LECTURE 20

24-FEB-2016

RTT FOR LINEAR MOMENTUM (CONTD...)

Yesterday, in the particle approach we saw how there will be local and convective rates for time derivative of a property defined with respect to an accelerating co-ordinate system.

⇒ Today, let us come back to the control volume approach.

Continuing the linear momentum principles,

Let us talk about acceleration defined in moving $o\,x,y,z$ co-ordinate. $OXYZ$ is fixed inertial co-ordinate

The absolute acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} + \vec{a}_{rel}$$

where $\vec{a}_{rel}$ is the relative acceleration due to accelerating co-ordinate system.

Newton's second law is applied to actual or absolute acceleration.

i.e. $$\Sigma\vec{F} = m\vec{a} = m\left[\frac{d\vec{v}}{dt} + \vec{a}_{rel}\right]$$

or $$\Sigma\vec{F} - m\,\vec{a}_{rel} = m\frac{d\vec{v}}{dt}$$

(Note that yesterday $\frac{d\vec{v}}{dt}(x,y,z,t) = \vec{a}(x,y,z,t)$.

The control volume formulation using RTT for non-inertial coordinates will be:

$$\frac{d}{dt}\left[(m\vec{V})_{system}\right] = \Sigma\vec{F} - \int_{CV} \vec{a}_{rel}\, dm$$

$$= \frac{d}{dt}\left[\int_{CV} \vec{V}\rho\, d\upsilon\right] + \int_{CS} \vec{V}\rho(\vec{V}_r\cdot\hat{n})\, dA$$

Frictionless Flow and the Bernoulli Equation

→ We know all fluids are viscous

→ Therefore, there will be some sort of friction.

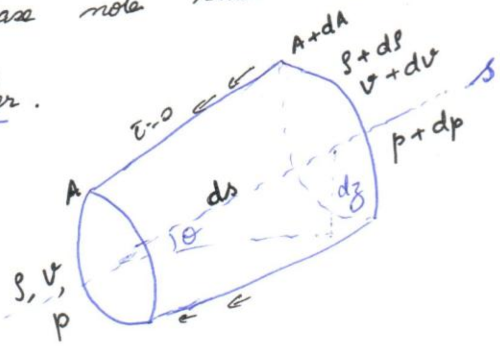
→ However, in certain cases, some flows can be classified as frictionless.

Bernoulli's equation relate pressure, velocity and elevation in a frictionless flow.

⇒ Consider a small elemental streamtube.

A streamtube is a tube of collection of streamlines.

Please note that no two streamlines intersect each other.



→ The streamtube is the control volume of interest

→ Frictionless mean, assuming $\tau = 0$ (shear stress).

The streamtube has one inlet and ~~two~~ one outlet.
The streamtube orientation is arbitrary. Here it is at an angle θ with the horizontal.

From figure, $dz = ds \sin\theta$

The weight of fluid, $dW = \rho g \, d\upsilon$

If we limit the cross sectional area, the streamtube becomes a streamline.

Conservation of mass for elemental control volume:
in such limiting situation, when $ds \to 0$.

$$\frac{d}{dt}\left[m_{system}\right] = 0 = \frac{d}{dt}\left[\int_{cv} \rho \, d\upsilon\right] + \dot{m}_{out} - \dot{m}_{in}$$

$$\text{i.e. } 0 \approx \frac{\partial \rho}{\partial t} d\upsilon + d\dot{m}$$

where $\dot{m} = \rho A V$ and $d\upsilon \approx A\,ds$

$$d\dot{m} = d(\rho A V) = -\frac{\partial \rho}{\partial t} A\,ds$$

Linear momentum equation will be:

$$\sum dF_s = \frac{d}{dt}\left[\int_{cv} \vec{V} \rho \, d\upsilon\right] + \left[\dot{m} V\right]_{out} - \left[\dot{m} V\right]_{in}$$

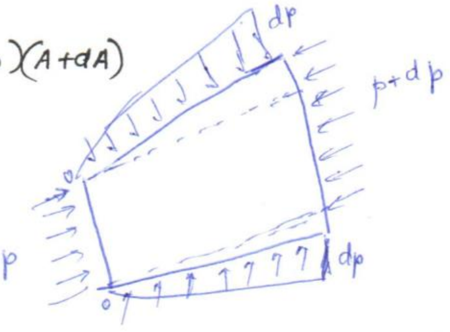
$$\approx \frac{\partial}{\partial t}(\rho V) A\,ds + d(\dot{m} V)$$

Note here V_s = velocity in streamline direction
$= V$.

If we neglect shear force along the walls, Then the force components

$$dF_{s\_grav} = -dW \sin\theta = -\rho g A \, ds \sin\theta$$

$$dF_{s\_pressure} = pA - (p+dp)(A+dA) + \frac{1}{2} dp \, dA$$



$$\cong -A \, dp$$

In linear momentum,

$$\sum dF_s = -\rho g A \, ds \sin\theta - A \, dp$$

$$= \frac{\partial}{\partial t}(\rho v) A \, ds + d(\dot{m} v)$$

$$= \frac{\partial \rho}{\partial t} v A \, ds + \frac{\partial v}{\partial t} \rho A \, ds + \dot{m} \, dv + v \, d\dot{m}$$

From continuity equation $\quad d\dot{m} = -\frac{\partial \rho}{\partial t} A \, ds$

$$\dot{m} = \rho A v$$

$\therefore$ ~~$\sum dF_s =$~~

$$-\rho g A \, dz - A \, dp = \frac{\partial v}{\partial t} \rho A \, ds + \dot{m} \, dv$$

$$\text{or} \quad \frac{\partial v}{\partial t} ds + g \, dz + \frac{dp}{\rho} + v \, dv = 0$$

This is Bernoulli's equation for unsteady frictionless flow along a streamline.

If we integrate this differential equation between any two points 1 and 2 on a streamline, we get

$$\int_{1}^{2} \frac{\partial v}{\partial t}\, ds + \int_{1}^{2} \frac{dp}{\rho} + \frac{1}{2}\left(v_2^2 - v_1^2\right) + g(z_2 - z_1) = 0$$

For steady, incompressible, flow,

$\frac{\partial v}{\partial t} = 0$, ρ is constant

$$\therefore \quad \frac{(p_2 - p_1)}{\rho} + \frac{1}{2}\left(v_2^2 - v_1^2\right) + g(z_2 - z_1) = 0$$

or $$\frac{p_2}{\rho} + \frac{1}{2} v_2^2 + g z_2 = \frac{p_1}{\rho} + \frac{1}{2} v_1^2 + g z_1 = \text{constant}$$