Angular Momentum

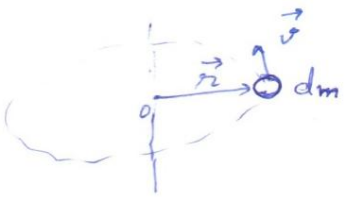
Control volume analysis can be done for angular momentum principles.

If $\vec{H}$ is angular-momentum vector

$\vec{r}$ is position vector from axis to point of elemental mass dm.

$\vec{v}$ is velocity of elemental mass dm

Then in RTT



Adopt $B = \vec{H}_o = \int_{syst} (\vec{r} \times \vec{v}) dm$

$$\therefore \beta = \frac{d\vec{H}_o}{dm} = \vec{r} \times \vec{v}$$

RTT becomes:

$$\left.\frac{dH_o}{dt}\right|_{system} = \frac{d}{dt}\left[\int_{cv} (\vec{r} \times \vec{v}) \rho \, dV\right] + \int_{cs} (\vec{r} \times \vec{v}) \rho (\vec{V}_r \cdot \hat{n}) dA$$

The rate of change of angular-momentum of a system should be equal to the sum of all moments about the point O

$$\therefore \quad \frac{dH_o}{dt} = \Sigma M_o = \Sigma (\vec{r} \times \vec{F})_o$$

i.e. If the control volume is non-deformable and there is no motion of control volume, then

$$\left.\frac{dH_o}{dt}\right|_{system} = \cancel{\frac{d}{dt}} \frac{\partial}{\partial t}\left[\int_{cv} (\vec{r} \times \vec{v})\, \rho \, dU\right] + \int_{cs} (\vec{r} \times \vec{v})\, \rho (\vec{v} \cdot \hat{n})\, dA$$

⇒ If there are one-dimensional inlets and outlets on the control surfaces of the volume.

$$\left.\frac{dH_o}{dt}\right|_{system} = \Sigma M_o = \Sigma (\vec{r} \times \vec{F})_o = \frac{\partial}{\partial t}\left[\int_{cv} (\vec{r} \times \vec{v})\, \rho \, dU\right] + \Sigma \left[(\vec{r} \times \vec{v})_{out} \, \dot{m}_{out}\right] - \Sigma \left[(\vec{r} \times \vec{v})_{in} \, \dot{m}_{in}\right]$$

Energy Equation

First law of thermodynamics, $\mathcal{B} \to$ energy E

$\beta \to$ energy per unit mass $= \frac{dE}{dm} = e$

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\left(\int_{cv} e \rho \, dv\right) + \int_{cs} e \rho (\vec{V}\cdot\hat{n}) \, dA$$

$Q \to$ denotes heat added to the system

$W \to$ denotes work done by the system

$$e_{\text{internal}} + e_{\text{kinetic}} + e_{\text{potential}} + e_{\text{other}} \to e$$

$$e = \hat{u} + \frac{1}{2}V^2 + gz$$

$\frac{dQ}{dt} \to$ conduction, convection, radiation

$$\dot{W} = \dot{W}_{\text{shaft}} + \dot{W}_{\text{press}} + \dot{W}_{\text{visc stress}} = \dot{W}_s + \dot{W}_p + \dot{W}_v$$

$\to$ The work of gravitational

Shaft work isolates portion of work that is deliberately done by a machine protruding through control surfaces.

$$d\dot{W}_p = -(p\, dA) V_{n\,in} = -p(-\vec{V}\cdot\hat{n})\, dA \; ; \quad \dot{W}_p = \int_{cs} p(\vec{V}\cdot\hat{n})\, dA$$

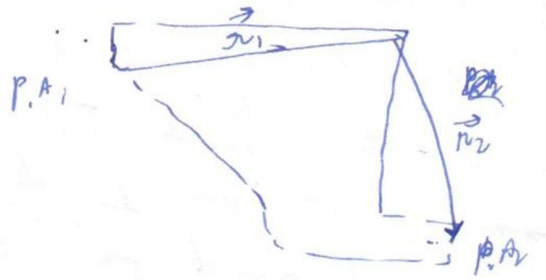
$\dot{W}_s =$

Pipe bend supported at point A. and connected to a flow system by flexible coupling at sections ① and ②. Fluid is incompressible and ambient pressure P_a is zero. a) Find an expression for the torque T that must be ~~supported~~ resisted by the support at A.

b) Compute T if $D_1 = D_2 = 8$ cm, $p_1 = 0.689476 \times 10^6$ Pa

$p_2 = 0.551581 \times 10^6$ Pa, $V_1 = 15$ m/s

$h_1 = 5$ cm, $h_2 = 25$ cm, $\rho = 1000$ kg/m³ (water).



T_A is the desired.

Flexible coupling at (1) and (2) suggest that there is no torque at section (1) and (2)

If we cut sections though them, there won't be any moment there.

Angular Momentum, $\vec{\omega} = \vec{r} \times \vec{v}$

$\vec{r}$ should be taken from A to section (1) and (2).

gage pressure forces. $p_1 A_1$ & $p_2 A_2$ also have moments about A.

$$\Sigma M_A = T_A + \vec{r}_1 \times (-p_1 A_1 \hat{n}_1) + \vec{r}_2 \times (-p_2 A_2 \hat{n}_2)$$

$=$

Note. $$\Sigma M_o = \frac{\partial}{\partial t}\left[\int_{cv} (\vec{r} \times \vec{v}) \rho \, dV\right] + \int_{cs} (\vec{r} \times \vec{v}) \rho (\vec{v} \cdot \hat{n}) \, dA$$

$$\int_{cs} (\vec{r} \times \vec{v}) \rho (\vec{v} \cdot \hat{n}) \, dA = \Sigma (\vec{r} \times \vec{v})_{out} \dot{m}_{out} - \Sigma (\vec{r} \times \vec{v})_{in} \dot{m}_{in}$$

$$\Rightarrow T_A + \vec{r}_1 \times (-p_1 A_1 \hat{n}_1) + \vec{r}_2 \times (-p_2 A_2 \hat{n}_2)$$
$$= (\vec{r}_2 \times \vec{v}_2) \dot{m}_{out} - (\vec{r}_1 \times \vec{v}_1) \dot{m}_{in}$$

$r_1 \sin\theta_1 = h_1$, $r_2 \sin\theta_2 = h_2$

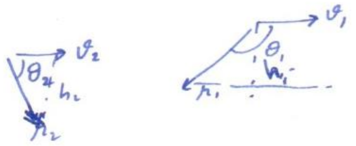
$\dot{m}_{in} = \dot{m}_{out}$ for steady flow condition.

$$\therefore \quad T_A + p_1 A_1 h_1 - p_2 A_2 h_2 = \dot{m}\,(h_2 v_2 - h_1 v_1)$$

$$\text{or} \quad T_A = h_2\,(p_2 A_2 + \dot{m}\, v_2) - h_1\,(p_1 A_1 + \dot{m}\, v_1)$$

where p_1 and p_2 are gage pressures.

⇒ This result is independent of shape of pipe bend.



⇒ For the case here, given $D_1 = D_2 = 8\text{ cm} = 0.08\text{ m}$

$$A_1 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.08^2 = 5.0265 \times 10^{-3}\text{ m}^2$$

$$v_1 = 15\text{ m/s}$$

$$\therefore \quad Q = A_1 v_1 = 5.0265 \times 10^{-3} \times 15 = 0.075398\text{ m}^3$$

$$\therefore v_2 = \cancel{5.0265 \times 10^{-3}}\ 15\text{ m/s.}$$

$$\dot{m} = \rho A v = 1000 \times 5.0265 \times 10^{-3} \times 15 = 75.3975\text{ kg/s}$$

$$\Rightarrow \therefore \text{Torque} \quad T_A = h_2\,(p_2 A_2 + \dot{m}\, v_2) - h_1\,(p_1 A_1 + \dot{m}\, v_1)$$

$$= 0.25\,(0.551581 \times 10^6 \times 5.0265 \times 10^{-3} + 75.3975 \times 15)$$

$$- 0.05\,(0.689476 \times 10^6 \times 5.0265 \times 10^3 + 75.3975 \times 15)$$

$$= \qquad \text{Nm}$$

Clockwise or counter-clockwise ??