LECTURE 16

17-02-2016

CONSERVATION OF MASS, LINEAR MOMENTUM

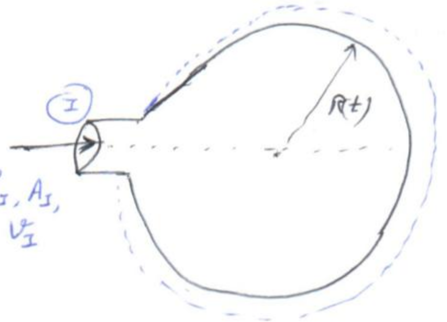
As stated earlier, Reynolds Transport Theorem helps to convert system analysis to control volume approach. The general form of RTT, i.e. for any <u>deformable moving control volume</u>.

$$\frac{d}{dt}\left(B_{system}\right) = \frac{d}{dt}\left[\int_{cv} \beta\rho \, dU\right] + \int_{cs} \beta\rho\left(\vec{V}_r \cdot \hat{n}\right) dA$$

If the control volume is non-deformable and fixed in space, then you can write RTT as:

$$\frac{d}{dt}\left[B_{system}\right] = \int_{cv} \frac{\partial}{\partial t}(\beta\rho)\, dU + \int_{cs} \beta\rho(\vec{V}\cdot\hat{n})\, dA$$

Subsequently, we worked on an example problem on baloon.



$\Rightarrow$ We have suggested baloon as deformable control volume

B = mass of air in control volume

Since mass conservation principle suggests $\frac{d}{dt} m_{system} = 0$

We worked out, At inlet $|\vec{V}_r| = V_I$

At outlet (i.e. other surface) $|\vec{V}_r| = 0$.

$$\therefore \quad 0 = \frac{d}{dt}\left[\int_{cv} \rho_b(t)\, dU\right] + \int_{out} \rho * 0 - \int_{inlet} \rho\,(\vec{V}_r \cdot \hat{n})\, dA$$

$$\text{i.e.} \quad 0 = \frac{d}{dt}\left[\rho_b \frac{4\pi}{3} R^3\right] - \rho_I A_I V_I$$

$$\text{or} \quad \frac{d}{dt}\left[\rho_b R^3\right] = \frac{3}{4\pi} \rho_I A_I V_I$$

Application of RTT to conservation of mass principle

For conservation of mass principle

your B = extensive property = mass of fluid in control volume

$\therefore B_{system}$ = mass of fluid at instant 't' = m_{system}

$\beta = 1$, $\quad B_{cv} = \int_{cv} \rho \, dV$

$$\therefore \frac{dm_{system}}{dt} = 0 = \frac{d}{dt}\left[\int_{cv} \rho \, dV\right] + \int_{cs} \rho(\vec{V}_r \cdot \hat{n}) \, dA$$

$\Rightarrow$ For a fixed, non-deformable CV,

$$0 = \int_{cv} \frac{\partial \rho}{\partial t} dV + \int_{cs} \rho(\vec{V} \cdot \hat{n}) \, dA$$

$\Rightarrow$ For steady flow, $\frac{\partial}{\partial t}(\,) = 0$

$$\therefore \int_{cs} \rho(\vec{V} \cdot \hat{n}) \, dA = 0$$

This means that the Net Outflow is zero. (Or Outflow = Inflow)

If your control surfaces have one dimensional inlets and outlets, then

$$\sum_{Inlets} A_i V_i = \sum_{outlets} A_j V_j$$

$\Rightarrow$ For incompressible liquids or fluids, the density remains constant. In that case, in a non-deformable fixed control volume

$$0 = \int_{cv} \frac{\partial \rho}{\partial t}^{\,0} dV + \int_{cs} \rho(\vec{V} \cdot \hat{n}) \, dA$$

$$\therefore \int_{cs} \rho[\vec{V} \cdot \hat{n}] \, dA = 0$$

$$\text{"} \quad \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$ (If ① is inlet and ② is outlet).

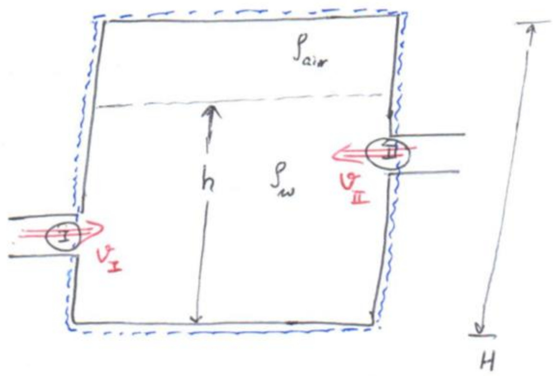
Example (From FM White's "Fluid Mechanics")

A tank closed at the top is filled with water by two one-dimensional inlets. The water height is 'h' and the air is trapped at top. a) Find the expression for $\frac{dh}{dt}$, b) Compute $\frac{dh}{dt}$ if diameter at section (I) is $D_I = 3$ cm., $V_I = 1.5$ m/s, and at section (II), $D_{II} = 8$ cm, $V_{II} = 1.0$ m/s. The tank top area is $A_t = 0.25\ m^2$. The water is at $20°c$.

Solution

The blue dotted lines demark the control volume of interest.

There are two inlet sections (I) and (II).



$$\frac{d}{dt}\left[\int_{cv} \rho\, dU\right] - \rho_I A_I V_I - \rho_{II} A_{II} V_{II} = 0$$

(Conservation of Mass)

Here $\rho_I = \rho_{II} = \rho_w$, $\frac{d}{dt}\left[\int_{cv} \rho\, dU\right] = \rho_w\left(A_I V_I + A_{II} V_{II}\right)$

Now $\frac{d}{dt}\left[\int_{cv} \rho\, dU\right] = \frac{d}{dt}\left(\rho_w A_t h\right) + \frac{d}{dt}\left[\rho_a A_t (H-h)\right]$

As the air is trapped, the time rate of change of mass of air will be zero.

$$\therefore \frac{d}{dt}\left[\int_{cv} \rho\, dU\right] = \frac{d}{dt}\left[\rho_w A_t h\right] = \rho_w A_t \frac{dh}{dt}$$

($\because$ Water is incompressible).

Hence $\rho_w A_t \frac{dh}{dt} = \rho_w \left(A_I v_I + A_{II} v_{II} \right)$

or $\frac{dh}{dt} = \frac{1}{A_t} \left[A_I v_I + A_{II} v_{II} \right]$

As $A_{II} = \frac{\pi}{4} * (0.08)^2 \cancel{\approx 0.063 \; m^2}$, $V_I = 150$ cm/s

or $\frac{\pi}{4} \times 8^2 \approx \underline{50.3} \; cm^2$ $\quad V_{II} = 100$ cm/s

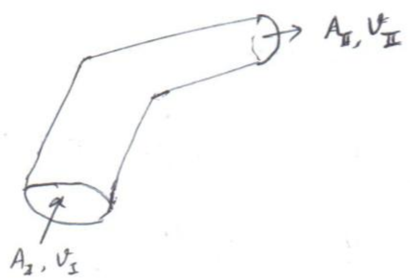
$A_I = \frac{\pi}{4} \times 3^2 \approx \underline{7 \; cm^2}$ $\quad A_t = 2500 \; cm^2$

$\therefore \frac{dh}{dt} = \frac{1}{2500} \left[7 * 150 + 50.3 * 100 \right]$

$= \underline{2.432}$ cm/s

Quiz

A pipe bend is as shown below. Water flows through the pipe. There is one inlet and one outlet. The ~~discharge in~~ flow is steady in pipe. The discharge



$Q = 100 \; cm^3/s$

$D_I = 5$ cm, $D_{II} = 3$ cm

Find v_I and v_{II} in cm/s.