LECTURE 10
02-FEB-2016

# HYDROSTATIC FORCES ON SURFACES - 2

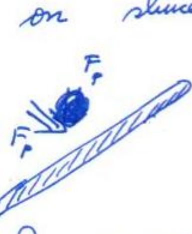
Yesterday we discussed about hydrostatic forces on plane surfaces

↳ The centroid of the plane surface
↳ The equivalent point force
↳ The center of pressure, etc.

Subsequently, we stated a problem on sluice gate. Hope, you all have gone through last day's class notes on that problem description.

To summarise the methods that were involved in solving that problem.

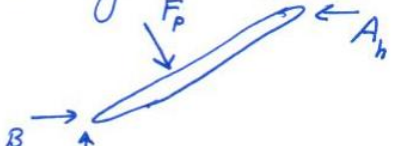
i) First you computed the total hydrostatic force due to water that will act on sluice gate




ii) Then you find the center of pressure $(x_{cp}, y_{cp})$

$$y_{cp} = -\frac{I_{xx}\sin\theta}{h_{CG}A} \quad ; \quad x_{cp} = -\frac{I_{xy}\sin\theta}{h_{CG}A}$$

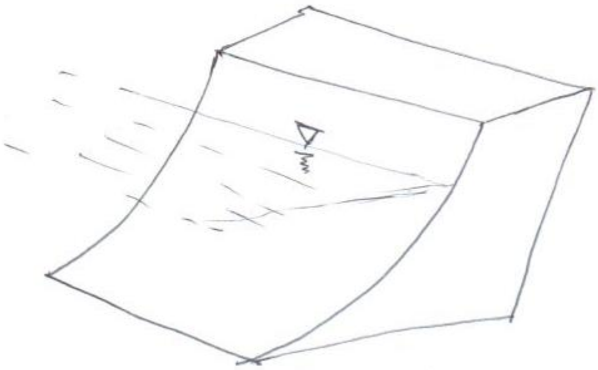
iii) Develop free-body diagram of sluice gate

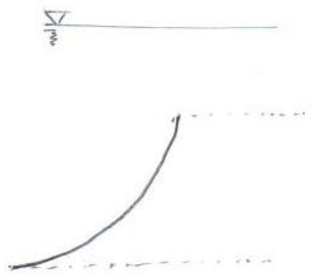


Using static principles, evaluate the forces $A_h$, $B_h$, and $B_v$

## Hydrostatic Forces on Curved Surfaces

Rather than a plane surface, if you have curved surfaces, on the retaining structures, how will you compute the hydrostatic forces?



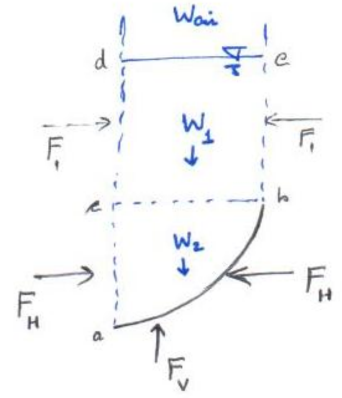


→ You know that at each elemental area, on the curved surface, the hydrostatic force will act into the surface perpendicularly.

→ Theoretically you need to integrate the elemental force throughout the whole curved surface area.

→ Sometimes, the computations may be tedious. Using static principles, we can compute the components of the total hydrostatic force in vertical and horizontal directions.

→ Take the free-body diagram of the water column enclosed by the curved surface.




→ The desired hydrostatic force that fluid exerts on the solid surface will be reversed as we are taking the FBD of water column.

$F_H$ and $F_V$ are horizontal and vertical components of hydrostatic force (on the curved surface, ofcourse).

Balancing forces,

$\Sigma F_V = 0$; i.e. $F_V - W_1 - W_2 - W_{air} = 0$

or $F_V = W_1 + W_2 + W_{air}$

Looking into the figure, in the portion bcde, the horizontal forces $F_1$ balances.

∴ Same balancing on the portion abc is also to be done. i.e. $F_H$ acts into on left side and $F_H$ acts into on right side.

Example (As adopted from FM White's "Fluid Mechanics")

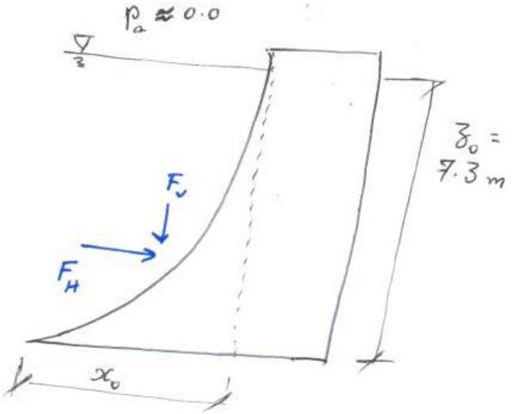
A dam has a parabolic shape $z/z_0 = \left(\frac{x}{x_0}\right)^2$ with $x_0 = 3.0$ m, $z_0 = 7.3$ m.

The fluid is water with density = 1000 kg/m³

Atmospheric pressure ≈ 0.0

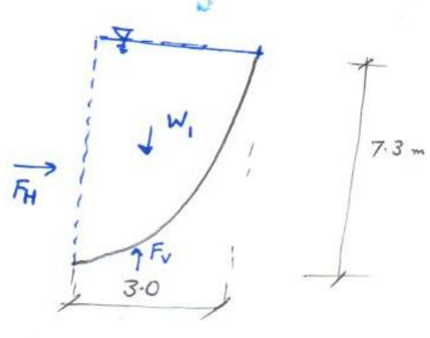
Compute components of hydrostatic force on the dam.

The width of the dam is 15.25 m




Solution

Take FBD of water column.




→ The width of dam = 15.25 m

→ Therefore, the vertical rectangular plane that is projected area of the curved surface

$= 15.25 * 7.3$

$A_{proj} = 111.325 \text{ m}^2$

→ The centroid of this vertically projected plane will be at height $h_{CG} = \frac{7.3}{2} = 3.65$ m

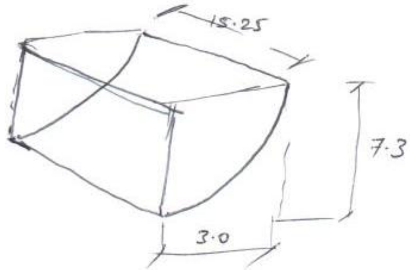
$\therefore F_H = \rho g h_{CG} A_{proj} = 1000 \times 9.81 \times 3.65 \times 111.325$

$= 3986.16$ kN

To find $F_v$,

→ The vertical force $F_v$ equals the weight of water above the parabola.

→ To determine the volume of water enclosed by the parabolic surface at one end and vertical plane at the other end




$$\text{Volume} = \frac{2}{3} * z_0 * x_0 * b$$

$$= \frac{2}{3} * 7.3 * 3.0 * 15.25$$

$$= 222.65 \ m^3$$

$$\therefore \text{Weight} = \rho g * \text{Volume} = 1000 \times 9.81 \times 222.65$$

$$= \underline{2184.2} \ kN$$

$$\therefore F = \sqrt{F_H^2 + F_v^2} = \sqrt{(2184.2)^2 + (3986.16)^2}$$

$$= \underline{4545.35} \ kN$$

## Buoyancy and Stability

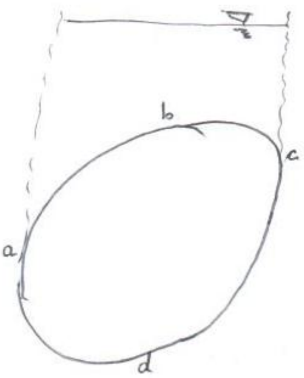
Till now, we were describing computing hydrostatic forces on surfaces. That is, those forces were computed on one side of the surfaces.

→ If a body is submerged, or floating,

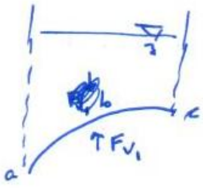
its all sides will be subjected to hydrostatic forces.

→ Therefore, there will be net pressure force acting on such a body. This force is called the <u>buoyant force</u>.

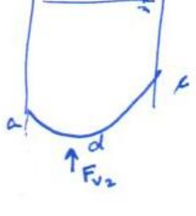
→ Recall our explanation for <u>curved surfaces</u>. Now consider a submerged body




→ The body can be now suggested to consist of two curved surfaces abc and adc




→ Along curve abc, the vertical component of hydrostatic force $F_{V1}$




→ Along curve adc, the vertical component of hydrostatic force is $F_{V_2}$

$\therefore$ Net vertical force $= F_{V2} - F_{V1}$

$= F_B$

= weight of fluid equivalent to body volume.

It is based on Archimedes <u>principle</u>.