LECTURE 12

10-FEB-2016

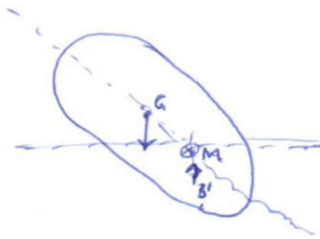
METACENTER & STABILITY

In the last lecture, we discussed about buoyancy (or buoyant force) and also introduced the concept of <u>metacenter</u>.

→ Metacentric height is the distance between metacenter and centre of gravity of the body.

→ Also we have seen that if metacenter M is above centre of gravity G, for a body then there is restoring moment present, that will allow the body to return to earlier stable position.

→ Also, if M is below center of gravity G, then there is a rotating moment present, which ~~that~~ will overturn the body



→ We will describe about it in more detail today.

→ For that, we need to relate stability with respect to <u>waterline area</u>.

Let us take a cross section of a vessel.

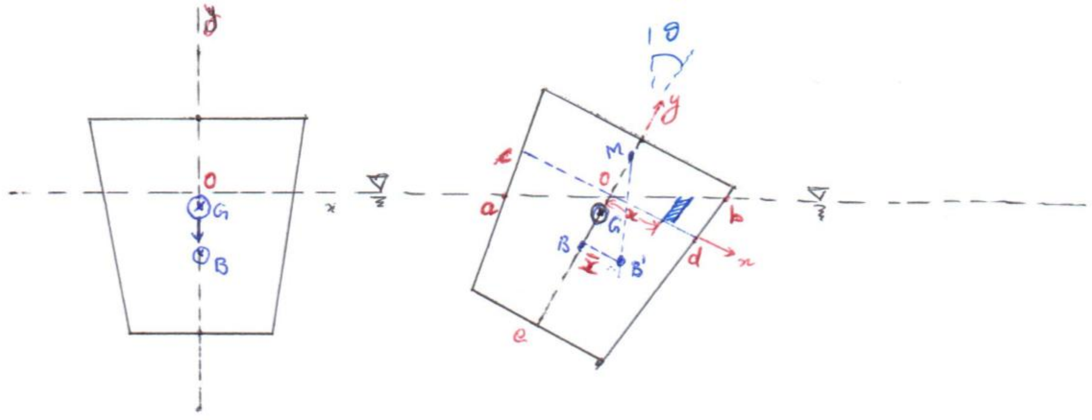


Fig 1A Fig 1B

→ The original position of the vessel is shown in Fig 1A. It is symmetric over y- axis. Let O be the origin of ~~co-ordinate~~ xy co-ordinate system.

→ Let the vessel have a length L into the plane of this paper.

→ When we tilt this vessel by an angle θ, the submerged ~~volume~~ shape of the volume changes. In that case, the center of buoyancy shifts to the new point B'. It is the centroid of the submerged volume aObde (Refer FM WHITE's Fluid Mech..)

→ To calculate this centroid, let it be at a distance $\bar{x}$ from y-axis.

As this is a body, let the volume of the submerged portion be denoted as V_{aObde}

$$\bar{x}\, V_{aObde} = \int_{cOdea} x\, dV + \int_{Obd} x\, dV - \int_{cOa} x\, dV$$

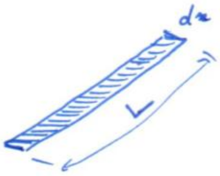
$$= 0 + \int_{Obd} x\, L\, dA - \int_{cOa} x\, L\, dA$$

where $dV = L\, dA$

Now $dA = x \tan\theta\, dx$

$$\therefore \bar{x}\, V_{aObde} = 0 + \int_{Obd} x\, L(x \tan\theta\, dx) - \int_{cOa} x\, L(-x\tan\theta)\, dx$$

Now note that $L\, dx$ = area of elemental waterline area or the strip $= dA_{waterline}$



$$\therefore \bar{x}\, V_{aObde} = \cancel{I_0 \tan\theta}\ \tan\theta \int_{waterline} x^2\, dA_{waterline}$$

$$= I_0 \tan\theta$$

where $I_0 \rightarrow$ area moment of inertia of the waterline footprint of the body. This area moment of inertia is about the tilt axis passing through O.

$\therefore$ The distance between M and B can be determined as such.

$$\bar{x}\, V_{aObde} = \bar{x}\, V_{submerged} = I_0 \tan\theta$$

$\overline{MB}$ → represents distance between M and B

$\overline{MG}$ → represents distance between M and G (metacentric height).

$$\therefore \quad \overline{MB} = \frac{\bar{x}}{\tan\theta} = \frac{I_0}{V_{submerged}}$$

Note $\overline{MB} = \overline{MG} + \overline{GB}$

$$\therefore \text{ Metacentric height } \quad \overline{MG} = \frac{I_0}{V_{submerged}} - \overline{GB}$$

→ An engineer has to design the body (vessel here) in such a way that $\overline{MG}$ is positive

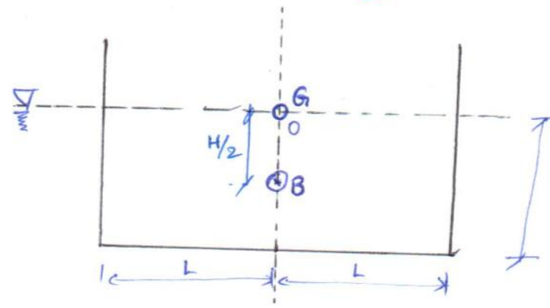
That is, if $\overline{MG}$ is positive, body is stable for small disturbances.

→ If $\overline{GB}$ is negative, then your $\overline{MG}$ is always positive

Example (As adapted from FM WHITE's Fluid Mechanics)

A barge has a uniform rectangular cross section of width 2L and vertical draft height H. a) Determine the metacentric height for a small tilt angle,

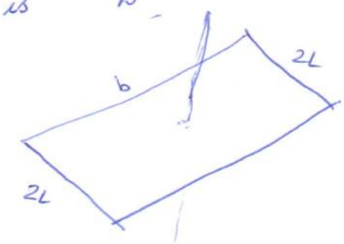
b) The range of ratio L/H for which the barge is statically stable, if G is exactly at the waterline.



→ Let us assume the barge has length 'b' into the paper

→ We need to suggest suitable dimension for L and H, such that barge is stable.

→ As the width into the paper is 'b'.



$$I_O = \frac{b(2L)^3}{12}$$

→ $$V_{submerged} = 2bLH$$

$$\therefore \quad \overline{MG} = \frac{I_O}{V_{submerged}} - \overline{GB}$$

$$= \frac{8bL^3/12}{2bLH} - \frac{H}{2}$$

$$= \frac{L^2}{3H} - \frac{H}{2}$$

The barge will be stable if

$$L^2 > \frac{3H^2}{2}$$

i.e. If the barge is wider relative to its draft height, then it is more stable.

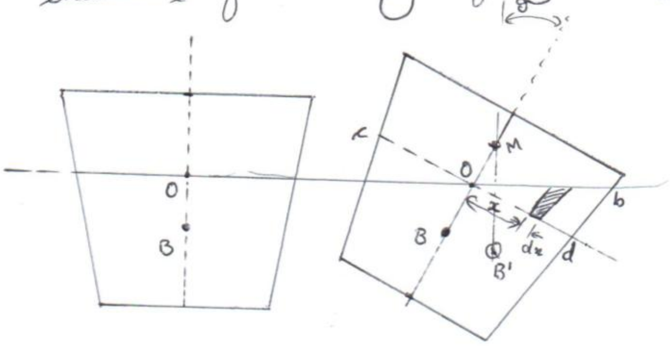
Please note that in the above example problem, the barge was of regular shape. That is cross-sectional area is retained throughout.

For irregular shape bodies, it is very difficult to determine metacentric height.

Lecture 12

Clarification

To resolve the ambiguity regarding moment of inertia evaluation of a tilting floating body.



Assume the center of gravity to be on waterline initially and it coincides with O.

For the integral

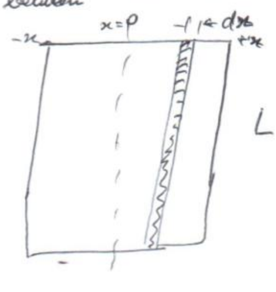
$$\bar{x}\, V_{aObde} = \int_{cOdea} x\,dV + \int_{Obd} x\,dV - \int_{cOa} x\,dV$$

$$= 0 + \int_{Obd} x\, L\,dA - \int_{cOa} x\, L\,dA$$

where $dV = L\,dA$ and $dA = x \tan\theta\, dx$

$$\therefore\ \bar{x}\, V_{aObde} = 0 + \int_{Obd} x\, L\,(x \tan\theta\, dx) - \int_{cOa} x\, L(-x \tan\theta)\,dx$$

Now note that if we integrate $L\,dx$ between $-x$ to $+x$ we get the initial waterline area.



$$\therefore \int x^2 \tan\theta\, L\,dx = \tan\theta \int x^2\, dA_{waterline}$$

$$= \tan\theta \cdot I_O$$