LECTURE 23

09 - MARCH - 2016

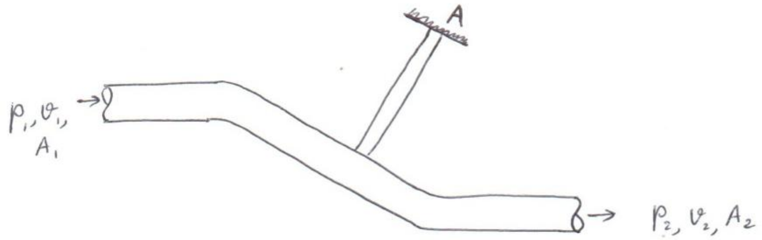
ANGULAR MOMENTUM, ENERGY CONSERVATION

Yesterday, we were discussing on applying RTT to the principle of angular momentum.

For a non-deformable, stationary control volume:

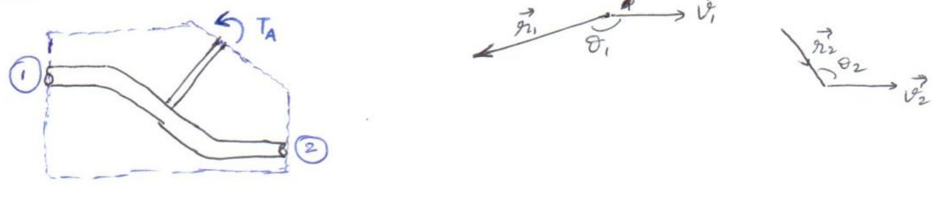
$$\left.\frac{dH_0}{dt}\right|_{system} = \Sigma M_0 = \Sigma(\vec{r} \times \vec{F})_0 = \frac{\partial}{\partial t}\left[\iint_{cv}(\vec{r}\times\vec{v})\rho\, dV\right] + \int_{cs}(\vec{r}\times\vec{v})\rho(\vec{v}\cdot\hat{n})\,dA$$

⇒ Subsequently we were working on an example problem: on pipe bend fixed to a pipe network using flexible couplings



→ ~~The control~~ We were asked to find the torque that has to be resisted by the fixed joint at A due to the fluid motion in the pipe bend.

→ The control volume was drawn as:



Considering counter clock-wise moment as positive, we have

$$\Sigma M_A = T_A + \vec{r}_1 \times (-p_1 A_1 \hat{n}_1) + \vec{r}_2 \times (-p_2 A_2 \hat{n}_2)$$

From RTT,

$$\Sigma M_A = \frac{\partial}{\partial t}\left[\int_{cv} (\vec{r} \times \vec{v})\, \rho \, dV\right] + \int_{cs} (\vec{r} \times \vec{v})\, \rho \,(\vec{v}\cdot\hat{n})\, dA$$

$\frac{\partial}{\partial t}[\dots] \to 0$ Steady state

i.e. $\Sigma M_A = (\vec{r}_2 \times \vec{v}_2)\, \dot{m}_{out} - (\vec{r}_1 \times \vec{v}_1)\, \dot{m}_{in}$

Again as flow is steady $\dot{m}_{out} = \dot{m}_{in}$

$\therefore$ ~~$\Sigma M_A =$~~ and $|\vec{r}_2 \times \vec{v}_2| = h_2 v_2$

$|\vec{r}_1 \times \vec{v}_1| = h_1 v_1$

$$\Sigma M_A = \dot{m}\,(h_2 v_2 - h_1 v_1)$$

Also $T_A + \vec{r}_1 \times (-p_1 A_1 \hat{n}_1) + \vec{r}_2 \times (-p_2 A_2 \hat{n}_2)$

$$= T_A + p_1 A_1 h_1 - p_2 A_2 h_2$$

$$\therefore \; T_A + p_1 A_1 h_1 - p_2 A_2 h_2 = \dot{m}\,[h_2 v_2 - h_1 v_1]$$

or $$T_A = h_2 (p_2 A_2 + \dot{m} v_2) - h_1 (p_1 A_1 + \dot{m} v_1)$$

In the given example: $D_1 = D_2 = 8$ cm $= 0.08$ m, $\therefore A_1 = A_2 = 5.0265 \times 10^{-3}\ m^2$

From continuity, $v_1 = v_2 = 15$ m/s, Mass flux, $\dot{m} = \rho A v$

$= 1000 \times 5.0265 \times 10^{-3} \times 15$

$= 75.3975$ kg/s

Torque, $T_A = h_2 (p_2 A_2 + \dot{m} v_2) - h_1 (p_1 A_1 + \dot{m} v_1)$

Given $h_1 = 5$ cm, $h_2 = 25$ cm

$=$

Energy Principle

Again revisiting the general RTT

$$\left.\frac{dB}{dt}\right|_{system} = \frac{d}{dt}\left[\int_{cv} \beta \rho \, dU\right] + \int_{cs} \beta \rho (\vec{V}_r \cdot \hat{n}) \, dA$$

Let E be the energy of a system

$$\therefore B = E$$

$$\text{and } \beta = \frac{dE}{dm} = e$$

$$\therefore \left.\frac{dE}{dt}\right|_{system} = \frac{d}{dt}\left(\int_{cv} e \rho \, dU\right) + \int_{cs} e \rho (\vec{V} \cdot \hat{n}) \, dA$$

(Ofcourse, it is assumed control volume is stationary)

Now what is $\left.\frac{dE}{dt}\right|_{system}$??

Let us suggest $Q \rightarrow$ heat added to the system

$W \rightarrow$ work done by the system

$$\therefore \left.\frac{dE}{dt}\right|_{system} = \frac{dQ}{dt} - \frac{dW}{dt}$$

$\Rightarrow$ The system energy per unit mass

$$e = e_{internal} + e_{kinetic} + e_{potential} + e_{other}$$

$\rightarrow$ In simple cases, we can neglect e_{other}

Then we have

$$e = \hat{u} + \frac{1}{2} v^2 + gz$$

where $\hat{u}$ is internal energy ~~stored per~~ in system per unit mass

$\Rightarrow$ The change in Q i.e. $\frac{dQ}{dt}$ can be associated with respect convection, conduction, radiation, etc.

$\Rightarrow$ The rate of change ~~by~~ of work $\frac{dW}{dt}$ can be due to shaft work change, pressure work change, viscous work, etc.

$$\therefore \frac{dW}{dt} = \mathring{W}_{shaft} + \mathring{W}_{press} + \mathring{W}_{visco}$$

$$= \mathring{W}_s + \mathring{W}_p + \mathring{W}_v$$

$\rightarrow$ In shaft work, you should not incorporate portion of work deliberately done by a machine.

$\rightarrow$ Rate of work $\mathring{W}_p$ done by pressure forces occur only on control surfaces.

$$\mathring{W}_p = \int_{cs} p\,(\vec{v}\cdot\hat{n})\,dA$$

$\therefore$ The control volume for non-deformable stationary equation becomes

$$\frac{dQ}{dt} - \mathring{W}_s - \int_{cs} p(\vec{v}\cdot\hat{n})\,dA - \mathring{W}_v$$

$$= \frac{\partial}{\partial t}\left[\int_{cv}\left(\hat{u} + \frac{v^2}{2} + gz\right)\rho\,dV + \int_{cs} e\,\rho(\vec{v}\cdot\hat{n})\,dA\right.$$

i.e. $$\frac{dQ}{dt} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\left[\int_{cv}\left(\hat{u} + \frac{v^2}{2} + gz\right)\rho \, dV\right] + \int_{cs}\left(\hat{h} + \frac{v^2}{2} + gz\right)\rho\,(\vec{v}\cdot\hat{n})\, dA$$

where $\hat{h} = \hat{u} + \frac{p}{\rho}$ and is called <u>enthalpy</u>.

~~[illegible]~~: <u>Steady flow Energy Equation</u>

For one inlet and one outlet, for steady flow:

$$\frac{dQ}{dt} - \dot{W}_s - \dot{W}_v = -\dot{m}_1\left[\hat{h}_1 + \frac{v_1^2}{2} + gz_1\right] + \dot{m}_2\left[\hat{h}_2 + \frac{v_2^2}{2} + gz_2\right]$$

As $\dot{m}_1 = \dot{m}_2 = \dot{m}$, we have

$$\hat{h}_1 + \frac{v_1^2}{2} + gz_1 = \left(\hat{h}_2 + \frac{v_2^2}{2} + gz_2\right) - q + w_s + w_v$$

where $q = \dfrac{dQ/dt}{\dot{m}\ \cancel{dm/dt}} = \dfrac{\dot{Q}}{\dot{m}} = dQ/dm$

$w_s = \dfrac{\dot{W}_s}{\dot{m}} = dW_s/dm$

$w_v = \dfrac{\dot{W}_v}{\dot{m}} = dW_v/dm$