6. Plane waves reflection from media interface

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6.1 Introduction

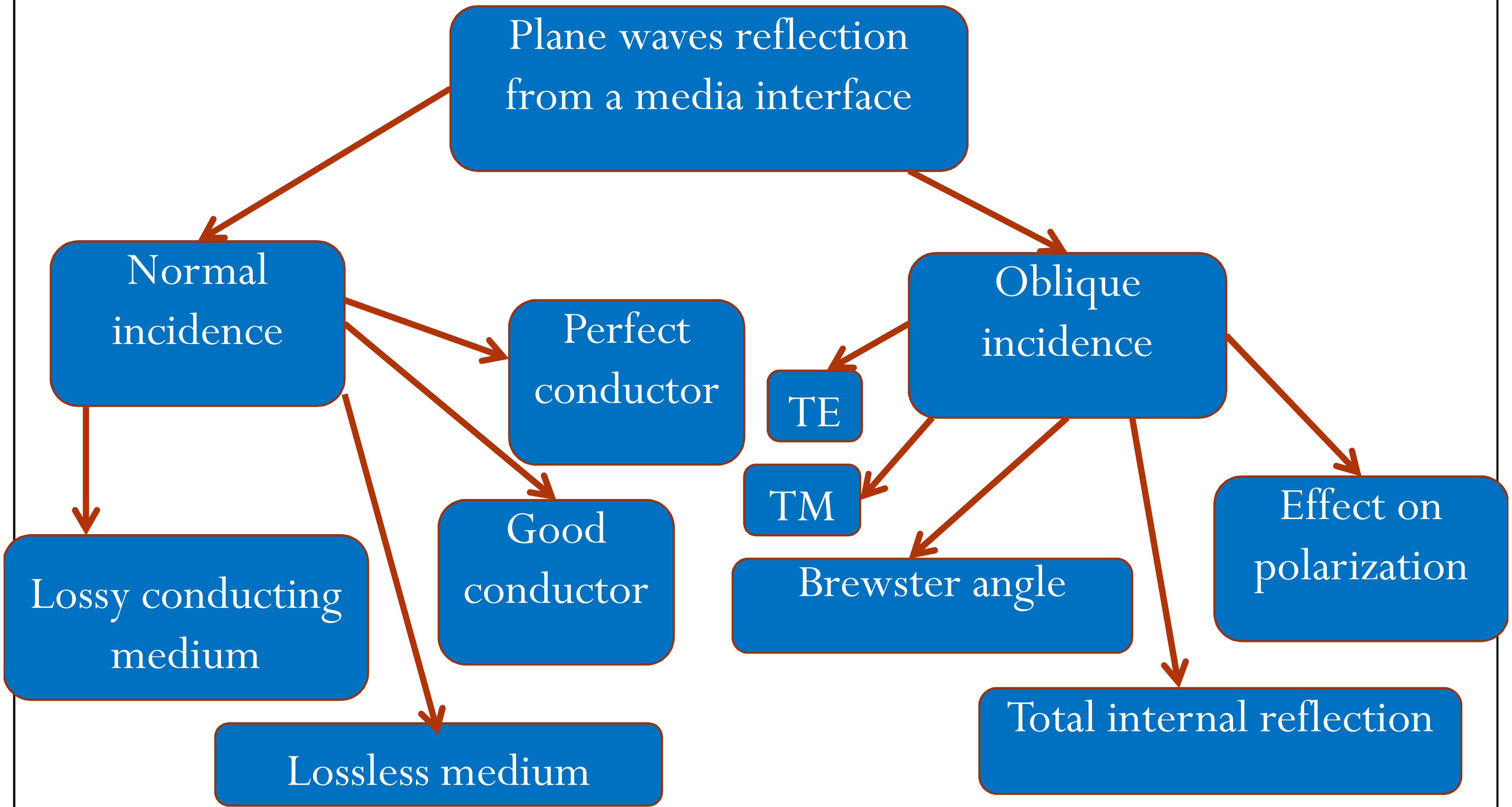

Fig. 6.1 Plane waves reflection from media interface

6.1 Introduction

- Till now, we have studied plane waves in various medium
- Let us try to explore how plane waves will behave at a media interface
- In practical scenarios of wireless and mobile communications,
  - radio wave will reflect from
    - walls &
    - other obstacles on its path
- When a radio wave reflects from a surface,
  - the strength of the reflected waves is less than that of the incident wave

6.1 Introduction

- The ratio of the two (reflected wave/incident wave) is known as the '***reflection coefficient***' of the surface
- This ratio depends on the
    - conductivity (σ),
    - permittivity (ε) and
    - permeability (μ)
  - of the material that forms the reflective surface
  - as well as material properties of the
    - air
  - from which the radio wave is incident

6.1 Introduction

- Some part of the wave will be transmitted through the material
- How much of the incident wave has been transmitted through the material is
  - also dependent on the material parameters mentioned above
- It is given by another ratio known as '***transmission coefficient***'
- It is the ratio of the transmitted wave divided by the incident wave

6.1 Introduction

- In plane wave reflection from media interface,
- what we will be doing is
  - basically writing down the
    - electric and
    - magnetic field expressions
  - in all the regions of interest and
- apply the boundary conditions
  - to get the values of the
    - transmission and
    - reflection coefficients

6.1 Introduction

- One of the possible applications of such an exercise is in wireless communication,
  - where we have to find the
    - transmission and
    - reflection coefficients of
  - multipaths
- Electromagnetic waves are often
  - reflected or
  - scattered or
  - diffracted
- at one or more obstacles before arriving at the receiver

6.2 Plane wave reflection from media interface at normal incidence

- For smooth surfaces,
  - EM waves are reflected;
- For rough surfaces,
  - EM waves are scattered;
- For edges of surfaces,
  - EM waves are diffracted
- Fig. 6.2 depicts multipath for a mobile receiver in the car from
  - direct,
  - reflected and
  - diffracted waves
- of the signal sent from the base station

6.1 Introduction

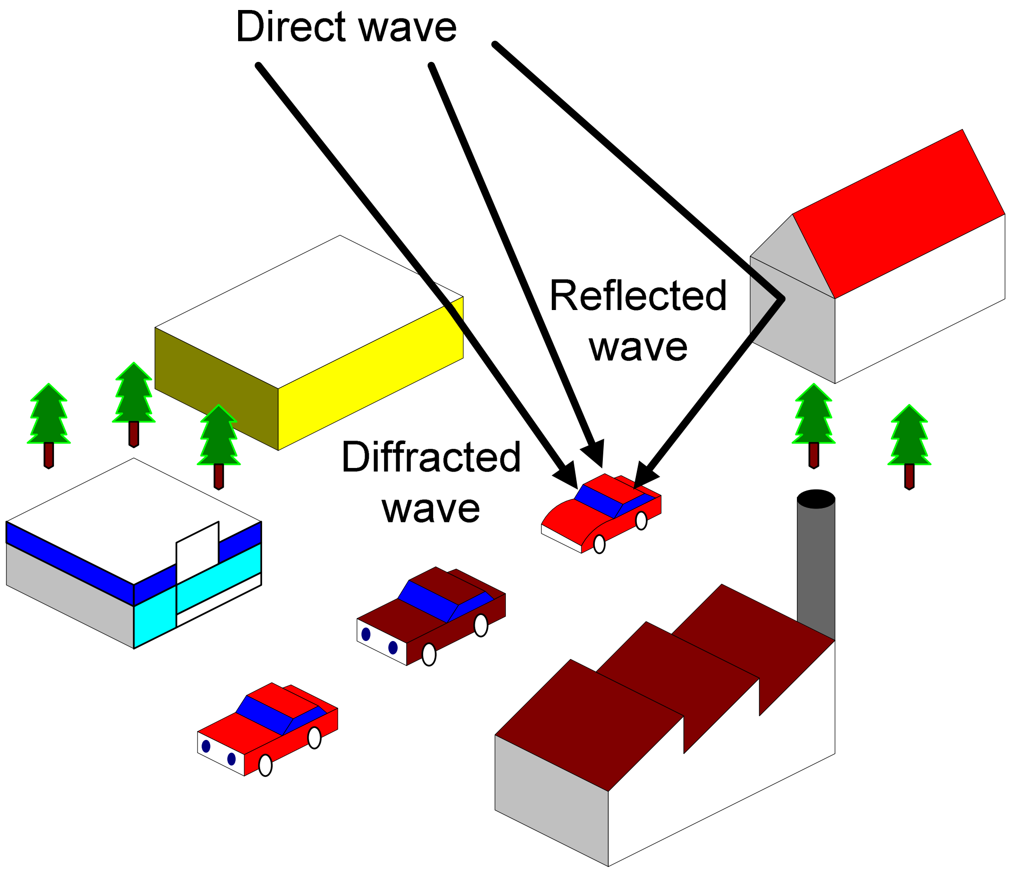

Fig. 6.2 Illustration of reflected and diffracted wave in mobile communications (this chapter's study will be very helpful in understanding wireless and mobile communications)

6.2 Plane wave reflection from media interface at normal incidence

- We will consider the case of normal incidence,
  - when the incident wave propagation vector is along the normal to the interface between two media

6.2.1 Lossy conducting medium

- We will assume plane waves with electric field vector oriented along the x-axis and
- propagating along the positive z-axis without loss of generality
- For $z<0$ (we will refer this region as region I and it is assumed to be a lossy medium)

6.2 Plane wave reflection from media interface at normal incidence

- Let us do a more generalized analysis by assuming this region I as a lossy medium
- It could be a lossless medium like free space
- The incident electric and magnetic fields can be expressed as

$$\vec{E}_i = \hat{x} E_o e^{-\gamma_1 z}$$

$$\vec{H}_i = \hat{y} \frac{1}{\eta_1} E_o e^{-\gamma_1 z}$$

6.2 Plane wave reflection from media interface at normal incidence

- where η_1 is the medium 1 wave impedance and
- E_o is the arbitrary amplitude of the incident electric field
- The expression for intrinsic wave impedance and propagation constant

$$\eta_1 = \frac{j\omega\mu_1}{\gamma_1} = \frac{j\omega\mu_1}{\sqrt{j\omega\mu_1\left(j\omega\varepsilon_1 + \sigma_1\right)}} = \sqrt{\frac{j\omega\mu_1}{j\omega\varepsilon_1 + \sigma_1}}; \gamma_1 = \alpha_1 + j\beta_1 = j\omega\sqrt{\mu_1\varepsilon_1}\sqrt{1 - \frac{j\sigma_1}{\omega\varepsilon_1}}$$

6.2 Plane wave reflection from media interface at normal incidence

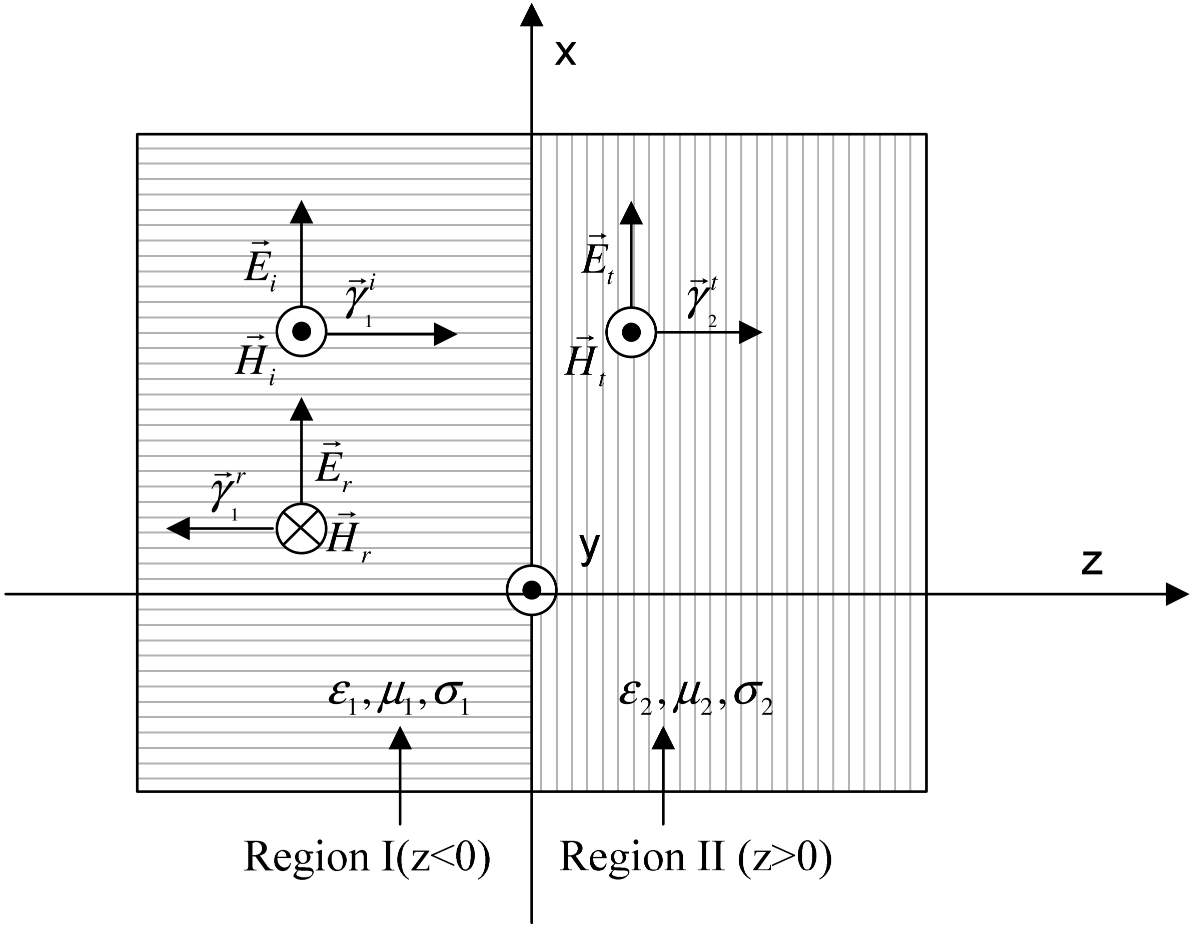

Fig. 6.3 A plane EM wave is incident from region I or medium 1 for z<0

6.2 Plane wave reflection from media interface at normal incidence

- Convention:
  - a circle with a dot in the center → an arrow pointing perpendicularly out of the page and
  - a circle with a cross → an arrow pointing perpendicularly into the page)
- Notation for fields:
  - subscript i→ incident, r→ reflected and t→transmitted

6.2 Plane wave reflection from media interface at normal incidence

- Note that the incident wave from region I will be partially reflected and transmitted at the media interface between two regions
- For reflected wave, z<0, wave direction is in the –z axis and the reflected electric field is expressed as

$$\vec{E}_r = \hat{x}\Gamma E_o e^{+\gamma_1 z}$$

- where the Γ is the reflection coefficient

6.2 Plane wave reflection from media interface at normal incidence

- The reflection coefficient is defined as the ratio of amplitude of the reflected electric field divided by amplitude of the incident electric field as follows:

$$\Gamma = \frac{|E_r|}{|E_i|}$$

- The reflected magnetic field can be obtained from the reflected electric field using the Maxwell's curl equations as

6.2 Plane wave reflection from media interface at normal incidence

$$\nabla \times \vec{E}_r = -j\omega\mu_1 \vec{H}_r$$

$$\Rightarrow \vec{H}_r = -\frac{\nabla \times \vec{E}_r}{j\omega\mu_1} = \frac{j\nabla \times \vec{E}_r}{\omega\mu_1} = \frac{j}{\omega\mu_1}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \Gamma E_o e^{+\gamma_1 z} & 0 & 0 \end{vmatrix}$$

$$= \hat{y}\frac{j}{\omega\mu_1}\left\{\frac{\partial}{\partial z}\left(\Gamma E_o e^{+\gamma_1 z}\right)\right\} = \hat{y}\frac{j}{\omega\mu_1}\left(\gamma_1 \Gamma E_o e^{+\gamma_1 z}\right) = -\hat{y}\frac{\gamma_1}{j\omega\mu_1}\left(\Gamma E_o e^{+\gamma_1 z}\right)$$

$$\therefore \vec{H}_r = -\hat{y}\frac{\Gamma}{\eta_1} E_o e^{+\gamma_1 z}$$

6.2 Plane wave reflection from media interface at normal incidence

- Therefore, the Poynting vector for reflected wave of region I is

$$\vec{S}_r = \vec{E}_r \times \vec{H}_r^* == -|\Gamma|^2 |E_o|^2 e^{+2\alpha z} \frac{\hat{z}}{\eta_1^*}$$

- shows power is traveling in the -z axis for the reflected wave
- For transmitted wave, z>0, wave is propagating in lossy medium 2 (we will refer this region as region II),

6.2 Plane wave reflection from media interface at normal incidence

$$\vec{E}_t = \hat{x}\tau E_o e^{-\gamma_2 z}$$

$$\vec{H}_t = \hat{y}\tau \frac{E_o}{\eta_2} e^{-\gamma_2 z}$$

- where η_2 is the wave impedance of lossy medium 2 and τ is the transmission coefficient
- intrinsic wave impedance and propagation constant are

6.2 Plane wave reflection from media interface at normal incidence

$$\eta_2 = \frac{j\omega\mu_2}{\gamma_2} = \frac{j\omega\mu_2}{\sqrt{j\omega\mu_2\left(j\omega\varepsilon_2 + \sigma_2\right)}} = \sqrt{\frac{j\omega\mu_2}{j\omega\varepsilon_2 + \sigma_2}}; \gamma_2 = \alpha_2 + j\beta_2 = j\omega\sqrt{\mu_2\varepsilon_2}\sqrt{1 - \frac{j\sigma_2}{\omega\varepsilon_2}}$$

- transmission coefficient is defined as the ratio of amplitude of the transmitted electric field divided by amplitude of the incident electric field as follows:

$$\tau = \frac{|E_t|}{|E_i|}$$

6.2 Plane wave reflection from media interface at normal incidence

- Our intention here is to find the transmission and reflection coefficients
- Let us rewrite the fields in the two regions: region I ($z<0$) and region II ($z>0$) (see Table 6.1) and
- Apply the boundary conditions to obtain the two unknown coefficients

6.2 Plane wave reflection from media interface at normal incidence

Table 6.1 Fields in the two lossy regions (normal incidence)

Region I (lossy medium 1)	Region II (lossy medium 2)
$\vec{E}_i = \hat{x} E_o e^{-\gamma_1 z}$ $\vec{H}_i = \hat{y} \frac{1}{\eta_1} E_o e^{-\gamma_1 z}$ $\vec{E}_r = \hat{x} \Gamma E_o e^{+\gamma_1 z}$ $\vec{H}_r = -\hat{y} \frac{\Gamma}{\eta_1} E_o e^{+\gamma_1 z}$	$\vec{E}_t = \hat{x} \tau E_o e^{-\gamma_2 z}$ $\vec{H}_t = \hat{y} \tau \frac{E_o}{\eta_2} e^{-\gamma_2 z}$

6.2 Plane wave reflection from media interface at normal incidence

- This is basically boundary value problem with boundary conditions at z=0
- Note that total electric and magnetic fields (both incident and reflected) are tangential to the interface at z=0
- Similarly, the transmitted electric and magnetic field are also tangential to the interface at z=0
- Also note that there are no surface current density at the interface
- Hence, the tangential components of electric and magnetic fields must be continuous at the interface z=0

6.2 Plane wave reflection from media interface at normal incidence

$$\vec{E}_i + \vec{E}_r = \vec{E}_t\Big|_{z=0} \Rightarrow (1+\Gamma)E_0 = \tau E_0 \Rightarrow 1+\Gamma = \tau$$

$$\vec{H}_i + \vec{H}_r = \vec{H}_t\Big|_{z=0} \Rightarrow E_0\left(\frac{1-\Gamma}{\eta_1}\right) = E_0\left(\frac{\tau}{\eta_2}\right) \Rightarrow \frac{1-\Gamma}{\eta_1} = \frac{\tau}{\eta_2}$$

- Therefore,

$$\frac{1-\Gamma}{\eta_1} = \frac{\tau}{\eta_2} \Rightarrow \frac{1-\Gamma}{\eta_1} = \frac{1+\Gamma}{\eta_2} \Rightarrow (1-\Gamma)\eta_2 = \eta_1(1+\Gamma) \Rightarrow \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

- Hence,

$$\tau = 1+\Gamma = 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

6.2 Plane wave reflection from media interface at normal incidence

6.2.2 Lossless medium

- If the regions are lossless dielectric,
  - then, $\sigma = 0$ and μ and ε are real quantities
- The propagation constant for this case is purely imaginary and can be written as
  - $\gamma = j\beta = j\omega\sqrt{(\mu\varepsilon)}$
- The wave impedance of the dielectric is
- $\eta = \dfrac{j\omega\mu}{\gamma} = \dfrac{j\omega\mu}{j\beta} = \dfrac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\dfrac{\mu}{\varepsilon}} = \eta_0\sqrt{\dfrac{\mu_r}{\varepsilon_r}}$

6.2 Plane wave reflection from media interface at normal incidence

- For a lossless medium,
- η is real, so,
- both Γ and τ are real
- Electric and magnetic fields are in phase with each other
- The wavelength in the dielectric is
- $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{\omega\sqrt{\mu\varepsilon}}$
- And the phase velocity is
- $v_p = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{c}{\sqrt{\mu_r\varepsilon_r}}$
- where c is the speed of light in free space

6.2 Plane wave reflection from media interface at normal incidence

Table 6.2 Fields in the two lossless regions (normal incidence)

Region I (lossless medium 1)	Region II (lossless medium 2)
$\vec{E}_i = \hat{x}E_o e^{-j\beta_1 z}$ $\vec{H}_i = \hat{y}\frac{1}{\eta_1}E_o e^{-j\beta_1 z}$ $\vec{E}_r = \hat{x}\Gamma E_o e^{+j\beta_1 z}$ $\vec{H}_r = -\hat{y}\frac{\Gamma}{\eta_1}E_o e^{+j\beta_1 z}$	$\vec{H}_t = \hat{y}\tau\frac{E_o}{\eta_2}e^{-j\beta_2 z}$ $\vec{E}_t = \hat{x}\tau E_o e^{-j\beta_2 z}$

6.2 Plane wave reflection from media interface at normal incidence

- Applying the boundary conditions like before,
- we can get the expression for
  - reflection and
  - transmission coefficients,
- we will get the same expression as before

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}; \quad \tau = \frac{2\eta_2}{\eta_1 + \eta_2}$$

6.2 Plane wave reflection from media interface at normal incidence

- Poynting vector in region I

$$\vec{S}^1 = \vec{E}_{tot}^1 \times \left(\vec{H}_{tot}^1\right)^* = \left(\vec{E}_i + \vec{E}_r\right) \times \left(\vec{H}_i + \vec{H}_r\right)^* = \left(E_0 e^{-j\beta_1 z}\hat{x} + \Gamma E_0 e^{+j\beta_1 z}\hat{x}\right) \times \left(\frac{E_0^*}{\eta_1} e^{j\beta_1 z}\hat{y} - \Gamma \frac{E_0^*}{\eta_1} e^{-j\beta_1 z}\hat{y}\right)$$

$$= \left\{\frac{|E_0|^2}{\eta_1} - \frac{\Gamma |E_0|^2 e^{-2j\beta_1 z}}{\eta_1} + \frac{\Gamma |E_0|^2 e^{+2j\beta_1 z}}{\eta_1} - \frac{\Gamma^2 |E_0|^2}{\eta_1}\right\}\hat{z} = \left\{\frac{|E_0|^2}{\eta_1}\left(1-\Gamma^2\right) + \frac{\Gamma |E_0|^2}{\eta_1}\left(e^{+2j\beta_1 z} - e^{-2j\beta_1 z}\right)\right\}\hat{z}$$

$$= \left\{\frac{|E_0|^2}{\eta_1}\left(1-\Gamma^2\right) - \frac{\Gamma |E_0|^2}{\eta_1} 2j\sin\left(2\beta_1 z\right)\right\}\hat{z}$$

6.2 Plane wave reflection from media interface at normal incidence

- Poynting vector in region II

$$\vec{S}^2 = \vec{E}_t \times \vec{H}_t^* = \hat{x}\left(\tau E_0 e^{-j\beta_2 z}\right) \times \left(\frac{\tau E_0}{\eta_2} e^{-j\beta_2 z}\right)^* \hat{y} = \hat{z}\frac{\tau^2 \left|E_0\right|^2}{\eta_2}$$

$$\because 1-\Gamma^2 = 1-\frac{(\eta_2-\eta_1)^2}{(\eta_2+\eta_1)^2} = \frac{4\eta_2\eta_1}{(\eta_2+\eta_1)^2} \Rightarrow \frac{1-\Gamma^2}{\eta_1} = \frac{4\eta_2}{(\eta_2+\eta_1)^2}$$

$$\tau = \frac{2\eta_2}{\eta_2+\eta_1} \Rightarrow \tau^2 = \frac{4\eta_2^2}{(\eta_2+\eta_1)^2} \Rightarrow \frac{1-\Gamma^2}{\eta_1} = \frac{\tau^2}{\eta_2} \therefore \vec{S}^2 = \hat{z}\frac{\left(1-\Gamma^2\right)\left|E_0\right|^2}{\eta_1}$$

6.2 Plane wave reflection from media interface at normal incidence

Power conservation

- Compare the time average power flow in the two regions
- For z <0, the time average power flow through 1m^2 cross section is

$$S_{avg}^{1}=\frac{1}{2}\mathrm{Re}\left(\vec{S}^{1}\bullet\hat{z}\right)=\frac{1}{2}\left|E_0\right|^2\frac{1-\Gamma^2}{\eta_1}$$

- And for z >0, the time average power flow through 1m^2 cross section is

$$S_{avg}^{2}=\frac{1}{2}\mathrm{Re}\left(\vec{S}^{2}\bullet\hat{z}\right)=\frac{1}{2}\left|E_0\right|^2\frac{1-\Gamma^2}{\eta_1}$$

6.2 Plane wave reflection from media interface at normal incidence

- Hence,

$$S_{avg}^{1} = S_{avg}^{2}$$

- So real power is conserved

6.2.3 Good conductor:

- If the region II (z >0) is a good (but not perfect) conductor and
- region I is a lossless medium like free space,
- the propagation constant for medium 2 is

6.2 Plane wave reflection from media interface at normal incidence

$$\gamma_2 = \alpha_2 + j\beta_2 = (1+j)\sqrt{\frac{\omega\mu_2\sigma_2}{2}} = (1+j)\frac{1}{\delta_s^2}$$

- where δ_s^2 is the skin depth for the medium 2,
- which is a good conductor
- Similarly, the intrinsic impedance of the conducting medium 2 (see also example 6.2) simplifies to

$$\eta_2 = \frac{j\omega\mu_2}{\gamma_2} = \frac{j\omega\mu_2}{(1+j)\sqrt{\frac{\omega\mu_2\sigma}{2}}} = \frac{j}{1+j}\sqrt{\frac{2(\omega\mu_2)^2}{\omega\mu_2\sigma}} = \frac{j}{1+j}\sqrt{\frac{2\omega\mu_2}{\sigma}}$$

$$= \frac{j(1-j)}{(1+j)(1-j)}\sqrt{\frac{2\omega\mu_2}{\sigma}} = \frac{(j+1)}{(1-j^2)}\sqrt{\frac{2\omega\mu_2}{\sigma}} = \frac{(j+1)}{2}\sqrt{\frac{2\omega\mu_2}{\sigma}} = (j+1)\sqrt{\frac{\omega\mu_2}{2\sigma}}$$

6.2 Plane wave reflection from media interface at normal incidence

- Now, the wave impedance of medium 2 is complex, with a phase angle of 45^o so the electric
- and magnetic field will be 45^o out of phase and Γ and τ will be complex.
- Now, let us write down the field expressions in region I and II for lossless dielectric (region I) and good conductor (region II) interface

6.2 Plane wave reflection from media interface at normal incidence

Table 6.4 Fields in the two regions (normal incidence)

Region I (lossless medium)	Region II (good conductor)
$\vec{E}_i = \hat{x}E_o e^{-j\beta_1 z}$ $\quad \vec{H}_i = \hat{y}\dfrac{1}{\eta_1}E_o e^{-j\beta_1 z}$ $\vec{E}_r = \hat{x}\Gamma E_o e^{+j\beta_1 z}$ $\quad \vec{H}_r = -\hat{y}\dfrac{\Gamma}{\eta_1}E_o e^{+j\beta_1 z}$	$\vec{E}_t = \hat{x}\tau E_o e^{-\gamma_2 z}$ $\vec{H}_t = \hat{y}\tau\dfrac{E_o}{\eta_2}e^{-\gamma_2 z}$

6.2 Plane wave reflection from media interface at normal incidence

- Applying the boundary conditions like before,
- we can get the expression for
  - reflection and
  - transmission coefficients,
- we will get the same expression as before

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}; \quad \tau = \frac{2\eta_2}{\eta_1 + \eta_2}$$

6.2 Plane wave reflection from media interface at normal incidence

- Note that we have to use the previous expressions for intrinsic wave impedances for lossless medium and good conductor

$$\eta_1 = \sqrt{\frac{\mu}{\varepsilon}}; \qquad \eta_2 = (1+j)\sqrt{\frac{\omega\mu_2}{2\sigma_2}}$$

Power conservation

- let us find out the Poynting vector in the two regions
- Noting that the field in region I comprise of incident and reflected wave,
- we can write the Poynting vector for z <0

6.2 Plane wave reflection from media interface at normal incidence

$$\vec{S}^1 = \vec{E}_{tot}^1 \times \left(\vec{H}_{tot}^1\right)^* = \left(\vec{E}_i + \vec{E}_r\right) \times \left(\vec{H}_i + \vec{H}_r\right)^* = \left(E_0 e^{-j\beta_1 z}\hat{x} + \Gamma E_0 e^{+j\beta_1 z}\hat{x}\right) \times \left(\frac{E_0^*}{\eta_1} e^{j\beta_1 z}\hat{y} - \Gamma^* \frac{E_0^*}{\eta_1} e^{-j\beta_1 z}\hat{y}\right)$$

$$= \frac{\left|E_0\right|^2}{\eta_1}\left(1 - \Gamma^* e^{-2j\beta_1 z} + \Gamma e^{+2j\beta_1 z} - \left|\Gamma\right|^2\right)\hat{z}$$

- For z >0, only transmitted fields exist and we can write down the Poynting vector as

$$\vec{S}^2 = \vec{E}_t \times \vec{H}_t^* = \hat{x}\left(\tau E_0 e^{-\gamma z}\right) \times \left(\frac{\tau^* E_0^*}{\eta_2^*} e^{-\gamma^* z}\right)\hat{y} = \hat{z}\,\frac{\left|\tau\right|^2 \left|E_0\right|^2}{\eta_2^*} e^{-2\alpha z}$$

6.2 Plane wave reflection from media interface at normal incidence

- At z=0,

$$\vec{S}^1 = \frac{|E_0|^2}{\eta_1}\left(1-\Gamma^* + \Gamma - |\Gamma|^2\right)\hat{z}$$

$$\vec{S}^2 = \frac{|E_0|^2 |\tau|^2}{\eta_2^*}\hat{z} = \frac{4|E_0|^2 \eta_2}{(\eta_1+\eta_2)^2}\hat{z} = \frac{|E_0|^2}{\eta_1}\left(1-\Gamma^* + \Gamma - |\Gamma|^2\right)\hat{z}$$

- Complex power is conserved
- Then the time average power flow through a 1m^2 cross section can be calculated as follows
- Since

6.2 Plane wave reflection from media interface at normal incidence

$$-\Gamma^* e^{-2j\beta_1 z} + \Gamma e^{+2j\beta_1 z}$$

- is purely an imaginary number, therefore,

$$S_{avg}^1 = \frac{1}{2}\mathrm{Re}\left(\vec{S}^1 \bullet \hat{z}\right) = \frac{1}{2}\left|E_0\right|^2 \frac{1-\left|\Gamma\right|^2}{\eta_1}$$

$$S_{avg}^2 = \frac{1}{2}\mathrm{Re}\left(\vec{S}^2 \bullet \hat{z}\right) = \frac{1}{2}\left|E_0\right|^2 \frac{1-\left|\Gamma\right|^2}{\eta_2} e^{-2\alpha z}$$

- which shows power balance at z=0 only

6.2 Plane wave reflection from media interface at normal incidence

- Note that the Poynting vector for z>0,
- this power density in the lossy conductor decays exponentially according to the attenuation factor

6.2.4 Perfect conductor:

- Now assume that the region II (z >0) contains a perfect conductor and region I is a lossless medium
- Let us rewrite the expressions for propagation constant and wave impedance of a good conductor and deduce the expressions for perfect conductor

6.2 Plane wave reflection from media interface at normal incidence

$$\gamma_2 = \alpha_2 + j\beta_2 = (1+j)\sqrt{\frac{\omega\mu_2\sigma_2}{2}} = (1+j)\frac{1}{\delta_s^2}$$

$$\eta_2 = (1+j)\sqrt{\frac{\omega\mu_2}{2\sigma_2}}$$

- Perfect conductor implies $\sigma_2 \to \propto$, and
- correspondingly, $\alpha_2 \to \propto$, $\delta_s^{\,2} \to 0$, $\eta_2 \to 0$
- Note that $$\tau = \frac{2\eta_2}{\eta_1 + \eta_2}, \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
- Therefore, $\tau \to 0$ and $\Gamma \to -1$

6.2 Plane wave reflection from media interface at normal incidence

Table 6.5 Fields in the two regions (normal incidence)

Region I (lossless medium)	Region II (perfect conductor)
$\vec{E}_i = \hat{x}E_o e^{-j\beta_1 z}$ $\quad \vec{H}_i = \hat{y}\frac{1}{\eta_1}E_o e^{-j\beta_1 z}$ $\vec{E}_r = \hat{x}\Gamma E_o e^{+j\beta_1 z}$ $\quad \vec{H}_r = -\hat{y}\frac{\Gamma}{\eta_1}E_o e^{+j\beta_1 z}$	$\vec{E}_t \cong 0 \quad \vec{H}_t \cong 0$

6.2 Plane wave reflection from media interface at normal incidence

- The field for z >0 thus decay infinitely fast and
- are identically zero in the perfect conductor as we have seen in example 6.2 as well
- The perfect conductor can be thought of as 'shorting out' the incident electric field
- Now let us write down the field expressions in region I and II for lossless dielectric and perfect conductor interface (see Table 6.5)

6.2 Plane wave reflection from media interface at normal incidence

Power conservation

- In order to see the power conservation,
- let us find out the Poynting vector in the two regions
- Noting that the field in region I comprise of
  - incident and
  - reflected wave,
- we can write the Poynting vector for z <0 (Γ=-1)

$$\vec{E}_{tot}^{1} = \vec{E}_i + \vec{E}_r = \hat{x}E_0\left(e^{-j\beta_1 z} - e^{j\beta_1 z}\right) = -\hat{x}2jE_0 \sin\beta_1 z$$

6.2 Plane wave reflection from media interface at normal incidence

$$\vec{H}_{tot}^{1} = \overline{H}_{i} + \vec{H}_{r} = \hat{y}\frac{E_0}{\eta_1}\left(e^{-j\beta_1 z} + e^{j\beta_1 z}\right) = \hat{y}2\frac{E_0}{\eta_1}\cos\beta_1 z$$

- Therefore, the Poynting vector for z<0 is

$$\overline{S}^{1} = \vec{E}_{tot}^{1} \times \left(\vec{H}_{tot}^{1}\right)^{*} = -\hat{z}j\frac{4\left|E_0\right|^2}{\eta_1}\sin\beta_1 z\cos\beta_1 z$$

- which has a zero real part and
- thus indicates that no real power is delivered to the perfect conductor
- Fields in region II is anyway zero and hence power is also zero

6.3 Plane wave reflection from media interface at oblique incidence

- We will consider the problem of a plane wave
  - obliquely incident on a plane interface
  - between two lossy conducting regions
- We will first consider two particular cases of this problem as follows:
  - the electric field is in the xz plane (parallel polarization)
  - the electric field is in normal to the xz plane (perpendicular polarization)

6.3 Plane wave reflection from media interface at oblique incidence

- Any arbitrary incident plane wave can be expressed
  - as a linear combination of these two principal polarizations
- The plane of incidence is that plane containing
  - the normal vector to the interface and
  - the direction of propagation vector of the incident wave

6.3 Plane wave reflection from media interface at oblique incidence

- For Fig. 6.4, this is the xz plane
- For perpendicular polarization (TE),
  - electric field is perpendicular to the plane of incidence
- For parallel polarization (TM),
  - electric field is parallel to the plane of incidence

6.3 Plane wave reflection from media interface at oblique incidence

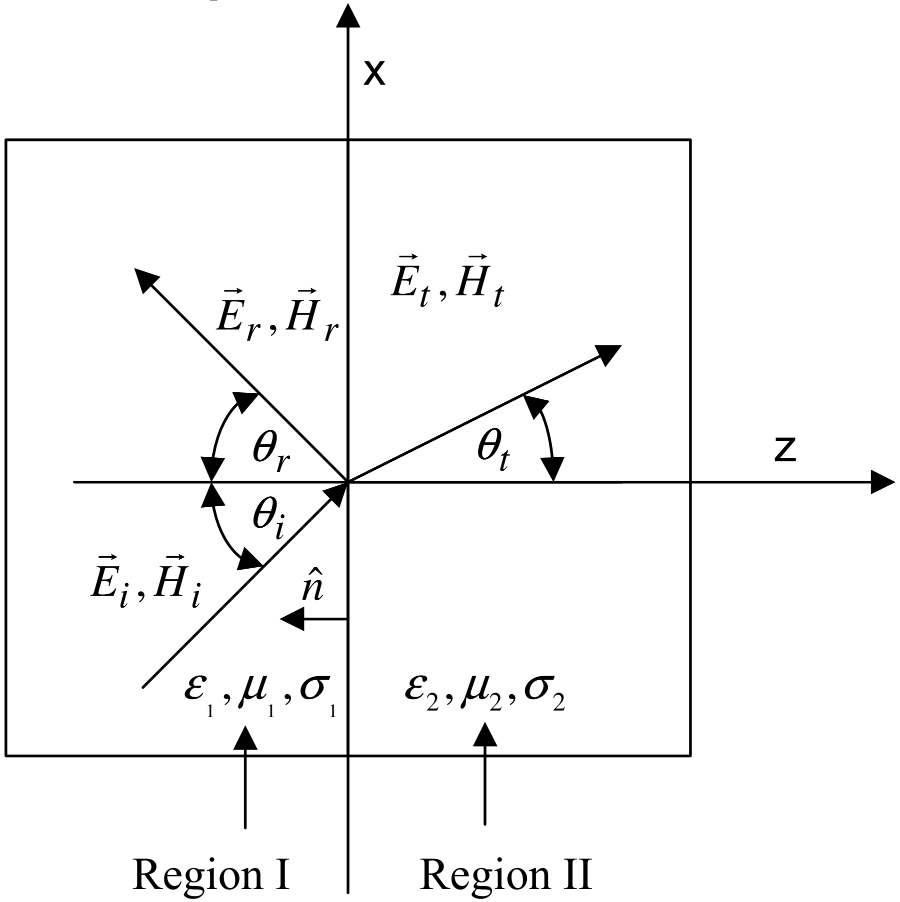

Fig. 6.5 Oblique incidence of plane EM wave at a media interface

6.3 Plane wave reflection from media interface at oblique incidence

6.3.1 Perpendicular polarization (TE):

- In this case, electric field vector is perpendicular to the xz plane,
- Hence, it will have component along the y-axis
- Since the electric field is transversal to the plane of incidence
- They are also known transverse electric (TE) waves

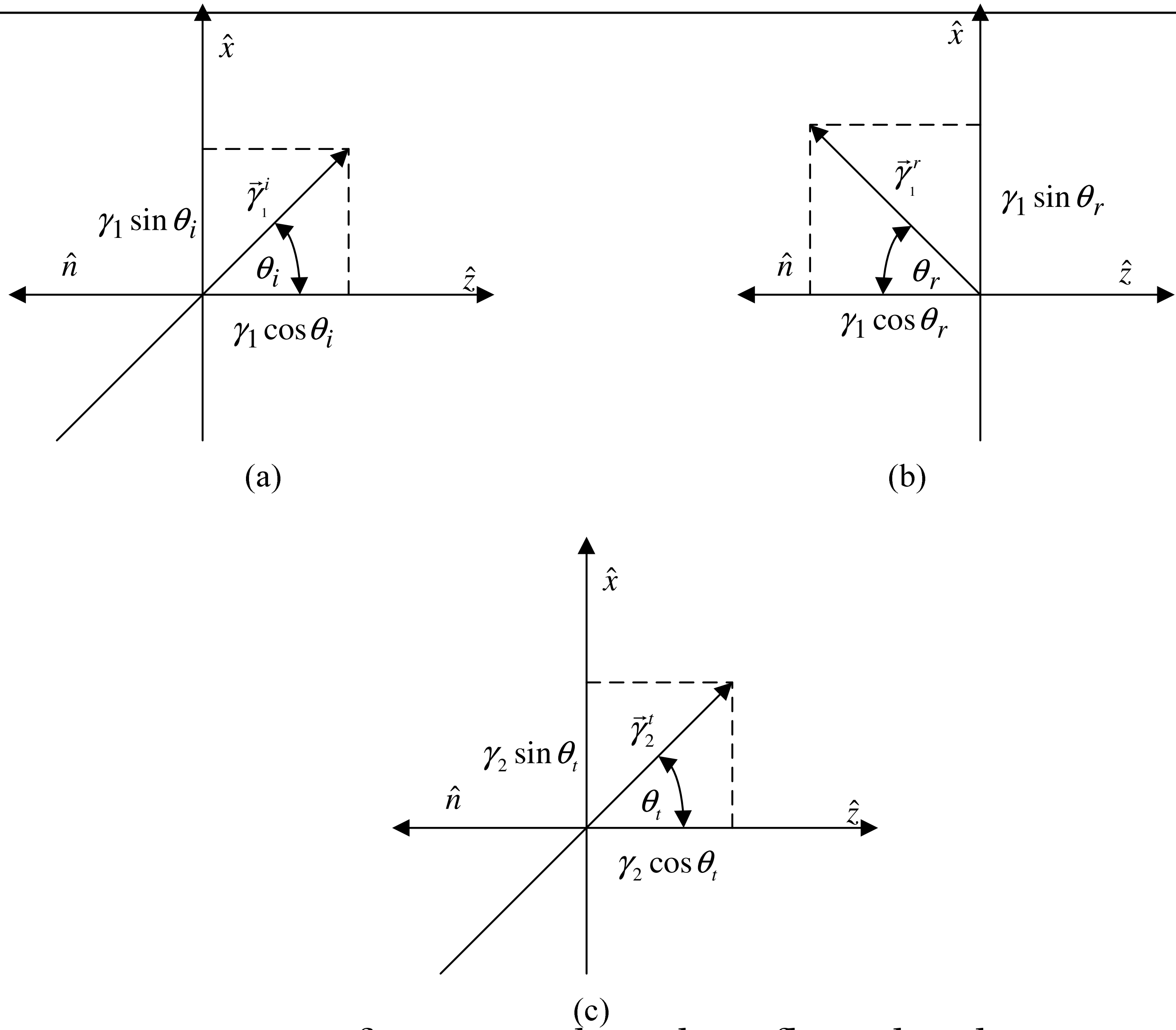

Fig. 6.6 Wave propagation vector for (a) incident (b) reflected and (c) transmitted EM waves at oblique incidence

6.3 Plane wave reflection from media interface at oblique incidence

- Let us assume that the incident wave propagates in the first quadrant of xz plane without loss of generality and
- $\vec{\gamma}_1{}^i$ (incident propagation vector) makes an angle θ_i with the normal (see Fig. 6.6 (a))

$$\vec{\gamma}_1^i \bullet \vec{z}^{'} = \left(\gamma_1 \cos\theta_i \hat{z} + \gamma_1 \sin\theta_i \hat{x}\right) \bullet \left(z\hat{z} + x\hat{x}\right) = \gamma_1 \cos\theta_i z + \gamma_1 \sin\theta_i x = \gamma_1\left(z\cos\theta_i + x\sin\theta_i\right)$$

$$\vec{E}_i = E_0 e^{-\gamma_1\left(z\cos\theta_i + x\sin\theta_i\right)}\hat{y}$$

$$\because \nabla\times\vec{E}_i = -j\omega\mu_1\vec{H}_i \Rightarrow \vec{H}_i = \frac{\nabla\times\vec{E}_i}{-j\omega\mu_1}$$

6.3 Plane wave reflection from media interface at oblique incidence

$$= \frac{1}{-j\omega\mu_1}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)} & 0 \end{vmatrix} = \frac{E_0}{-j\omega\mu_1}\left\{\left(-\frac{\partial e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}}{\partial z}\hat{x}\right) + \left(\frac{\partial e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}}{\partial x}\hat{z}\right)\right\}$$

$$= \frac{E_0}{j\omega\mu_1}\left\{\left(\frac{\partial e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}}{\partial z}\hat{x}\right) - \left(\frac{\partial e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}}{\partial x}\hat{z}\right)\right\} = \frac{E_0\gamma_1}{j\omega\mu_1} e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}\left\{-\cos\theta_i\hat{x} + \hat{z}\sin\theta_i\right\}$$

$$= \frac{E_0}{\eta_1} e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}\left(-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i\right)$$

6.3 Plane wave reflection from media interface at oblique incidence

- Let us assume that the reflected wave propagates in the second quadrant of xz plane and
- $\vec{\gamma}_1^{\,r}$ (reflected propagation vector) makes an angle θ_r with the normal (see Fig. 6.6 (b))

$$\vec{\gamma}_1^{\,r} \bullet \vec{z}\,' = \left(-\gamma_1 \cos\theta_r \hat{z} + \gamma_1 \sin\theta_r \hat{x}\right) \bullet \left(z\hat{z} + x\hat{x}\right) = -\gamma_1 \cos\theta_r z + \gamma_1 \sin\theta_r x = \gamma_1 \left(-z\cos\theta_r + x\sin\theta_r\right)$$

$$\vec{E}_r = E_0 \Gamma_{TE} e^{-\gamma_1 \left(-z\cos\theta_r + x\sin\theta_r\right)} \hat{y}$$

6.3 Plane wave reflection from media interface at oblique incidence

- Note that $\vec{\gamma}_1^{\,r}$ and $\vec{\gamma}_1^{\,i}$ will have the same magnitude
  - since both the waves are still in the same region I,
  - only their direction changes
- Since the Poynting vector must be negative like the previous case of normal incidence,

$$\vec{H}_r = \frac{E_0}{\eta_1}\Gamma_{TE}e^{-\gamma_1(-z\cos\theta_r + x\sin\theta_r)}\left(\hat{x}\cos\theta_r + \hat{z}\sin\theta_r\right)$$

- You could also use the Maxwell's curl equation below to find this

$$\vec{H}_r = \frac{\nabla\times\vec{E}_r}{-j\omega\mu_1}$$

6.3 Plane wave reflection from media interface at oblique incidence

- The transmitted fields will have similar expression with the incident fields except
  - that now the θ_i should be replaced by θ_t (angle that transmitted propagation vector makes with the normal),
  - γ_1 should be replaced by γ_2 (wave is in region II now) and
  - multiplication by (transmission coefficient)
- The transmitted fields are $\vec{E}_t = \hat{y} E_0 \tau_{TE} e^{-\gamma_2 (z\cos\theta_t + x\sin\theta_t)}$

$$\vec{H}_t = \frac{\nabla \times \vec{E}_t}{-j\omega\mu_2} = \frac{E_0 \tau_{TE}}{\eta_2} e^{-\gamma_2 (z\cos\theta_t + x\sin\theta_t)} \left(-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t\right)$$

6.3 Plane wave reflection from media interface at oblique incidence

Table 6.5 Fields in two regions (oblique incidence: perpendicular polarization)

Region I (lossy medium 1)	Region II (lossy medium 2)
$\vec{E}_i = E_0 e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}\hat{y}$ $\vec{H}_i = \frac{E_0}{\eta_1} e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}(-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)$ $\vec{E}_r = E_0\Gamma_{TE} e^{-\gamma_1(-z\cos\theta_r + x\sin\theta_r)}\hat{y}$ $\vec{H}_r = \frac{E_0\Gamma_{TE}}{\eta_1} e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}(\hat{x}\cos\theta_r + \hat{z}\sin\theta_r)$	$\vec{E}_t = \hat{y}E_0\tau_{TE} e^{-\gamma_2(z\cos\theta_t + x\sin\theta_t)}$ $\vec{H}_t = \frac{E_0\tau_{TE}}{\eta_2} e^{-\gamma_2(z\cos\theta_t + x\sin\theta_t)}(-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)$

6.3 Plane wave reflection from media interface at oblique incidence

- Equating the tangential components of electric field
  - (electric field has only E_y component and it is tangential at the interface z=0) and
- magnetic field
  - (magnetic field has two components: H_x and H_z and only H_x is tangential at the interface z=0)
- at z=0 gives

$$e^{-\gamma_1 x \sin\theta_i} + \Gamma_{TE} e^{-\gamma_1 x \sin\theta_r} = \tau_{TE} e^{-\gamma_2 x \sin\theta_t}$$

$$\frac{-1}{\eta_1}\cos\theta_i e^{-\gamma_1 x \sin\theta_i} + \frac{\Gamma_{TE}}{\eta_1}\cos\theta_r e^{-\gamma_1 x \sin\theta_r} = -\frac{\tau_{TE}}{\eta_2}\cos\theta_t e^{-\gamma_2 x \sin\theta_t}$$

6.3 Plane wave reflection from media interface at oblique incidence

- If E_x and H_y are to be continuous at the interface z = 0 for all x,
- then, this x variation must be the same on both sides of the equations (also known as *phase matching condition*)

$$\gamma_1 \sin\theta_i = \gamma_1 \sin\theta_r = \gamma_2 \sin\theta_t$$

$$\Rightarrow \theta_i = \theta_r ; \gamma_1 \sin\theta_i = \gamma_2 \sin\theta_t$$

6.3 Plane wave reflection from media interface at oblique incidence

- The first is Snell's law of reflection
  - which states that the angle of incidence equals the angle of reflection
- The second result is the Snell's law of refraction
  - (refraction is the change in direction of a wave due to change in velocity from one medium to another medium)
- Also note that refractive index of a medium is defined as

$$n = \frac{c}{v_p} = \frac{\sqrt{\mu_r \varepsilon_r \mu_0 \varepsilon_0}}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\mu_r \varepsilon_r}$$

6.3 Plane wave reflection from media interface at oblique incidence

- hence, for a lossless dielectric media,

$$\frac{\sin\theta_i}{\sin\theta_t}=\frac{\gamma_2}{\gamma_1}=\frac{\beta_2}{\beta_1}=\frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_1}}=\frac{v_1}{v_2}=\frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}=\frac{n_2}{n_1}$$

- Now we can simplify above two equations by applying Snell's two laws as follows

$$1+\Gamma_{TE}=\tau_{TE}$$

$$-\frac{\cos\theta_i}{\eta_1}+\Gamma_{TE}\frac{\cos\theta_r}{\eta_1}=-\frac{\tau_{TE}}{\eta_2}\cos\theta_t$$

6.3 Plane wave reflection from media interface at oblique incidence

- The above two equations has two unknowns τ_{TE} and Γ_{TE} and it can be solved easily as follows

$$\tau_{TE} = \left(\frac{\cos\theta_i}{\eta_1} - \Gamma_{TE} \frac{\cos\theta_r}{\eta_1} \right) \frac{\eta_2}{\cos\theta_t} \qquad 1 + \Gamma_{TE} = \tau_{TE}$$

- Therefore,

$$1 + \Gamma_{TE} = \left(\frac{\eta_2}{\eta_1} \frac{\cos\theta_i}{\cos\theta_t} - \Gamma_{TE} \frac{\eta_2}{\eta_1} \frac{\cos\theta_r}{\cos\theta_t} \right) \Rightarrow \Gamma_{TE} \left(1 + \frac{\eta_2 \cos\theta_r}{\eta_1 \cos\theta_t} \right) = \left(\frac{\eta_2 \cos\theta_i}{\eta_1 \cos\theta_t} - 1 \right)$$

$$\Rightarrow \Gamma_{TE} \left(\frac{\eta_1 \cos\theta_t + \eta_2 \cos\theta_r}{\eta_1 \cos\theta_t} \right) = \left(\frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_1 \cos\theta_t} \right) \Rightarrow \Gamma_{TE} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_r}$$

6.3 Plane wave reflection from media interface at oblique incidence

$$\therefore \tau_{TE} = 1 + \Gamma_{TE} = 1 + \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_r} = \frac{\eta_1 \cos\theta_t + \eta_2 \cos\theta_r + \eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_r} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_t + \eta_2 \cos\theta_r}$$

- Hence, the reflection and transmission coefficients for oblique incidence (*Fresnel coefficients*) for perpendicular polarization are given as follows:

$$\Gamma_{TE} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad \tau_{TE} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

- For normal incidence, it is a particular case and put

$$\theta_i = \theta_r = \theta_t = 0$$

6.3 Plane wave reflection from media interface at oblique incidence

6.3.2 Parallel Polarization (TM):

- In this case, electric field vector lies in the xz plane
- Since the magnetic field is transversal to the plane of incidence such waves are also called as transverse magnetic (TM) waves
- So let us start with H vector which will have only y component,

$$\vec{H}_i = \hat{y}\frac{E_0}{\eta_1}e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}$$

6.3 Plane wave reflection from media interface at oblique incidence

$$\because \nabla\times\vec{H} = j\omega\varepsilon_1\vec{E} \Rightarrow \vec{E}_i = \frac{\nabla\times\vec{H}_i}{j\omega\varepsilon_1}$$

$$= \frac{1}{j\omega\varepsilon_1}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{E_0}{\eta_1}e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)} & 0 \end{vmatrix} = \frac{E_0}{j\omega\varepsilon_1\eta_1}\left\{-\left(\frac{\partial e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}}{\partial z}\hat{x}\right) + \left(\frac{\partial e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}}{\partial x}\hat{z}\right)\right\}$$

$$= \frac{E_0\gamma_1}{j\omega\varepsilon_1\eta_1}e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}\left\{\cos\theta_i\hat{x} - \hat{z}\sin\theta_i\right\} = E_0 e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}\left(\cos\theta_i\hat{x} - \hat{z}\sin\theta_i\right)$$

6.3 Plane wave reflection from media interface at oblique incidence

- Similar to the previous case of parallel polarization, we can write the reflected magnetic field vector as

$$\vec{H}_r = -\hat{y}\frac{E_0\Gamma_{TM}}{\eta_1}\hat{y}e^{-\gamma_1(-z\cos\theta_r + x\sin\theta_r)}$$

- The transmitted fields will have similar expression with the incident fields except that
  - now the θ_i should be replaced by θ_t (angle that transmitted propagation vector makes with the normal),
  - γ_1 should be replaced by γ_2 (wave is in region II) and
  - multiplied by a factor (transmission coefficient for TM case)

6.3 Plane wave reflection from media interface at oblique incidence

$$\vec{E}_t = E_0 \tau_{TM} e^{-\gamma_2 (z\cos\theta_t + x\sin\theta_t)} (\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)$$

$$\vec{H}_t = \hat{y} \frac{\tau_{TM} E_0}{\eta_2} e^{-\gamma_2 (z\cos\theta_t + x\sin\theta_t)}$$

Table 6.6 Fields in two regions (oblique incidence: parallel polarization) {next page}

6.3 Plane wave reflection from media interface at oblique incidence

Region I (lossy medium 1)	Region II (lossy medium 2)
$\vec{E}_i = E_0 e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}(\cos\theta_i\hat{x} - \hat{z}\sin\theta_i)$ $\vec{H}_i = \hat{y}\dfrac{E_0}{\eta_1}e^{-\gamma_1(z\cos\theta_i + x\sin\theta_i)}$ $\vec{E}_r = E_0\Gamma_{TM}e^{-\gamma_1(-z\cos\theta_r + x\sin\theta_r)}\{\cos\theta_r\hat{x} + \hat{z}\sin\theta_i\}$ $\vec{H}_r = -\hat{y}\dfrac{E_0\Gamma_{TM}}{\eta_1}\hat{y}e^{-\gamma_1(-z\cos\theta_r + x\sin\theta_r)}$	$\vec{E}_t = E_0\tau_{TM}e^{-\gamma_2(z\cos\theta_t + x\sin\theta_t)}(\hat{x}\cos\theta_t - \hat{z}\sin\theta_t)$ $\vec{H}_t = \hat{y}\dfrac{\tau_{TM}E_0}{\eta_2}e^{-\gamma_2(z\cos\theta_t + x\sin\theta_t)}$

6.3 Plane wave reflection from media interface at oblique incidence

- Equating the tangential components of magnetic field
  - (magnetic field has only H_y component and it is tangential at the interface z=0) and
- electric field
  - (electric field has two components: E_x and E_z and only E_x is tangential at the interface z=0)
- at z=0 gives

$$e^{-\gamma_1 x \sin\theta_i}\cos\theta_i + e^{-\gamma_1 x \sin\theta_r}\Gamma_{TM}\cos\theta_r = \tau_{TM} e^{-\gamma_2 x \sin\theta_t}\cos\theta_t$$

$$\frac{e^{-\gamma_1 x \sin\theta_i}}{\eta_1} - \frac{\Gamma_{TM} e^{-\gamma_1 x \sin\theta_r}}{\eta_1} = \frac{\tau_{TM} e^{-\gamma_2 x \sin\theta_t}}{\eta_2}$$

6.3 Plane wave reflection from media interface at oblique incidence

- Therefore,

$$\gamma_1 \sin\theta_i = \gamma_1 \sin\theta_r = \gamma_2 \sin\theta_t$$

$$\Rightarrow \theta_i = \theta_r ; \gamma_1 \sin\theta_i = \gamma_2 \sin\theta_t$$

- which is Snell's law of reflection and refraction and this implies that

$$\cos\theta_i + \Gamma_{TM} \cos\theta_r = \tau_{TM} \cos\theta_t$$

$$\frac{1}{\eta_1}\left(1-\Gamma_{TM}\right) = \frac{\tau_{TM}}{\eta_2}$$

6.3 Plane wave reflection from media interface at oblique incidence

- The above two equations has two unknowns τ_{TM} and Γ_{TM} and it can be easily solved as follows

$$\tau_{TM} = \frac{\cos\theta_i + \Gamma_{TM}\cos\theta_r}{\cos\theta_t} \qquad \tau_{TM} = \frac{\eta_2}{\eta_1}\left(1-\Gamma_{TM}\right)$$

- Therefore,

$$\frac{\cos\theta_i + \Gamma_{TM}\cos\theta_r}{\cos\theta_t} = \frac{\eta_2}{\eta_1}\left(1-\Gamma_{TM}\right) \Rightarrow \frac{\eta_2}{\eta_1}\left(1-\Gamma_{TM}\right) = \frac{\cos\theta_i}{\cos\theta_t} + \frac{\Gamma_{TM}\cos\theta_r}{\cos\theta_t}$$

$$\Rightarrow \Gamma_{TM}\left(-\frac{\eta_2}{\eta_1} - \frac{\cos\theta_r}{\cos\theta_t}\right) = \left(\frac{\cos\theta_i}{\cos\theta_t} - \frac{\eta_2}{\eta_1}\right) \Rightarrow \Gamma_{TM}\left(\frac{-\eta_2\cos\theta_t - \eta_1\cos\theta_r}{\eta_1\cos\theta_t}\right) = \left(\frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_t}\right)$$

$$\Rightarrow \Gamma_{TM} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_r} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

6.3 Plane wave reflection from media interface at oblique incidence

$$\therefore \tau_{TM} = \frac{\eta_2}{\eta_1}(1-\Gamma_{TM}) = \frac{\eta_2}{\eta_1}\left(1-\frac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}\right) = \frac{\eta_2}{\eta_1}\left(\frac{\eta_2\cos\theta_t+\eta_1\cos\theta_i-\eta_2\cos\theta_t+\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}\right)$$

$$= \frac{\eta_2}{\eta_1}\left(\frac{2\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}\right) = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}$$

- Hence, the reflection and transmission coefficients for oblique incidence (*Fresnel coefficients*) for parallel polarization are given as follows:

$$\therefore \Gamma_{TM} = \frac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i} \quad \tau_{TM} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}$$

- Note that cosine terms multiplication in the above equations is different from the previous expression for TE waves

6.3 Plane wave reflection from media interface at oblique incidence

6.3.3 Brewster angle:

- For $\Gamma=0$, the angle of incidence is known as Brewster angle (θ_B)
- This means at this angle of incidence of the EM wave from region I,
  - there will be no reflection from region II,
  - all the EM waves will be absorbed

6.3 Plane wave reflection from media interface at oblique incidence

- *For parallel polarization,*

$$\theta_i = \theta_B^{TM}$$

$$\Gamma_{TM} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = 0$$

$$\Rightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_B^{TM}$$

- Also we have from 2nd Snell's law

$$\gamma_1 \sin\theta_B^{TM} = \gamma_2 \sin\theta_t$$

6.3 Plane wave reflection from media interface at oblique incidence

- Using the above two equations, we can get (see textbook)

$$\sin^2\theta_B^{TM} = \left\{ \frac{\dfrac{\eta_1^2}{\eta_2^2} - 1}{\dfrac{\eta_1^2}{\eta_2^2} - \dfrac{\gamma_1^2}{\gamma_2^2}} \right\}$$

- For lossless media, we have,

$$\because \frac{\eta_1^2}{\eta_2^2} = \frac{\mu_1^2\gamma_2^2}{\gamma_1^2\mu_2^2} = \frac{\mu_1^2\mu_2\varepsilon_2}{\mu_1\varepsilon_1\mu_2^2} = \frac{\mu_1\varepsilon_2}{\varepsilon_1\mu_2}; \frac{\gamma_1^2}{\gamma_2^2} = \frac{\mu_1\varepsilon_1}{\varepsilon_2\mu_2} \therefore \sin^2\theta_B^{TM} = \frac{\dfrac{\mu_1\varepsilon_2}{\varepsilon_1\mu_2} - 1}{\dfrac{\mu_1\varepsilon_2}{\varepsilon_1\mu_2} - \dfrac{\mu_1\varepsilon_1}{\varepsilon_2\mu_2}}$$

6.3 Plane wave reflection from media interface at oblique incidence

- For magnetic media, $\varepsilon_1 = \varepsilon_2$
- $\sin^2 \theta_B^{TM}$ is of the form 1/0 and Brewster angle does not exist
- But, for dielectric media, it exist and can be calculated as follows

$$\Rightarrow \sin \theta_B^{TM} = \sqrt{\frac{\frac{\varepsilon_2}{\varepsilon_1} - 1}{\frac{\varepsilon_2}{\varepsilon_1} - \frac{\varepsilon_1}{\varepsilon_2}}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_2 + \varepsilon_1}} \Rightarrow \tan \theta_B^{TM} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}$$

6.3 Plane wave reflection from media interface at oblique incidence

- *For perpendicular polarization,*
- Similarly, Brewster's angle doesn't exist for perpendicular polarization at oblique incidence for a non-magnetic or dielectric medium (see textbook for derivation)
- For magnetic medium

$$\therefore \sin^2\theta_B^{TE} = \frac{\mu_2\mu_1 - (\mu_2)^2}{(\mu_1)^2 - (\mu_2)^2} = \frac{\mu_2}{\mu_1+\mu_2} \Rightarrow \sin\theta_B^{TE} = \sqrt{\frac{\mu_2}{\mu_1+\mu_2}}; \tan\theta_B^{TE} = \sqrt{\frac{\mu_2}{\mu_1}} = \frac{n_2}{n_1}$$

6.3 Plane wave reflection from media interface at oblique incidence

- Brewster angle exist for oblique incidence (perpendicular polarization) for interface between two magnetic media
- An incident wave consisting of both polarizations will have reflected wave for only one polarization for Brewster angle
- Brewster angle is also known as **Polarizing angle** (used in optics and lasers)
- In other words, an incident wave which is composed of both TE and TM waves will have reflected TE (TM) waves only for dielectric (magnetic) media interface
- Thus, a circularly polarized incident wave at polarizing angle will become a linearly polarized wave

6.3 Plane wave reflection from media interface at oblique incidence

6.3.4 Total internal reflection:

- Used in optical fibers
- Light strikes an angle of incidence greater than critical angle,
  - all of the light energy will be reflected back, and
  - none of them will be absorbed
- This critical angle is related to the refractive index of the two media and is given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

- where are the refractive index of medium 2 and 1 respectively

6.3 Plane wave reflection from media interface at oblique incidence

- In fact, this can be observed for both the polarizations we have discussed above
- From Snell's Second law, we have,

$$\gamma_1 \sin\theta_i = \gamma_2 \sin\theta_t \Rightarrow \sin\theta_i = \frac{\gamma_2 \sin\theta_t}{\gamma_1}$$

- If we consider non-magnetic materials (dielectrics) of , in that case,

$$\Rightarrow \sin\theta_i = \frac{\sqrt{\varepsilon_2}\sin\theta_t}{\sqrt{\varepsilon_1}}$$

6.3 Plane wave reflection from media interface at oblique incidence

- Hence, for

$$\theta_i = \theta_c, \theta_t = \frac{\pi}{2}$$

- (it means the transmitted wave travels along the interface, for our case, it is x-axis) which implies that

$$\Rightarrow \sin\theta_c = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}} \Rightarrow \theta_c = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

6.3 Plane wave reflection from media interface at oblique incidence

- We can write the above relation as

$$\theta_c = \sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right) = \sin^{-1}\left(\sqrt{\frac{\mu_0\mu_{r2}\varepsilon_0\varepsilon_{r2}}{\mu_0\mu_{r1}\varepsilon_0\varepsilon_{r1}}}\right) = \sin^{-1}\left(\sqrt{\frac{\mu_{r2}\varepsilon_{r2}}{\mu_{r1}\varepsilon_{r1}}}\right) = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

- Note that $\varepsilon_1 > \varepsilon_2$ to have a real value of critical angle
- Besides, we have assumed that for a dielectric, $\mu_{r1} = \mu_{r2} = 1$

6.3 Plane wave reflection from media interface at oblique incidence

- What will happen when the angle of incidence is greater than the critical angle?
- For all angles greater than the critical angle,
  - using Snell's second law, and
  - noting that waves are moving from a denser to a rarer dielectric medium,

$$\mu_{r1} = \mu_{r2} = 1 \qquad \varepsilon_1 > \varepsilon_2$$

$$\because \theta_i > \theta_c \therefore \sin\theta_t = \frac{\sqrt{\mu_1\varepsilon_1}\sin\theta_i}{\sqrt{\mu_2\varepsilon_2}} > 1 \Rightarrow \cos\theta_t = \pm\sqrt{1-\sin^2\theta_t}$$

  - The value of $\cos\theta_t$ should be imaginary

6.3 Plane wave reflection from media interface at oblique incidence

- It can be shown that (see textbook for derivation)

$$\therefore \vec{S} = \vec{E} \times \vec{H}^* = \hat{x}\frac{|E_0\tau|^2}{\eta_2} e^{-j2\beta_2 z\cos\theta_t} \sin\theta_t + \hat{z}\frac{|E_0\tau|^2}{\eta_2} e^{-j2\beta_2 z\cos\theta_t} \cos^*\theta_t$$

- For z>0, since $\cos\theta_t$ is an imaginary number, $\cos^*\theta_t$ is also purely imaginary
- Hence, the second term is imaginary, therefore, no real power is flowing along z-axis in region II
- The first term which has direction along x-axis is real and is exponentially decaying with z

6.3 Plane wave reflection from media interface at oblique incidence

- Hence,
- When a wave is incident at an angle of incidence greater than or equal to the critical angle from region I,
- the wave will be total internally reflected in region I
- Surface waves exist in the region II,
- which is propagating along x-axis (direction of power flow), and
- attenuating very fast along z-axis
- People have used this for communications using submarines

6.3 Plane wave reflection from media interface at oblique incidence

6.3.5 Effects on polarization:

- Since any arbitrary wave incident obliquely from region I can be decomposed into linear combinations of TE and TM waves and
- we also know that the expressions for reflection and transmission coefficients for parallel and perpendicular polarizations are different

$$\Gamma_{TE} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}; \qquad \Gamma_{TM} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

6.3 Plane wave reflection from media interface at oblique incidence

- So, any circularly polarized incident wave will
  - become elliptically polarized after reflection or transmission
- Similarly, a linearly polarized wave (note that linear polarization could be expressed as linear combination of RHCP and LHCP) may be
  - rotated after reflection

6.3 Plane wave reflection from media interface at oblique incidence

- For instance,
- If the region II is a perfect conductor, $\eta_2=0$, $\Gamma=-1$ for both TE and TM cases,
  - this means a RHCP wave after reflection will become LHCP wave and vice versa
  - This is why circular polarization is not used for indoor communication systems since we have many reflections inside a room

6.4 Summary

Plane waves reflection from a media interface

Normal incidence

Lossy conducting medium

$$\Gamma=\frac{\eta_2-\eta_1}{\eta_2+\eta_1};\quad \tau=\frac{2\eta_2}{\eta_1+\eta_2}$$

Lossless medium

$$S_{avg}^{1}=S_{avg}^{2}$$

$$S_{avg}^{1}=\frac{1}{2}\mathrm{Re}\left(\vec{S}^{1}\bullet\hat{z}\right)=\frac{1}{2}\left|E_0\right|^2\frac{1-\Gamma^2}{\eta_1}$$

Good conductor

$$S_{avg}^{2}=\frac{1}{2}\mathrm{Re}\left(\vec{S}^{+}\bullet\hat{z}\right)=\frac{1}{2}\left|E_0\right|^2\frac{1-\left|\Gamma\right|^2}{\eta_2}e^{-2\alpha z}$$

Perfect conductor

$\tau \to 0$ and $\Gamma \to -1$

Oblique incidence

TE

$$\Gamma_{TE}=\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}$$

$$\tau_{TE}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}$$

TM

$$\therefore\Gamma_{TM}=\frac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}$$

$$\tau_{TM}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}$$

Brewster angle

$$\tan\theta_B^{TM}=\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}=\frac{n_2}{n_1}$$

$$\tan\theta_B^{TE}=\sqrt{\frac{\mu_2}{\mu_1}}=\frac{n_2}{n_1}$$

Effect on polarization

CP→EP

Total internal reflection

$$\theta_c=\sin^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

$$\because\theta_i>\theta_c\ \therefore\sin\theta_t=\frac{\sqrt{\mu_1\varepsilon_1}\sin\theta_i}{\sqrt{\mu_2\varepsilon_2}}>1\Rightarrow\cos\theta_t=\pm\sqrt{1-\sin^2\theta_t}$$

Fig. 6.7 Plane waves reflection from media interface in a nutshell