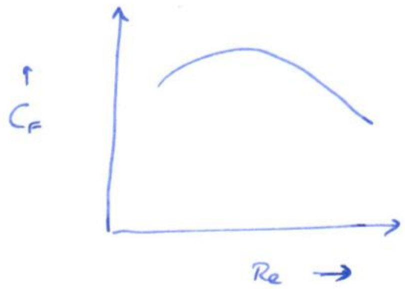
$\rightarrow$ Now to develop relations, we need to do several experiments. Each experiments may cost highly. If we do dimensional analysis, we can reduce the number of experiments and still get a relation of force with respect to length, velocity, density, and viscosity.

e.g. $$\frac{F}{\rho V^2 L^2} = g\left(\frac{\rho V L}{\mu}\right)$$

where $F/\rho V^2 L^2 = C_F$ is a dimensionless force coefficient.

$\frac{\rho V L}{\mu} \rightarrow$ is dimensionless Reynolds number.

i.e. You can now plot :



⇒ The dimensional analysis helps in thinking and planning the experiments, to validate theories, etc.

⇒ Dimensional analysis can provide scaling laws. It helps in converting data from <u>low cost models</u> to design information for <u>expensive prototypes</u>.

i.e. The dimensionless numbers for model and prototype should be same and from that the dimensions of prototype can be designed.

e.g.: If $Re_{model} = Re_{prototype}$

Then $C_{F_{model}} = C_{F_{prototype}}$

i.e. $$\frac{F_{model}}{\rho_{model} V_{model}^2 L_{model}^2} = \frac{F_{prototype}}{\rho_{proto} V_{proto}^2 L_{proto}^2}$$

where $$\frac{\rho_{proto} V_{proto} L_{proto}}{\mu_{proto}} = \frac{\rho_{model} V_{model} L_{model}}{\mu_{model}}$$

$\therefore$ We can get:

$$\frac{F_{proto}}{F_{model}} = \frac{\rho_{proto}}{\rho_{model}} \left(\frac{V_{proto}}{V_{model}}\right)^2 \left(\frac{L_{proto}}{L_{model}}\right)^2$$

Principle of dimensional homogeneity (PDH)

The principle as stated in FM White's Fluid Mechanics text book is

"If an equation truly expresses a proper relationship between variables in a physical process, it will be dimensionally homogeneous; that is, each of its additive terms will have the same dimensions."