LECTURE - 11

03-FEB-2016

# BUOYANCY & STABILITY

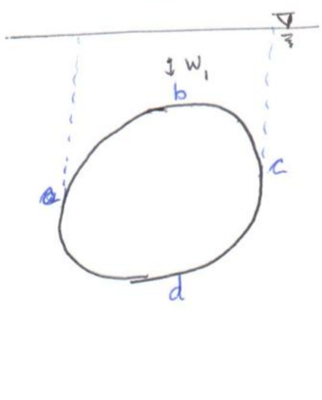
Yesterday, we started discussing on buoyancy.

Archimedes principles state: (As taken from FM White's "Fluid Mechanics")

(i) A body immersed in a fluid experiences a vertical buoyant force equal to the weight of fluid it displaces.

(ii) A floating body displaces its own weight in the fluid in which it floats.

As discussed yesterday, we can describe it through a submerged body




→ Let us assume, the submerged body consist of two surfaces

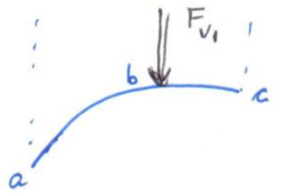
Upper surface - abc

Lower surface - adc

→ From our analysis on curved surfaces,

Now ~~the~~ take free-body diagrams of the solid body itself (not the water column).

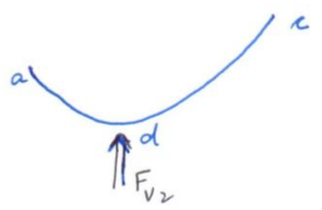
→ The upper surface - abc




It will have a ~~vertical~~ hydrostatic force ~~component~~ $F_1$ acting ~~perpendicular~~ into the ~~and~~ curved surface.

→ The vertical component of it is $F_{V_1}$ downwards, $F_{V_1} = W_1$

→ Similarly for lower surface - adc




→ The vertical component of hydrostatic force $F_2$ will be $F_{V_2}$ acting upwards. ~~$= W_2$~~

∴ The net vertical force $= F_{V_2} - F_{V_1} = F_b$

buoyant force.

⇒ If you want to do the same analysis by considering pressure forces on individual elements on the curved surfaces of the submerged body

$$F = \int p \, dA$$

→ This net force will be acting upwards.

This is because hydrostatic pressure at bottom is more compared to the top and at bottom hydrostatic force will act upwards.

$F_B$ = Weight of fluid equivalent to the volume of body that is submerged

⇒ The line of action of buoyant force is through the center of volume of the displaced body.
(Please note that it is ~~called~~ center of volume) ~~and~~

~~not the centr~~

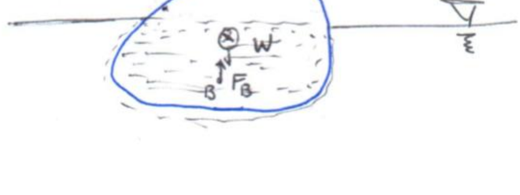
→ This point is called center of buoyancy (CB).

→ This point CB need not be the center of mass of the solid submerged. Why?

Floating Bodies

We have seen that for a fully submerged solid body, there will be a net pressure force acting upwards called the buoyant force. This force was equal to the weight of fluid that is displaced by the solid.

In floating bodies, only a portion of the body is submerged.



The dotted portion shows the displaced volume of the liquid by the floating object.

→ In this case, the net pressure force that will act on the body (or buoyant force)

$$F_B = \rho g * \text{Displaced Volume of liquid}$$
$$= \text{Weight of the floating body.}$$

→ The buoyant force and the weight of the body have to be collinear.

This is because in static condition, the net moment should be zero.

This floating body explanation is part of Archimede's principle.

<u>Example</u> (Adopted from FM White's "Fluid Mechanics")

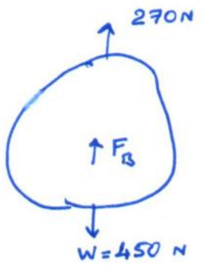
A block of concrete weighs 450 N in air and weighs 270 N when immersed in fresh water of density 1000 kg/m$^3$. What is the average specific weight of the block?

<u>Solution</u> The actual weight of a body is the one measured in air.

$\therefore$ Actual weight of concrete = 450 N

There is apparent weight loss in the concrete body, when it is submerged in water.

→ This apparent weight loss is due to buoyant force.




$$\Sigma F_z = 0$$

i.e. $W_2 - W_1 + F_B = 0$

or $F_B = W_1 - W_2$

$= 450 - 270 = \underline{\underline{180\ N}}$

Recall Buoyant force = (Specific gravity of water) * (Volume of the submerged solid)

i.e. ~~180~~ $180 = (1000 \times 9.81) * V$

∴ Volume of block, $V = 0.01835\ m^3$

$= \underline{18349}\ cm^3$

∴ Specific ~~gravity~~ weight of concrete $\gamma_{concrete} = \dfrac{450}{0.01835}$

$= 24523\ N/m^3$

$= \underline{\underline{24.5\ kN/m^3}}$

→ Here you can see the specific weight of concrete is much higher than that of water (9.81 kN/m³).

→ Sometimes, a body will have ~~exactly~~ its weight and volume distributed in such a way that it will equal the specific weight of water.

→ If it is equal to specific weight of water, then it is called <u>Neutrally Buoyant</u>.

Neutrally buoyant objects will remain at rest at any point where it is immersed.

Positive buoyant objects remain in floating conditions.

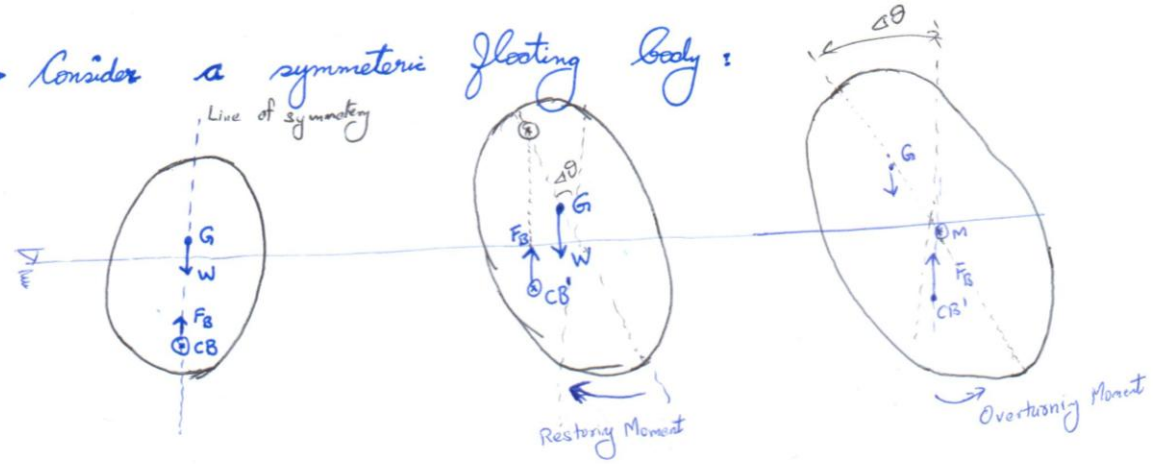
## Stability

We may not know whether a floating body is stable. That is, whether it will prevent over turning.

→ If the floating body frequently overturns, then it is unstable

→ As engineers, we should design floating bodies that do not overturn.

⇒ Consider a symmeteric floating body:




We already described that for a floating body

$$F_B = (\text{Sp. gravity of fluid}) \times (\text{Volume of fluid displaced by floating body})$$
$$= \text{Weight of the floating body}.$$

(i) Using this relation, the body's center of mass G and center of buoyancy CB are computed.
We told that ~~at~~ both the forces should be collinear.

(ii) If we tilt the body by a small angle $\Delta\theta$, then the vertical buoyant force's ~~action~~ line of action will change. This is because of change in surface area that is in contact with water.

The new position CB' of the center of buoyancy is to be calculated.

→ A vertical line drawn upwards from CB' will intersect the line of symmetery. This point M is called Metacenter.

(iii) If the point M is above point G, the body has a restoring moment and the original position is stable. (i.e. Metacenteric height is positive).

→ If the point M is below point G, the body is unstable and will overturn if disturbed.