LECTURE 18

22-FEB-2016

Conservation of Linear Momentum

Recall in last class, we discussed on RTT for conservation of linear momentum

That is $\quad \mathcal{B} = m\vec{v}, \quad \therefore \beta = \vec{v}$

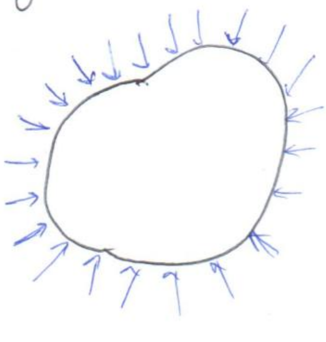
$$\frac{d}{dt}\left[(m\vec{v})_{system}\right] = \Sigma\vec{F} = \frac{d}{dt}\left[\int_{CV} \vec{v}\,\rho\, dU\right] + \int_{CS} \vec{v}\,\rho\,(\vec{V}_r \cdot \hat{n})\, dA$$

The net force $\Sigma\vec{F}$ is arrived at by balancing all the

→ surface forces and body forces

Again, one particular type of surface force is pressure force

$$\vec{F}_{pressure} = \int_{CS} p(-\hat{n})\, dA$$



If p is a constant, then if the ~~closed~~ control surface is closed, we get

$$\vec{F}_{pres} = -p\int_{CS} \hat{n}\, dA = 0$$

Also recall, for one-dimensional openings in the control volume say opening ① as inlet and opening ② as outlet,

Then in steady state conditions.

$$\Sigma\vec{F} = \dot{m}_2\vec{v}_2 - \dot{m}_1\vec{v}_1$$

i.e. $\Sigma\vec{F} = \left(\rho_2 A_2 v_2\right)\vec{v_2} - \left(\rho_1 A_1 v_1\right)\vec{v_1}$

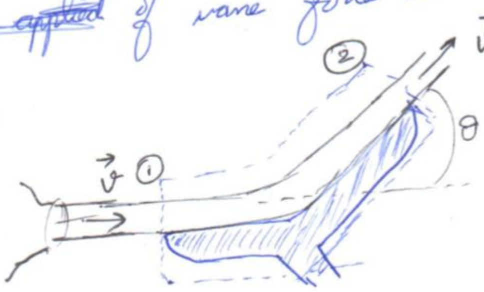
Also recall from conservation of mass in steady state

$$\rho_2 A_2 v_2 - \rho_1 A_1 v_1 = 0$$

$$\dot{m}_2 - \dot{m}_1 = 0 \quad \text{or} \quad \dot{m}_2 = \dot{m}_1 = \dot{m}$$

Example (As adopted from FM White's ... Fluid Mechanics)

A fixed vane turns a water jet of cross sectional area A through an angle θ without changing its velocity magnitude. The flow is steady, pressure is p_a everywhere and friction on the vane is negligible. a) Find the components ~~F_x and F_y of the applied~~ of vane force in x and y-direction.



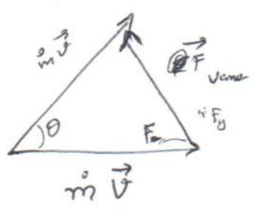
Solution

→ The dotted lines demark the control volume of interest.

→ It is clear from the control volume that the outside of control volume have the atmospheric pressure p_a. Pressure force is neglected.

→ The inlet velocity is $|\vec{v}|$ and outlet velocity is $|\vec{v}|$ (It has been told the magnitude is same).

→ Since the flow is steady, $\Sigma\vec{F} = \dot{m}\left[\vec{v_2} - \vec{v_1}\right]$



$$\vec{F}_{vane} = \dot{m}\left(\vec{V}_2 - \vec{V}_1\right)$$

$$\dot{m} = \rho A V$$

$$F_x = \dot{m}\left(V_{2x} - V_{1x}\right)$$
$$= \dot{m} V\left(\cos\theta - 1\right)$$

$$F_y = \dot{m}\left(V_{2y} - V_{1y}\right)$$
$$= \dot{m} V \sin\theta$$

The vane force $\vec{F} = F_x \hat{i} + F_y \hat{j}$

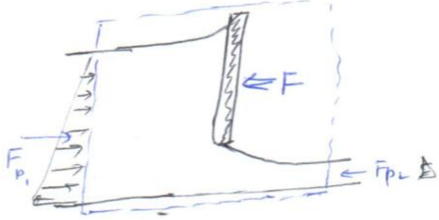
Example

A sluice gate controls flow in open channels. The flow is uniform at sections ① and ②. Pressure of liquid is assumed as hydrostatic. Neglect bottom friction and atmospheric pressure and derive a formula for the horizontal force F required to hold the gate.

Solution.

We can frame a control volume for this problem. Since sections ① and ② have uniform flow, it will be easy for us in calculations if we incorporate CV through those sections.

→ As the flow is uniform at ① and ②, it automatically



implies steady state conditions as 1 and 2 are part of the control surfaces.

Let 'b' the width of section into the paper.

Using mass conservation, $\dot{m} = \rho v_1 h_1 b = \rho v_2 h_2 b$

$$v_2 = v_1 \frac{h_1}{h_2}$$

→ As the pressure is hydrostatic, a section 1, bottom

$$p_{1_b} = p_a + \rho g h_1$$

Similarly, at section 2 bottom, $p_{2_b} = p_a + \rho g h_2$

As atmospheric pressure p_a is assumed same, we can compute net pressure force through gage pressure.

→ As we want to find horizontal force F on the gate, we will use RTT for force component in x-direction.

$$\text{i.e.} \quad \Sigma F_x = \cancel{\frac{d}{dt}\left[\int_{cv} v_x \rho \, dU\right]} + \int_{cs} v_x \rho (\vec{v}_r \cdot \hat{n}) dA$$

$$= v_{2_x} \rho v_2 A_2 - v_{1_x} \rho v_1 A_1$$

As $\vec{v}_1 = v_{1_x} \hat{i} + 0 \hat{j}$ and $\vec{v}_2 = v_{2_x} \hat{i} + 0 \hat{j}$

$$\therefore \quad \Sigma F_x = \rho v_2^2 A_2 - \rho v_1^2 A_1$$

$$= \rho \left(A_2 v_2^2 - A_1 v_1^2\right)$$

Now $\Sigma F_x = \rho g \frac{h_1}{2} * h_1 b - \rho g \frac{h_2}{2} * h_2 b - F_{gate}$

$$\therefore \quad \Sigma F_x = \frac{\rho g b}{2}\left[h_1^2 - h_2^2\right] - F_{gate} = \rho\left[A_2 v_2^2 - A_1 v_1^2\right]$$

$$\therefore \quad F_{gate} = \frac{\rho g b}{2}\left(h_1^2 - h_2^2\right) - \rho\left(A_2 v_2^2 - A_1 v_1^2\right)$$

As $\quad v_2 = v_1 \dfrac{h_1}{h_2}$

$$F_{gate} = \frac{\rho g b}{2}\left(h_1^2 - h_2^2\right) - \rho\left(A_2 \, v_1^2 \frac{h_1^2}{h_2^2} - A_1 v_1^2\right)$$

$$A_2 = h_2 b \quad , \quad A_1 = h_1 b$$

i.e. $$F_{gate} = \frac{\rho g b}{2}\left(h_1^2 - h_2^2\right) - \frac{\rho}{h_2^2}\left(h_2 b \, v_1^2 \, h_1^2 - h_2^2 b \, h_1 v_1^2\right)$$

$$= \rho b\left[\frac{g}{2}\left(h_1^2 - h_2^2\right) - \frac{v_1^2}{h_2^2} h_1 h_2 \left(h_1 - h_2\right)\right]$$

$$F_{gate} = \rho b\left(h_1 - h_2\right)\left[\frac{g}{2}\left(h_1 + h_2\right) - v_1^2 \frac{h_1}{h_2}\right]$$