14. AMPLIFIERS

14.1 Introduction

Microwave amplifiers combine active elements with passive transmission line circuits to provide functions critical to microwave systems and instruments. The history of microwave amplifiers begins with electron devices using resonant or slow-wave structures to match wave velocity to electron beam velocity. The design techniques used for BJT and FET amplifiers employ the full range of concepts we have developed in the study of microwave transmission lines, two-port networks and Smith chart presentation. In microwave systems, amplifiers play a significant role. We will start with the definitions of two port power gains and discuss the stability circles. Then we will design single-stage amplifiers for maximum gain, specified gain and low noise figure. The S parameters of a given microwave transistor can be derived from transistor equivalent circuit models based on device physics, or they can be measured directly. Generally, a manufacturer of a device intended for microwave applications will provide extensive S-parameter data to permit accurate design of microwave amplifiers. This can be verified by measurement, a step that has proven important on many occasions. For a bipolar junction transistor, in addition to intrinsic device parameters such as base resistance and collector-base capacitance, amplifier performance is strongly affected by the so-called parasitic elements associated with the device package, including base-lead and emitter-lead inductance internal to the package. Similar considerations apply to microwave field-effect transistors. The magnitude and phase angle of each of the S parameters typically vary with frequency, and characterization over the complete range of interest is necessary.

Solid-state microwave amplifiers play an important role in communication where it has different applications, including low noise, high gain, and high power amplifiers. The high gain and low noise amplifiers are small signal low power amplifiers and are mostly used in the receiver side where the signal level is low. The small signal S parameter can be used in designing these low power amplifiers. The high power amplifier is used in the transmitter side where the signal should be at a high level to cross the desired distance. The intent of this chapter is to give an overview of some basic principles used in the analysis and design of the small signal low power amplifier and we will restrict ourselves to single-stage amplifier designs only.

14.2 Power Gains

In this section, we will define various terms for power relating to the performance of the amplifier. Basically there are three types of power gains:
Let us try to get the various expressions of power of an amplifier, viz., $P_L$, $P_{in}$, $P_{avn}$ and $P_{avs}$ so that we can get their ratios to get the expressions of three power gains we have defined above.

1) By voltage division at the input port 1 (refer to Fig. 14.1),

$V_1 = V_s \frac{Z_{in}}{Z_{in} + Z_s} = V_1^+ + V_1^- = V_1^+ (1 + \Gamma_{in})$

$\therefore \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}, \Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}$

$\Rightarrow \Gamma_{in} Z_{in} + \Gamma_{in} Z_0 = Z_{in} - Z_0$

$\Rightarrow Z_{in} (\Gamma_{in} - 1) = Z_0 (-1 - \Gamma_{in})$

$\Rightarrow \frac{Z_{in}}{Z_0} = \frac{-1 - \Gamma_{in}}{-1 + \Gamma_{in}} = \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$

**Fig. 14.1 A general two-port network**

- Operating power gain, $G \equiv \frac{P_L}{P_{in}}$, is defined as the ratio of the power delivered to the load $P_L$ to the power input to the amplifier $P_{in}$.

- Available power gain, $G_A \equiv \frac{P_{avn}}{P_{avs}}$, is defined as the ratio of the power available from the amplifier $P_{avn}$ to the power available from the source $P_{avs}$.

- Transducer power gain, $G_T \equiv \frac{P_L}{P_{avs}}$, is defined as the ratio of the power delivered to the load $P_L$ to the power available from the source $P_{avs}$. The transducer power gain is equal to the operating power gain only when the amplifier is conjugate matched to the source, i.e., $\Gamma_{in} = \Gamma_s^\ast$. Therefore, $G_p > G_T$ in general.
Hence, \( V_i^+ = \frac{V_i}{1+\Gamma_{in}} \cdot \frac{1}{1+\Gamma_{in}} \cdot \frac{Z_{in}}{Z_{in}+Z_s} \cdot \frac{1+\Gamma_{in}}{1-\Gamma_{in}} \cdot \frac{1+\Gamma_{in}}{1-\Gamma_{in}} \cdot \frac{V_s \cdot (1-\Gamma_{s})}{(1+\Gamma_{in})(1-\Gamma_{s})+(1+\Gamma_{s})(1-\Gamma_{in})} \)

\( = \frac{V_s \cdot (1-\Gamma_{s})}{2(1-\Gamma_{s} \Gamma_{in})} \)

Hence, the power delivered into the amplifier, i.e., the input power of the amplifier is

\[ P_{in} = \frac{1}{2} \frac{|V_i|^2}{Z_0} \cdot (1-|\Gamma_{in}|^2) \]

\[ = \frac{|V_s|^2}{8Z_0} \cdot \frac{(1-|\Gamma_{in}|^2) \cdot |1-\Gamma_{s}|^2}{|1-\Gamma_{s} \Gamma_{in}|^2} \]

Note that \( P_{av}(\Gamma_{in}) = P_{in} \mid \Gamma_{in} = \Gamma_{s} \cdot \Gamma_{in} \). Therefore, \( P_{av} = P_{in} \mid \Gamma_{av} = \Gamma_{s} \cdot \Gamma_{in} = \frac{|V_s|^2}{8Z_0} \cdot \frac{|1-\Gamma_{s}|^2}{(1-|\Gamma_{s}|^2)} \)

2) From scattering matrix analysis at the output port 2 (refer to Fig. 14.1),

\( V_2^- = S_{21} V_1^+ + S_{22} V_2^+ \) and \( V_2^+ = \Gamma_L V_2^- \)

\( \Rightarrow V_2^- = S_{21} V_1^+ + S_{22} \Gamma_L V_2^- \)

\( \Rightarrow V_2^- = \frac{S_{21} V_1^+}{1-S_{22} \Gamma_L} \)

\( = \frac{S_{21} V_1^+}{(1-S_{22} \Gamma_L)(1-\Gamma_{in})} \)

Therefore, \( P_L = \frac{1}{2} \frac{|V_2|^2}{Z_0} (1-|\Gamma_L|^2) = \frac{1}{8Z_0} \frac{|V_s|^2}{|S_{21}|^2} \frac{|1-\Gamma_L|^2}{|1-S_{22} \Gamma_L|^2} (1-|\Gamma_L|^2) \)

Note that \( P_{av}(\Gamma_{out}) = P_{L} \mid \Gamma_{in} = \Gamma_{out} \).

\[ = \frac{|V_s|^2}{8Z_0} \cdot \frac{|S_{21}|^2}{|1-S_{22} \Gamma_{out}|^2} \cdot \frac{|1-\Gamma_{in}|^2}{|1-\Gamma_{s} \Gamma_{in}|^2} (1-|\Gamma_{out}|^2) \]

We also know that, \( \Gamma_{in} = S_{11} + \frac{S_{12} S_{22} \Gamma_L}{1-S_{22} \Gamma_L} = \frac{S_{11} - S_{12} S_{22} \Gamma_L + S_{12} S_{22} \Gamma_L}{1-S_{22} \Gamma_L} \)

putting this value in \( |1-\Gamma_{s} \Gamma_{in}|^2 \mid_{\Gamma_{in} = \Gamma_{out}} \), we get
\[
\left| \frac{1 - S_{11}}{1 - S_{22}} \right|^2 = \left| \frac{1 - S_{11} \Gamma_{in}}{1 - S_{22} \Gamma_{in}} \right|^2 = \left| 1 - \frac{S_{11} \Gamma_{in}^2}{1 - \Gamma_{in}^2} \right|^2 = \left| 1 - \frac{S_{11}}{1 - \Gamma_{in}^2} \right|^2
\]

\[
P_{av} = \frac{|V_i|^2 |S_{21}|^2 |1 - \Gamma_s|^2}{8Z_o |1 - S_{11} \Gamma_s|^2 (1 - |\Gamma_{out}|^2)^2}
\]

3) Let us know get all the power gain ratios.

**Operating power gain**, \( G \equiv \frac{P_L}{P_{in}} = \frac{1 - |\Gamma_L|^2 |S_{21}|^2}{|1 - S_{22} \Gamma_s|^2 (1 - |\Gamma_{in}|^2)}
\]

**Available power gain**, \( G_A \equiv \frac{P_{av}}{P_{avs}} = \frac{|S_{21}|^2 (1 - |\Gamma_{out}|^2)(1 - |\Gamma_s|^2)}{|1 - S_{22} \Gamma_s|^2 |1 - \Gamma_s \Gamma_{in}|^2} \) or \( \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)}{|1 - S_{11} \Gamma_s|^2 (1 - |\Gamma_{out}|^2)} \)

**Transducer power gain**, \( G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)(1 - |\Gamma_s|^2)}{|1 - S_{22} \Gamma_L|^2 |1 - \Gamma_s \Gamma_in|^2} \)

A special case for the transducer power gain occurs when \( S_{12} \) is zero or negligible. Then the device is non-reciprocal and hence \( \Gamma_{in} = S_{11} \) and \( S_{12} = 0 \), therefore the unilateral transducer gain is

\[
G_{TU} = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)(1 - |\Gamma_s|^2)}{|1 - S_{22} \Gamma_L|^2 |1 - \Gamma_s S_{11}|^2}
\]

**14.3 Stability**

![Fig. 14.2 A general amplifier two-port network](image)

The stability of an amplifier is a very important consideration in a microwave circuit design. Stability or resistance to oscillation in a microwave circuit can be determined by
the S-parameters. Oscillations are possible in a two-port network if either or both the input and the output port have negative resistance. In the above circuit, oscillation is possible if either input or output port impedances has a negative real part which would implies that $|\Gamma_{in}| > 1$ or $|\Gamma_{out}| > 1$.

There are two types of amplifier stability, unconditionally stable and conditionally stable. In the former, the real part of the input and output impedances of the amplifier is greater than zero for all passive load and source impedances. However, the amplifier is said to be conditionally stable or potentially unstable if the real part of the input or output impedances of the amplifier is less than zero for at least a passive load or source impedances. The stability test should be done for every frequency in the desired range.

- **Unconditional stability:** network is unconditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for all passive source and load impedances. For unconditionally stable:

  $$|\Gamma_{in}| = \left| \frac{S_{11} + S_{12}S_{22} \Gamma_{L}}{1 - S_{22} \Gamma_{L}} \right| < 1$$
  
  $$|\Gamma_{out}| = \left| \frac{S_{22} + S_{12}S_{22} \Gamma_{L}}{1 - S_{11} \Gamma_{L}} \right| < 1$$

  If the device is unilateral, $S_{12} = 0$, these condition reduce to $|S_{11}| < 1$ and $|S_{22}| < 1$. The necessary and sufficient conditions for a two-port network to be unconditional stable are:

  $$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1; |\Delta| < 1 \text{ also called as Rollet’s condition}$$

  Or, $\mu = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11} \Delta + S_{21}S_{12}|} > 1$ also known as $\mu$-test.

In practice, most of the microwave transistor amplifiers are potentially unstable because of the internal feedback. There are two ways to overcome the stability problem of the transistor amplifier. The first is to use some form of feedback to stabilize the amplifier. The second is to use a graphical analysis to determine the regions where the values of $\Gamma_S$ and $\Gamma_L$ (source and load reflection coefficients) are less than one, which means the real parts of $Z_{IN}$ and $Z_{OUT}$ are positive.

- **Conditional stability:** Network is conditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ only for certain range of passive source and load impedances. This case is referred to as potentially unstable. Note that the stability condition of network is frequency dependent as it is possible for an amplifier to be stable at its design frequency and unstable at other frequencies. For conditionally stable: Stability circles are defined as the loci of $\Gamma_S (\Gamma_L)$ plane for which $|\Gamma_{in}| = 1$ or $|\Gamma_{out}| = 1$. Then the stability circles defines the boundary between stable and potentially unstable of $\Gamma_S$ and $\Gamma_L$. 
1) \( C_L = \frac{(S_{22} - \Delta S_{11}^*)}{|S_{22}|^2 - |\Delta|^2} \)

\( R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \), where \( \Delta = S_{11}S_{22} - S_{12}S_{21} \)

Output stability circles

2) \( C_S = \frac{(S_{11} - \Delta S_{22})^*}{|S_{22}|^2 - |\Delta|^2} \)

\( R_S = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \)

Input stability circles
Given the S parameters of the device, we can plot the input and output stability circle to define where $|\Gamma_{in}|=1$ and $|\Gamma_{out}|=1$. On the other side of the input stability circle we will have $|\Gamma_{out}|<1$ or $|\Gamma_{in}|<1$. While on the other side we will have $|\Gamma_{out}|>1$ or $|\Gamma_{in}|>1$. These are defined in figure above.

Example 14.1
The S parameters for the HP-FET-102 GaAs FET at 2 GHz with a bias voltage $V_{gs}=0$ are given as follows:
\[
S = \begin{pmatrix}
0.894(60.6^\circ) & 0.020(62.4^\circ) \\
3.122(123.6^\circ) & 0.781(-27.6^\circ)
\end{pmatrix}.
\]

Determine the stability of this transistor by calculating \(K\) and \(|\Delta|\), and plot the stability circles.

**Solution:**

\[
\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.696(-83^\circ) \Rightarrow |\Delta| = 0.696 < 1
\]

\[
K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 0.607 < 1
\]

Hence the device is conditionally stable. We need to draw the input and output stability circles to find the regions of stability on the Smith chart.

**Output stability circles**

\[
C_L = \frac{(S_{22} - \Delta S_{11}^*)}{|S_{22}|^2 - |\Delta|^2} = 1.363(46.687^\circ)
\]

\[
R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| = 0.5, \text{ where } \Delta = S_{11}S_{22} - S_{12}S_{21}
\]

Since \(|S_{11}| < 1\), the region of stability will be as in Fig. 14.4 (a).

**Input stability circles**

\[
C_S = \frac{(S_{11} - \Delta S_{22}^*)}{|S_{22}|^2 - |\Delta|^2} = 1.132(68.461^\circ)
\]

\[
R_S = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| = 0.199
\]

Since \(|S_{22}| < 1\), the region of stability will be as in Fig. 14.4 (b).
14.4 Single-Stage Amplifier Design

**Conjugate matching (maximum gain)**

After the stability of the transistor has been determined and the stable regions for $\Gamma_s$ and $\Gamma_L$ have been located on the Smith chart, the input and output matching sections can be designed. Since $G_0$ for Fig. 14.2 is fixed for a given transistor, the overall gain of the amplifier will be controlled by the gains $G_s$ and $G_L$ of the matching sections. Maximum gain will be realized when these sections provide a conjugate match between the amplifier source or load impedance and the transistor. If $|\Delta| < 1$ and $K > 1$, then are input and output simultaneously, conjugate match for $G_{T_{\text{max}}}$.

If $K < 1$, then draw input and output stability circles to see if input and output conjugate match possible. For maximum gain, input and output should be simultaneously conjugate matched, i.e, $\Gamma_{in}^* = \Gamma_s^*$ and $\Gamma_{out}^* = \Gamma_L^*$.

$$G_T = G_{T_{\text{max}}} = \frac{1}{1-|\Gamma_s|^2} \left| S_{21} \right|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$= \frac{S_{21}}{|S_{12}|} (K - \sqrt{K^2 - 1})$$

From input conjugate matching,

$$\Gamma_s^* = \Gamma_{in}^* = S_{11} + \frac{S_{12}^* S_{21} \Gamma_L}{1 - S_{22}^* \Gamma_L} \Rightarrow \Gamma_s^* = S_{11} + \frac{S_{12}^* S_{21}}{1/\Gamma_L^* - S_{22}^*}$$

From output conjugate matching,

$$\Gamma_L^* = \Gamma_{out}^* = S_{22}^* + \frac{S_{12} S_{21} \Gamma_s^*}{1 - S_{11}^* \Gamma_s^*} = \frac{S_{22} (1 - S_{11} \Gamma_s^*) + S_{12} S_{21} \Gamma_s}{1 - S_{11} \Gamma_s} = \frac{S_{22} - \Delta \Gamma_s}{1 - S_{11} \Gamma_s}$$

Substituting this value of $\Gamma_L^*$ in the previous expression for $\Gamma_s^*$ and expanding gives
\[ \Gamma_s (1/\Gamma_s^* - S_{12}^*) = S_{11}' (1/\Gamma_s - S_{12}^*) + S_{12}' S_{21}' \]

\[ \Rightarrow \Gamma_s (1 - S_{11}' \Gamma_s - S_{22}' \Delta \Gamma_s) = S_{11}' (1 - S_{11} \Gamma_s - S_{12}' (S_{22} - \Delta \Gamma_s)) + S_{12}' S_{21}' (S_{22} - \Delta \Gamma_s) \]

\[ \Rightarrow \Gamma_s (1 - S_{11} \Gamma_s - S_{22}^* (S_{22} - \Delta \Gamma_s)) = S_{11}' (1 - S_{11} \Gamma_s - S_{12}' (S_{22} - \Delta \Gamma_s)) + S_{12}' S_{21}' (S_{22} - \Delta \Gamma_s) \]

\[ \Rightarrow \Gamma_s (1 - S_{11} \Gamma_s - S_{22}^* (S_{22} - \Delta \Gamma_s)) = S_{11}' (1 - S_{11} \Gamma_s - S_{12}' (S_{22} - \Delta \Gamma_s)) + S_{12}' S_{21}' (S_{22} - \Delta \Gamma_s) \]

\[ \Gamma_s (1 - S_{12}' \Gamma_s - S_{11}' (S_{11} - \Delta S_{11}')) = S_{11}' (1 - S_{11} \Gamma_s - S_{12}' (S_{22} - \Delta \Gamma_s)) + S_{12}' S_{21}' (S_{22} - \Delta \Gamma_s) \]

\[ \Rightarrow \Gamma_s (1 - S_{12}' \Gamma_s - S_{11}' (S_{11} - \Delta S_{11}')) = S_{11}' (1 - S_{11} \Gamma_s - S_{12}' (S_{22} - \Delta \Gamma_s)) + S_{12}' S_{21}' (S_{22} - \Delta \Gamma_s) \]

\[ \Rightarrow \Gamma_s (1 - S_{12}' \Gamma_s - S_{11}' (S_{11} - \Delta S_{11}')) = S_{11}' (1 - S_{11} \Gamma_s - S_{12}' (S_{22} - \Delta \Gamma_s)) + S_{12}' S_{21}' (S_{22} - \Delta \Gamma_s) \]

\[ \Rightarrow \Gamma_s (1 - S_{12}' \Gamma_s - S_{11}' (S_{11} - \Delta S_{11}')) = S_{11}' (1 - S_{11} \Gamma_s - S_{12}' (S_{22} - \Delta \Gamma_s)) + S_{12}' S_{21}' (S_{22} - \Delta \Gamma_s) \]

Hence the solution for \( \Gamma_s = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \)

Similarly the solution for \( \Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \)

where

\[ B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \]

\[ B_2 = 1 + |S_{12}|^2 - |S_{11}|^2 - |\Delta|^2 \]

\[ C_1 = S_{11} - \Delta S_{22} \]

\[ C_2 = S_{22} - \Delta S_{11} \]

Example 14.2

Design an amplifier for maximum gain at 4.0GHz using single-stub matching sections.

The S parameters at 4.0GHz for the GaAs FET are given as follows:

\[ S = \begin{pmatrix} 0.72(116^0) & 0.030(57^0) \\ 0.030(57^0) & 2.60(76^0) \end{pmatrix} \]

\[ 0.72(116^0) & 0.030(57^0) \\ 0.030(57^0) & 2.60(76^0) \]

Solution:

\[ \Delta = S_{11} S_{22} - S_{12} S_{21} = 0.488(-162^0) \Rightarrow |\Delta| = 0.488 < 1 \]

\[ K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12} S_{21}|} = 1.195 > 1 \]

Hence the device is unconditionally stable. We don’t need to draw the input and output stability circles to find the regions of stability on the Smith chart.

For maximum gain, we should design the matching sections for a conjugate match to the transistor (\( \Gamma_{in} = \Gamma_s^* \) and \( \Gamma_{out} = \Gamma_L^* \)).

Hence the solution for \( \Gamma_s = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \) and \( \Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \)
where
\[ B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \]
\[ B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \]
\[ C_1 = S_{11} - \Delta S_{22}^* \]
\[ C_2 = S_{22} - \Delta S_{11}^* \]

We need to choose the proper sign so that \( \Gamma_s < 1 \) and \( \Gamma_L < 1 \) since \( S_{11} < 1 \).

Hence the solution for \( \Gamma_s = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \) is 0.872(123.407°) and \( \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} = 0.876(61.026°) \). Therefore the three gain factors in Fig. 14.2 can be calculated as follows:

\[ G_T = G_{T_{\text{max}}} G_S G_L = \frac{1}{1 - |\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \]

\[ \Rightarrow G_{T_{\text{max} \text{dB}}} = G_{S_{\text{dB}}} + G_{0_{\text{dB}}} + G_{L_{\text{dB}}} = (6.197 + 8.299 + 2.213)\text{dB} = 16.71\text{dB} \]

To match the source (input matching network), we first need to locate the \( \Gamma_s \) on the Smith chart. The impedance \( Z_s \) represented by the reflection coefficient is the impedance seen looking into the matching section toward the source impedance \( Z_0 \) as depicted in Fig. 14.2. Thus matching section basically transforms \( Z_0 \) to the impedance \( Z_s \). You can design a single stub matching network for this as well as for the output matching network.

Also refer to lecture slides

### 15. Control Circuits

#### 15.1 PIN diode based control circuits

PIN diode is a useful control element at microwave frequencies. It is used in realizing switches, phase shifters, attenuators, etc. which provide control functions at microwave frequencies. PIN diode differs considerably from ordinary p-n junction diode. It has a heavily doped p-region and a heavily doped n-region separated by a high resistively material that is nearly intrinsic. Figure 15.1 shows the schematic of the PIN diode and the impurity profile (doping profile).
The presence of the intrinsic layer $i$ (in practice a high resistive layer) under reverse bias leads to relatively higher diode impedance. Under forward bias condition, holes and electrons are injected from p$^+$ and n$^+$ layers to the intrinsic region. If the carrier lifetimes in the $i$ layer are relatively long, this layer becomes flooded with carriers at a reasonable forward bias level. When this happens, the diode exhibits low resistance and in fact appears as a virtual short circuit. For this characteristic of the PIN diode, it is essential that the lifetime of carriers in the $i$ layer is greater than the time period of the operating frequency. The effect of controlling the carrier density and hence the resistance of the $i$ layer by varying the forward bias is known as the conductivity modulation. The equivalent circuits for the PIN diode in the reverse bias (Off state) and forward (On state) are shown in the Figure 15.2.

The PIN diode differs from a conventional PN junction diode by having a thin layer of Intrinsic semiconductor material between the usual Positive and Negative doped regions. The addition of the intrinsic layer reduces the junction capacitance, since P and N regions are further apart. It also makes the forward conductivity of the diode a much more linear function of the diode bias current.

To sum up, when the diode is forward biased, it acts like a current controlled resistance. The PIN diode’s series resistance decreases as its forward bias current increases. When reverse biased, the PIN diode acts like a fixed capacitor.

The typical values for the various parameters are: (a) Junction capacitance $C_j = 1$ pF (b) Lead inductance $L_i = 0.5$ nH (c) $R_r = 5 \Omega$ $R_f = 1 \Omega$. The equivalent circuits shown above are simplified equivalent circuits and do not include the parasitic effect of the packages.
15.1.1 PIN diode switches

Switches are very important control circuit component and used extensively in microwave systems.

Single Pole Switches

An SPST (single pole single throw) switch can be realized using PIN diode operating in series or shunt configuration.

![Figure 15.3 SPST switch (a) Series (b) Shunt](image)

In the absence of the diode, the voltage that would appear across the load is $V_L' = \frac{V_s}{2}$.

With the diode present, the voltages that appear across the load are: (a) series case:
\[ V_L = \frac{Z_d}{2Z_0 + Z_d} V_s \] and (b) shunt case: \[ V_L = \frac{Z_d}{2Z_d + Z_0} V_s \]. The insertion loss is given by

\[ IL = -20 \log \left| \frac{V_{in}}{V_{out}} \right| \]. Therefore, the insertion loss is given by:

\[ IL_{series} = -20 \log \left| \frac{2Z_0}{2Z_0 + Z_d} \right| \]

\[ IL_{shunt} = -20 \log \left| \frac{2Z_d}{2Z_d + Z_0} \right| \]

For both the cases, the diode impedance for the forward and reverse biased states are given by:

\[ Z_d = \begin{cases} 
Z_r = R_r + j(\omega L - \frac{1}{\omega C}) \\
Z_f = R_f + j\omega L
\end{cases} \]

\[ Z_r \] and \[ Z_f \] respectively represents the impedance of the diode in the reverse and forward biased condition.

**SPDT switches**

The series and shunt configuration for SPDT (Single pole double throw) switches are shown below.

![Circuits for single pole double throw switches](image-url)

Figure 15.5 Circuits for single pole double throw switches (a) series (b) shunt
In operating the series switch, one diode is forward biased while the other diode is reverse biased. In case of shunt switch, the forward biased diode creates a short circuit that is transformed into an open circuit by the $\frac{\lambda}{4}$ section. When left side PIN diode of Figure 15.5 (a) is forward biased, it behaves like a short circuit; the $Z_{inl}$ looking towards left at the input junction is infinite due to the $\frac{\lambda}{4}$ transformers. But right side PIN diode of Figure 15.5 (b) is reverse biased, it behaves like a open circuit, the $Z_{inr}$ looking towards right at the input junction is zero due to the $\frac{\lambda}{4}$ transformers. Hence the power coming from input port goes to outport port 2 while the output port 1 is isolated. Similar operating principle for shunt switch for the other case when the right PIN diode is forward biased and the left diode is reverse biased. Use of $\frac{\lambda}{4}$ section limits the bandwidth of operation of such circuits.

15.1.2 Phase shifters using PIN diodes

The phase shifter is a two-port network in which the phase shift of a signal traveling from one port to the other can be controlled. PIN diodes are utilized in realizing electronically controlled phase shifters. Phase shifters where discrete values of phase shifts are available are called the digital phase shifters. Digital phase shifters are designed using switches and transmission line sections. These phase shifters can be designed as transmission line type or reflection type.

Transmission line type phase shifter configuration is shown in Figure 15.6. Depending on the bias applied to the two switches, the signal can reach the output port by two alternative paths. The differential phase shifts between the two paths are given by:

$$\Delta \phi = \beta \Delta L$$

![Figure 15.6 Switched line phase shifter](image)

This is also called a one-bit digital phase shifter.
Reflection type digital phase shifter is shown in Figure 15.7, which uses a circulator and a SPST switch.

![Figure 15.7 Reflection type phase shifter](image)

The relative phase difference between the signals at the output port when the switch is open and the switch is closed is given by:

\[
\phi = \beta [2(L_1 + L_2) - 2L_2] = 2\beta L_1.
\]

This arrangement thus results in a one-bit phase shifter.

### 15.1.3 PIN diode attenuator

The resistance of the \(i\)-layer of the forward biased PIN diode is a strong function of the bias current. This property is utilized in designing current controlled variable attenuators at the microwave frequencies. Under forward biased condition the intrinsic layer resistance of typical PIN diode varies according to the following relationship:

\[
R_i = 26I^{-0.87}, \quad I \text{ is the forward current in mA.}
\]

This relationship holds in the bias current range \(19 \mu A\) to \(10mA\). Above \(10mA\), the forward resistance approaches parasitic series resistance \(~0.5\ \Omega\). Towards zero bias and reverse bias, a limit of \(10,000\ \Omega\) is asymptotically approached. At frequency < \(10\ MHz\) the diode shows rectifying property whereas at frequency >> \(10\ MHz\) it behaves as a variable resistor or attenuator. Figure 15.8 shows schematic circuits of series and shunt attenuator configurations. In series circuit, the attenuation decreases with increase of biasing current with consequent decrease of RF resistance. In shunt circuit attenuation increases with biasing current because most of the RF energy is absorbed in the diode.
Microwave Tubes:

The efficiency of conventional tubes is largely independent of frequency up to a certain limit. When frequency increases beyond that limit, several factors combine to rapidly decrease tube efficiency. Tubes that are efficient in the microwave range usually operate on the theory of VELOCITY MODULATION, a concept that avoids the problems encountered in conventional tubes. Velocity modulation is more easily understood if the factors that limit the frequency range of a conventional tube are thoroughly understood. Therefore, the frequency limitations of conventional tubes will be discussed before the concepts and applications of velocity modulation are explained.

**Frequency Limitations of Conventional Tubes**

Three characteristics of ordinary vacuum tubes become increasingly important as frequency rises. These characteristics are interelectrode capacitance, lead inductance, and electron transit time. The INTERELECTRODE CAPACITANCES in a vacuum tube, at low or medium radio frequencies, produce capacitive reactances that are so large that no serious effects upon tube operation are noticeable. However, as the frequency increases, the reactances become small enough to materially affect the performance of a circuit. For example, in Figure 16.1 (a), a 1 pF capacitor has a reactance of 159,000 Ω at 1 MHz. If this capacitor was the interelectrode capacitance between the grid and plate of a tube, and the RF voltage between these electrodes was 500 V, then 3.15 mA of current would flow through the interelectrode capacitance. Current flow in this small amount would not seriously affect circuit performance. On the other hand, at a frequency of 100 MHz the reactance would decrease to approximately 1,590 Ω and, with the same voltage applied, current would increase to 315 mA (Figure 16.1(b)). Current in this amount would definitely affect circuit performance.
A good point to remember is that the higher the frequency, or the larger the interelectrode capacitance, the higher will be the current through this capacitance. If the signal frequency is 100 MHz or higher, this reactance is so small that much of the signal is short-circuited within the tube. Since the interelectrode capacitances are effectively in parallel with the tuned circuits, they will also affect the frequency at which the tuned circuits resonate.

Another frequency-limiting factor is the LEAD INDUCTANCE of the tube elements. Since the lead inductances within a tube are effectively in parallel with the interelectrode capacitance, the net effect is to raise the frequency limit. However, the inductance of the cathode lead is common to both the grid and plate circuits. This provides a path for degenerative feedback which reduces overall circuit efficiency.

A third limitation caused by tube construction is TRANSIT TIME. Transit time is the time required for electrons to travel from the cathode to the plate. While some small amount of transit time is required for electrons to travel from the cathode to the plate, the time is insignificant at low frequencies. In fact, the transit time is so insignificant at low frequencies that it is generally not considered to be a hindering factor. However, at high frequencies, transit time becomes an appreciable portion of a signal cycle and begins to hinder efficiency. For example, a transit time of 1 nanosecond, which is not unusual, is only 0.001 cycle at a frequency of 1 megahertz. The same transit time becomes equal to the time required for an entire cycle at 1,000 megahertz. Transit time depends on electrode spacing and existing voltage potentials. Transit times in excess of 0.1 cycle cause a significant decrease in tube efficiency.

If the tube is to operate efficiently, the plate current must be in phase with the grid-signal voltage and 180 degrees out of phase with the plate voltage. When transit time approaches 1/4 cycle, this phase relationship between the elements does not hold true. A positive swing of a high-frequency grid signal causes electrons to leave the cathode and flow to the plate. Initially this current is in phase with the grid voltage. However, since transit time is an appreciable part of a cycle, the current arriving at the plate now lags the grid-signal voltage. As a result, the power output of the tube decreases and the plate power dissipation increases.

**Fig. 16.1** Interelectrode capacitance in vacuum tube (a) at 1MHz and (b) at 100MHz
Several methods are available to reduce the limitations of conventional tubes, but none work well when frequency increases beyond 1,000 megahertz. Interelectrode capacitance can be reduced by moving the electrodes further apart or by reducing the size of the tube and its electrodes. Moving the electrodes apart increases the problems associated with transit time, and reducing the size of the tube lowers the power-handling capability. You can see that efforts to reduce certain limitations in conventional tubes are compromises that are often in direct opposition to each other. The net effect is an upper limit of approximately 1,000 megahertz, beyond which conventional tubes are not practical.

Review Question 16.1
What happens to the impedance of interelectrode capacitance as frequency increases?

Review Question 16.2
What undesirable effect is caused by the inductance of the cathode lead?

Review Question 16.3
How does transit time affect the relationship of the grid voltage and the plate current at high frequencies?

Review Question 16.4
Moving tube electrodes apart to decrease interelectrode capacitance causes an increase in the effect of what property?

**Velocity Modulation**

The microwave tube was developed when the use of the frequency spectrum went beyond 1,000 MHz and into the microwave range. The microwave tube uses transit time in the conversion of dc power to radio-frequency (RF) power. The interchange of power is accomplished by using the principle of electron VELOCITY MODULATION and low-loss resonant cavities in the microwave tube. A clear understanding of microwave tubes must start with an understanding of how electrons and electric fields interact. An electron has mass and thus exhibits kinetic energy when in motion. The amount of kinetic energy in an electron is directly proportional to its velocity; that is, the higher the velocity, the higher the energy level. The basic concept of the electron energy level being directly related to electron velocity is the key principle of energy transfer and amplification in microwave tubes. An electron can be accelerated or decelerated by an electrostatic field. Figure 16.2(a) shows an electron moving in an electrostatic field. The direction of travel (shown by the heavy arrow) is against the electrostatic lines of force which are from positive to negative. The negatively charged electron will be attracted to the positively charged body and will increase in velocity. As its velocity increases, the energy level of the electron will also increase. Where does the electron acquire its additional energy? The only logical source is from the electrostatic field. Thus, the conclusion is clear. An electron traveling in a direction opposite to electrostatic lines of force will absorb energy and increase in velocity (accelerate).

As Figure 16.2(b) illustrates, the opposite condition is also true. An electron traveling in the same direction as the electrostatic lines of force will decelerate by giving up energy to the field. The negatively charged body will repel the electron and cause it to decrease in velocity. When the velocity is reduced, the energy level is also reduced. The energy lost by the electron is gained by the electrostatic field.
Fig. 16.2 (a) Moving electron gaining energy and velocity and (b) Moving electron losing energy and velocity

The operation of a velocity-modulated tube depends on a change in the velocity of the electrons passing through its electrostatic field. A change in electron velocity causes the tube to produce BUNCHES of electrons. These bunches are separated by spaces in which there are relatively few electrons. Velocity modulation is then defined as that variation in the velocity of a beam of electrons caused by the alternate speeding up and slowing down of the electrons in the beam. This variation is usually caused by a voltage signal applied between the grids through which the beam must pass.

The first requirement in obtaining velocity modulation is to produce a stream of electrons which are all traveling at the same speed. The electron stream is produced by an electron gun. A simplified version of an electron gun is shown in Figure 16.3. Electrons emitted from the cathode are attracted toward the positive accelerator grid and all but a few of the electrons pass through the grid and form a beam. The electron beam then passes through a pair of closely spaced grids, called BUNCHER GRIDS. Each grid is connected to one side of a tuned circuit. The parallel-resonant tuned circuit is the doughnut-shaped resonant cavity surrounding the electron stream. The buncher grids are at the center of the cavity and are at the same dc potential as the accelerator grid. The alternating voltage which exists across the resonant circuit causes the velocity of the electrons leaving the buncher grids to differ from the velocity of the electrons arriving at the buncher grids. The amount of difference depends on the strength and direction of the electrostatic field within the resonant cavity as the electrons pass through the grids. The manner in which the buncher produces bunches of electrons is better understood by considering the motions of individual electrons, as illustrated in Figure 16.4.
Fig. 16.3 Electron gun with buncher grids

When the voltage across the grids is negative, as shown in Figure 16.4 (a), electron 1 crossing the gap at that time is slowed. Figure 16.4 (b) shows the potential across the gap at 0 volts; electron 2 is not affected. Electron 3 enters the gap (Figure 16.4 (c)) when the voltage across the gap is positive and its velocity is increased. The combined effect is shown in Figure 16.4 (d). All of the electrons in the group have been bunched closer together.

16. Sources

16.1 Gunn Diodes

In some materials (III-V compounds such as GaAs and InP), after an electric field in the material reaches a threshold level, the mobility of electrons decrease as the electric field is increased, thereby producing negative resistance. A two-terminal device made from such a material can produce microwave oscillations. In certain semiconductors, notably GaAs, electrons can exist in a high-mass low velocity state as well as their normal low-mass high-velocity state and they can be forced into the high-mass state by a steady electric field of sufficient strength. The Gunn diode is a so-called transferred electron device. Electrons are transferred from one valley in the conduction band to another valley. In order to understand the nature of the transferred electron effect exhibited by Gunn diodes, it is necessary to consider the electron drift velocity versus electric field (or current versus voltage) relationship for GaAs. Below the threshold field, $E_{th}$, of approximately 0.32 V/mm, the device acts as a passive resistance. However, above $E_{th}$ the electron velocity (current) decreases as the field (voltage) increases producing a region of negative differential mobility, NDM (resistance, NDR).
High electric fields: (a) Si-mobility saturation (b) GaAs-decrease in mobility

Increasing $E \rightarrow$ acceleration $\rightarrow$ electron transfer to the L valley, electron valley with higher effective mass ($m_L = 0.55m_0$ and $m_T = 0.067m_0$)

In solids, the valence band is the highest range of electron energies in which electrons are normally present at absolute zero temperature. The valence electrons are bound to
individual atoms, as opposed to conduction electrons (found in conductors and semiconductors), which can move freely within the atomic lattice of the material. On a graph of the electronic band structure of a material, the valence band is located below the conduction band, separated from it in insulators and semiconductors by a band gap. In metals, the conduction band has no energy gap separating it from the valence band.

\[ E_T < E < E_v \rightarrow \text{acceleration} \rightarrow \text{negative differential resistance} \]

The energy-momentum relationship contains two conduction band energy levels, \( \Gamma \) and \( L \) (also known as valleys) with the following properties:

- In the lower \( \Gamma \) valley, electrons exhibit a small effective mass and very high mobility, \( \mu_1 \).
In the satellite L valley, electrons exhibit a large effective mass and very low mobility, \( \mu_2 \).

The two valleys are separated by a small energy gap, \( \Delta E \), of approximately 0.31 eV. In equilibrium at room temperature most electrons reside near the bottom of the lower \( \Gamma \) valley. Because of their high mobility (~ 8000 cm\(^2\)V\(^{-1}\)s\(^{-1}\)), they can readily be accelerated in a strong electric field to energies in the order of the \( \Gamma - L \) intervalley separation, \( \Delta E \). Electrons are then able to scatter into the satellite L valley, resulting in a decrease in the average electron mobility, \( \mu \), as given below:

\[
\mu = \frac{(n_1\mu_1 + n_2\mu_2)}{(n_1 + n_2)}
\]

where \( n_1 \) = electron density in \( \Gamma \) valley, \( n_2 \) = electron density in L valley

Above the high field, \( E_H \), most electrons reside in the L valley and the device behaves as a passive resistance (of greater magnitude) once again.

In a practical Gunn diode, electrons are accelerated from the cathode by the prevailing electric field. When they have acquired sufficient energy, they begin to scatter into the low mobility satellite valley and slow down.

There are two types of resistances of a device: static resistance (\( R = V/I \)) and dynamic or differential resistance (\( r = dV/dI \)). In general current tends to increase with rising voltage but in some devices current decrease with increasing voltage which results in negative resistance.

**GUNN Diodes (Transferred Electron Devices):**

Gunn diodes are negative resistance devices which are normally used as low power oscillator at microwave frequencies in transmitter and also as local oscillator in receiver front ends. J. B. Gunn (1963) discovered microwave oscillation in Gallium arsenide (GaAs), Indium phosphide (InP) and Cadmium telluride (CdTe). These are semiconductors having a closely spaced energy valley in the conduction band as shown in Fig. 16.1(b) for GaAs. When a dc voltage is applied across the material as depicted in Fig. 16.1 (a), an electric field is established across it. At low \( E \)-field in the material, most of the electrons will be located in the lower energy central valley \( \Gamma \). At higher \( E \)-field, most of the electrons will be transferred in to the high-energy satellite L and X valleys where the effective electron mass is larger and hence electron mobility is lower than that in the low energy \( \Gamma \) valley. Since the conductivity is directly proportional to the mobility, the conductivity and hence the current decreases with an increase in \( E \)-field or voltage in an intermediate range, beyond a threshold value \( V_{th} \) as shown in Fig. 16.1(c). This is called the transferred electron effect and the device is also called ‘Transfer Electron Device (TED) or Gunn diode’. Thus the material behaves as negative resistance device over a range of applied voltages and can be used in microwave oscillators.
The basic structure of a Gunn diode is shown in Fig. 16.2 (a), which is of n-type GaAs semiconductor with regions of high doping (n⁺). The active n-layer is 10μm for 10GHz. Although there is no junction this is called a diode with reference to the positive end (anode) and negative end (cathode) of the dc voltage applied across the device. At low level of applied voltage or electric field to the GaAs, initially the current will increase with a rise in the voltage. When the diode voltage exceeds a certain threshold value, $V_{th}$ a high electric field (3.2 KV/m for GaAs) is produced across the active region and electrons are excited from their initial lower valley to the higher valley, where they become virtually immobile. If the rate at which electrons are transferred is very high, the current will decrease with increase in voltage, resulting in equivalent negative resistance effect. Since GaAs is a poor conductor, considerable heat is generated in the diode. The diode will be bonded into a heat sink (Au-stud). The electrical equivalent circuit of a Gunn diode is shown in Fig. 16.2 (b), where $C_j$ and $-R_j$ are the diode capacitance and resistance, respectively, $R_s$ includes the total resistance of lead, ohmic contacts, and bulk resistance of the diode, $C_p$ and $L_p$ are the package capacitance and inductance, respectively. The negative resistance has a value that typically lies in the range –5 to –20 ohms.
**Gunn Oscillator:**

In a Gunn Oscillator, the Gunn Diode is placed in a resonant cavity. In this case, the oscillation frequency is determined by cavity dimension rather than by the diode itself. Usually, the Gunn diode is mounted on a post structure between the waveguide walls, either $\lambda g/2$ from an iris or $\lambda_g/2$ from a short circuit. Some alteration is necessary to set the exact frequency to allow for diode and package parasitics and manufacturing tolerances. Tuning screws (either metal or dielectric) are used to modify the cavity resonant frequency. Power output variations are achieved by adjusting the coupling between diode and load using variations in post size or tuning screws.

![Diagram of Gunn Oscillator](image)

**16.2 Klystron tube (This section needs to be upgraded)**

Reflex Klystron is one of the most commonly used microwave (low power) generators. It converts D.C. power into microwave power.

*Reflex Klystrons Oscillator:*

The schematic diagram of a reflex klystron tube is shown in Fig. 16.3, which uses only a single re-entrants microwave cavity as resonator. The electron beam emitted from the cathode K is accelerated by the grid G and passes through the cavity anode A to the repeller space between the cavity anode and the repeller electrode.
Mechanism of Oscillation:
Due to dc voltage in the cavity circuit, RF noise is generated in the cavity. This electromagnetic noise field in the cavity becomes pronounced at cavity resonant frequency. The electrons passing through the cavity gap d experience this RF field and are velocity modulated in the following manner. The electrons as shown in Fig. 16.4 which encountered the positive half cycle of the RF field in the gap d will be accelerated, those (reference electrons) b which encountered zero RF field will pass with unchanged original velocity, and the electrons c which encountered the negative half cycle will be retarded on entering the repeller space.
All these velocity modulated electrons will be repelled back to the cavity by the repeller due to its negative potential. The repeller distance $L$ and the voltages can be adjusted to receive all the velocity modulated electrons at a same time on the positive peak of the cavity RF voltage cycle. Thus the velocity modulated electrons are bunched together and lose their kinetic energy when they encounter the positive cycle of the cavity RF field. This loss of energy is thus transferred to the cavity to conserve the total power. If the power delivered by the bunched electrons to the cavity is greater than the power loss in the cavity, the electromagnetic field amplitude at the resonant frequency of the cavity will increase to produce microwave oscillations. The RF power is coupled to the output load by means of a small loop which forms the center conductor of the coaxial line. When the power delivered by the electrons becomes equal to the total power loss in the cavity system, a steady microwave oscillation is generated at resonant frequency of the cavity.

**Mode of Oscillation:**
The bunched electrons in a Reflex Klystron can deliver maximum power to the cavity at any instant which corresponds to the positive peak of the RF cycle of the cavity oscillation. If $T$ is the time period at the resonant frequency, $t_0$ is the time taken by the reference electron to travel in the repeller space between entering the repeller space at $b$ and the returning to the cavity at positive peak voltage on formatting of the bunch, then

$$t_n = (n + 3/4) T = NT$$

where $N = n + 3/4$, $n = 0, 1, 2, 3, ...$, as shown in Fig. 16.4
Thus by adjusting repeller voltage for given dimensions of the Reflex Klystron, the bunching can be made to occur at \( N = \frac{1}{4}, \frac{3}{4}, \frac{2}{4}, \frac{3}{4}, \ldots \), etc. for modes \( n = 0, 1, 2, 3, \ldots \), respectively. It is obvious that the lowest order mode 3/4 occurs for a maximum value of repeller voltage when the transit time \( t_0 \) of the electrons in the repeller space is minimum. Higher modes occur at lower repeller voltages. Since at the highest repeller voltage, the acceleration of the bunched electrons of return is maximum, the power output of the lowest mode is maximum.

**Modulation:**
By varying the reflector voltage about a d.c. value, Klystron can be frequency and amplitude modulated simultaneously. For proper square wave modulation with 100% modulation index, the reflector voltage and amplitude of the square wave should be set as shown in Fig. 16.5. If the square wave peak to peak amplitude is \( V_m \) and \( V_o \) is the reflector d.c. voltage, the total reflector voltage will switch between \( (V_o+V_m) \) and \( (V_o-V_m) \). We have to choose \( V_o \) and \( V_m \) such that \( (V_o+V_m) \) is in the mode center and \( (V_o-V_m) \) is the non-oscillating region for proper square wave modulation.
Fig. 16.6 Square wave modulation
Fig. 16.7 Modes of Klystron