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To cite this article: S. Bhattacharjee, R.S. Kshetrimayum & R. Bhattacharjee (2015) Derivation of potential Green functions for ungrounded dielectric slab and its application in full wave analysis of PMAs, Journal of Electromagnetic Waves and Applications, 29:16, 2242-2256, DOI: 10.1080/09205071.2015.1095130

To link to this article: http://dx.doi.org/10.1080/09205071.2015.1095130

Published online: 13 Nov 2015.

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Derivation of potential Green functions for ungrounded dielectric slab and its application in full wave analysis of PMAs

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(Received 1 June 2015; accepted 9 September 2015)

In this paper, spatial domain potential Green’s functions of a horizontal electric dipole lying on an ungrounded dielectric slab is derived. The full wave technique using mixed potential integral equation (MPIE), involving the derived potential Green’s functions and method of moment (MoM), is used to calculate input impedance as well as reflection coefficient of printed monopole antennas (PMAs). The performance of rectangular and F-shaped PMAs is evaluated using the earlier mentioned full wave technique. The computed results for the input impedance and reflection coefficient of rectangular PMA are verified with high-frequency structure simulator (HFSS) results. Further, reflection coefficient of F-shaped PMA is computed using MPIE–MoM which is verified by HFSS simulation and experimental results.

Keywords: horizontal electric dipole; potential Green’s function; mixed potential integral equation (MPIE); printed monopole antennas

1. Introduction

The electromagnetic field of a horizontal electric dipole (HED) in multilayered dielectric media has applications in theoretical analysis of antenna, geophysical analysis, and radar. The present work is based on the derivation of potential (scalar and vector) Green’s function for a HED on an ungrounded dielectric slab for the analysis of printed monopole antennas (PMAs), using mixed potential integral equation and method of moments (MPIE–MoM). MPIE–MoM is preferred over traditional electric field integral equation (EFIE) because of the fact that both scalar and vector potentials vary as \(1/R\), where \(R\) represents the distance of the observation point from the antenna and therefore less singular compared to traditional EFIE.[1] The exact Green’s function formulation in different dielectric media required for full wave analysis of microstrip antenna (MSA) are available in [2–5]. In MPIE, Green’s function needs to satisfy the boundary condition on the patch metallization. A simple PMA, which is structurally similar to a microstrip line-fed patch antenna with the difference that the ground plane below the patch is removed. Thus, the metallic patch of PMA can be modeled analytically in terms of unit HED lying on a dielectric slab which is not backed by a ground plane. By applying boundary conditions on the tangential electric and magnetic field components at the two interfaces: first between air and the face of the dielectric slab on which the HED is kept and the second between the opposite face of the same dielectric slab and air, the field components are then determined in the spectral

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domain. The exact magnetic vector potential is obtained from magnetic field components
and then using Lorentz Gauge condition, one can evaluate the scalar potential. It may be
mentioned here that the scalar and vector potential Green’s function expressions which are
derived here are the main contribution of the paper, as existing literature to the best of the
knowledge of the authors does not report such expressions. After getting both scalar and
vector potential in spectral domain, spatial domain counterpart can be obtained using inverse
Fourier transform. Integral which represents inverse Fourier transform here is an integral of
Sommerfeld type. Solutions of such type of integral are discussed in literatures [6–8]. After
the integral is being evaluated, the MPIE involving spatial domain vector and scalar potential
Green’s function can be solved for unknown current distribution over the metallic patch
using Galerkin’s method. Resulting current distribution is used to calculate input impedance
and reflection coefficient of the rectangular PMA and the results are compared using Ansoft
high-frequency structure simulator (HFSS). HFSS results are in good agreement with the
one computed using the proposed MPIE–MoM approach. An F-shaped PMA is also analyzed
using the proposed MPIE–MoM approach and the theoretical results are in good agreement
with HFSS and experimental results reported in [9]. The rest of the paper is organized as
follows. In Section 2, by applying electric and magnetic field boundary conditions at the
interfaces, potentials are calculated from field components and then evaluated the same
in spatial domain using inverse Fourier transform which gives the standard Sommerfeld
Integral. In Section 3, we evaluated the Sommerfeld integral numerically, and then applied
it into MPIE to calculate unknown current over the metallic patch. The resulting current
distribution is used to calculate the input impedance as well as the reflection coefficient of
rectangular PMA. Reflection coefficient plot of F-shaped PMA (comparison of experi-
mental, HFSS and MPIE–MoM results) is presented in the Section 4. Conclusions are drawn in
Section 5.

2. Potential Green’s functions of a HED over ungrounded dielectric slab

Figure 1 depicts a PMA of arbitrary shape printed on a lossless dielectric slab of relative
permittivity $\epsilon_r$ and thickness $h$. It may be noted that PMA has a ground plane extending up
to the feed line only i.e. the ground plane under the patch has been removed. Hence, this
structure of PMA is different from the conventional MSA.

Therefore, the potential Green’s function for PMA will be inherently different from
that of MSA.[10] In the next section, we derive the spectral domain potential Green’s
functions.
2.1. Derivation of potential Green’s functions in spectral domain for an HED over ungrounded dielectric slab

Green’s function is used in full wave analysis to satisfy the boundary condition over the patch metallization, which is modeled in terms of a HED. So, in order to derive spectral domain Green’s function, an HED lying on a lossless and ungrounded dielectric slab is considered first since the ground plane under the patch has been removed for PMAs as shown in Figure 2. It may be noted that the case of an HED on a grounded dielectric substrate has been well investigated in the literature [10].

To derive potential Green’s function, let us consider an HED on a lossless dielectric slab located at \((x_0, y_0)\) shown in Figure 2, the \(x\)-directed current is defined as \(J_x = \delta(x - x_0)(y - y_0)\) and the effect of \(J_x\) is considered through boundary condition. The source-free Maxwell’s equations in frequency domain can be written as,

\[
\nabla \times \tilde{E} = -j\omega \mu_0 \tilde{H} \quad (1a)
\]

\[
\nabla \times \tilde{H} = j\omega \epsilon_0 \epsilon_r \tilde{E} \quad (1b)
\]

The above equations can be explicitly written after Fourier transform is applied to (1a) \((\frac{\partial}{\partial x} \rightarrow jk_x, \frac{\partial}{\partial y} \rightarrow jk_y)\) as,

\[
jk_y \tilde{E}_z - \frac{\partial \tilde{E}_y}{\partial z} = -j\omega \mu_0 \tilde{H}_x \quad (2a)
\]

\[
\frac{\partial \tilde{E}_x}{\partial z} - jk_x \tilde{E}_z = -j\omega \mu_0 \tilde{H}_y \quad (2b)
\]

\[
jk_x \tilde{E}_y - jk_y \tilde{E}_x = -j\omega \mu_0 \tilde{H}_z \quad (2c)
\]

Similarly for (1b) can be given as

\[
jk_y \tilde{H}_z - \frac{\partial \tilde{H}_y}{\partial z} = j\omega \epsilon_0 \epsilon_r \tilde{E}_x \quad (3a)
\]

\[
\frac{\partial \tilde{H}_x}{\partial z} - jk_x \tilde{H}_z = j\omega \epsilon_0 \epsilon_r \tilde{E}_y \quad (3b)
\]

\[
jk_x \tilde{H}_y - jk_y \tilde{H}_x = j\omega \epsilon_0 \epsilon_r \tilde{E}_z \quad (3c)
\]

Now, after differentiating (2b) w.r.t \(z\), the equation can be rewritten as

\[
\frac{\partial \tilde{H}_y}{\partial z} = \frac{jk_x}{\omega \mu_0} \frac{\partial \tilde{E}_z}{\partial z} - \frac{1}{\omega \mu_0} \frac{\partial^2 \tilde{E}_x}{\partial z^2} \quad (4)
\]

The term \(\frac{\partial \tilde{H}_y}{\partial z}\) in (4) can be replaced by (3a) and then after some simple algebraic calculation we get [2],

\[
k_p^2 \tilde{E}_x = jk_x \tilde{H}_z - \omega \mu_0 k_y \tilde{H}_z \quad (5)
\]

Similarly, the following expressions can also be obtained:

\[
k_p^2 \tilde{E}_y = jk_y \tilde{H}_z + \omega \mu_0 k_x \tilde{H}_z \quad (6a)
\]

\[
k_p^2 \tilde{H}_x = jk_x \tilde{H}_z + \omega \epsilon_0 \epsilon_r k_y \tilde{E}_z \quad (6b)
\]

\[
k_p^2 \tilde{H}_y = jk_y \tilde{H}_z - \omega \epsilon_0 \epsilon_r k_x \tilde{E}_z \quad (6c)
\]

Here, \(\tilde{E}\) and \(\tilde{H}\) are Fourier transform representation of the fields, \(\hat{E}\) and \(\hat{H}\) are spatial derivatives of Fourier transform representation of the fields. For the present case, since there
is no ground plane, we need to divide the equivalent antenna structure into three regions. Region 2 and Region 0 signifies region above the top and below the bottom region of the dielectric slab, respectively, whereas Region 1 denotes the dielectric slab itself as shown in Figure 2. The general solutions assumed for the Regions 0, 1, and 2 are as follows. For Region 2,

\[ \tilde{E}_z^2 = Ae^{-u^2z} \quad \text{for } z > h \]  
\[ \tilde{H}_z^2 = Be^{-u^2z} \quad \text{for } z > h \]

where \( u^2 = -k^2_z = k^2_p - k^2_0 \) and \( \text{Im}(u_2) > 0 \).

For Region 1,

\[ \tilde{E}_z^1 = C \cosh(u^1z) + D \sinh(u^1z) \quad \text{for } 0 \leq z \leq h \]  
\[ \tilde{H}_z^1 = E \sinh(u^1z) + F \cosh(u^1z) \quad \text{for } 0 \leq z \leq h \]

where \( u^1 = -k^2_z = k^2_p - \varepsilon_r k^2_0 \) and \( \text{Im}(u_1) > 0 \).

For Region 0,

\[ \tilde{E}_z^0 = Me^{\mu_0z} \quad \text{for } z < 0 \]  
\[ \tilde{H}_z^0 = Ne^{\mu_0z} \quad \text{for } z < 0 \]

where \( u_0^2 = -k^2_z = k^2_p - k^2_0 \), \( k^2_p = k^2_x + k^2_y \) and \( \text{Im}(u_0) > 0 \).

Now, in order to find the arbitrary constants \( A, B, C, D, E, F, M, \) and \( N \) respectively, the boundary conditions are applied as follows: (i) \( \tilde{E}_x, \tilde{E}_y \) and \( \tilde{H}_x, \tilde{H}_y \) are continuous at the interface between Region 0 and 1 (\( z = 0 \)), (ii) \( \tilde{E}_x, \tilde{E}_y, \) and \( \tilde{H}_x \) are continuous, whereas \( \tilde{H}_y \) is discontinuous by an amount of \( \tilde{J}_x \) at the interface between Regions 1 and 2 (\( z = h \)).

After applying the above boundary conditions in (5) to (6c) through (7) to (12), we get,

\[ \tilde{E}_z^2 = \frac{j_\chi k_x e^{-u^2(z-h)}}{\omega \varepsilon_0 DT_M} \]  
\[ \tilde{H}_z^2 = \frac{-j_\chi k_y e^{-u^2(z-h)}}{DT_E} \]

where \( DT_M = 1 + \frac{\mu_0 \varepsilon_\chi (u_1 + u_0 \varepsilon_\chi \tan(u_1 h))}{\varepsilon_\chi (u_1 + u_0 \varepsilon_\chi + u_1 \tan(u_1 h))} \) and \( DT_E = u_2 + \frac{(u_0 + u_1 \tan(u_1 h))}{(1 + \frac{u_0}{u_1} \tan(u_1 h))} \). It may be noted that the above expressions for \( DT_E \) and \( DT_M \) are different from that of the grounded dielectric slab in [10].
The relation required to derive vector potential Green’s functions using magnetic field components can be expressed as

\[ \mu_0 \vec{H} = \nabla \times \vec{A} \]  

(15)

Now, (15) can be expressed in frequency domain as

\[ \mu_0 \tilde{H}_x = -jk_y \tilde{A}_z \] (16a)

\[ \mu_0 \tilde{H}_y = \frac{\partial \tilde{A}_x}{\partial z} - jk_x \tilde{A}_z \] (16b)

\[ \mu_0 \tilde{H}_z = -jk_y \tilde{A}_x \] (16c)

Now, (13) and (14) using (16a) to (16c), spectral domain vector potential Green’s function at the interface \((z = h)\) can be found as:

\[ \tilde{G}_{xx}^A = \frac{\mu_0}{2\pi D_{TE}} \left[ \frac{1}{u_1} + \frac{u_0(1-\tanh^2(u_1h))}{(u_0\epsilon_r+u_1 \tanh(u_1h))(u_1+u_0 \tanh(u_1h))} \right] \]  

(17)

\[ \tilde{G}_{zx}^A = -j \mu_0 \left( 1 - \epsilon_r \right) k_x \tanh(u_1h) \left[ \frac{1}{u_1} + \frac{u_0(1-\tanh^2(u_1h))}{(u_0\epsilon_r+u_1 \tanh(u_1h))(u_1+u_0 \tanh(u_1h))} \right] \]  

(18)

Spectral domain scalar potential Green’s function can be derived from vector potential components using Lorentz Gauge condition \(\nabla \cdot \vec{A} + j\omega \mu_0 \epsilon_0 V = 0\), and the same can be obtained as

\[ \tilde{G}_V = \frac{1}{2\pi \epsilon_0} \left[ \frac{D_{TM} - (\epsilon_r - 1) u_2 \tanh(u_1h)}{D_{TE} D_{TM}} \left\{ \frac{1}{u_1} + \frac{u_0(1-\tanh^2(u_1h))}{(u_0\epsilon_r+u_1 \tanh(u_1h))(u_1+u_0 \tanh(u_1h))} \right\} \right] \]  

(19)

It may be pointed out here that the above expressions for vector potential \((\tilde{G}_{xx}^A \text{ and } \tilde{G}_{zx}^A)\) and scalar potential Green’s functions \((\tilde{G}_V)\) are not readily available in existing literature and they are different from potential Green’s function of HED on a grounded dielectric substrate.

The above expressions in (17) and (19) are the spectral domain Green’s function for ungrounded dielectric slab. But in order to solve MPIE spatial domain Green’s function is required. To get spatial domain Green’s function from its spectral domain counterpart, inverse Fourier transform or Fourier–Bessel transform is performed, and the same is given by

\[ f(\rho) = \int_0^\infty J_0(k\rho) \tilde{f}(k) k \rho dk \rho \]  

(20)

where \(\rho\) is the radial distance between source and the observer and \(J_0(\cdot)\) is the Bessel function of first kind. Now, using (20) spatial domain vector and scalar potential Green’s function can be expressed as
\[ G^{xx}_A = \frac{\mu_0}{2\pi} \int_0^\infty \tilde{G}^{xx}_A J_0(k_\rho \rho) k_\rho d\rho \] (21)

\[ G_V = \frac{1}{2\pi\epsilon_0} \int_0^\infty \tilde{G}_V J_0(k_\rho \rho) k_\rho d\rho \] (22)

3. Numerical evaluation of spatial domain vector and scalar potential Green’s functions

Both (21) and (22) are spatial domain vector and scalar potential Green’s function, for HED on a lossless ungrounded dielectric slab as shown in Figure 2 and have the nature of Sommerfeld integral. Generally, such integrals are numerically computed as they cannot be solved analytically. Many authors have already discussed about the solution of such type of integral using different methods. One such method is matrix pencil method discussed in detail in [11,12]. Another method used complex residue approach for singularity extraction,[6,13] in which poles appear due to zeros in denominator of the integrand. These poles make the integrand highly oscillatory, thus poles are eliminated from the integrand using residue theorem to make the integration smooth. In the next section, spatial domain integrals of Sommerfeld type which arises in (21) and (22) are discussed in detail.

3.1. Approximation of vector and scalar potential Green’s function for a HED on an ungrounded dielectric slab

Since spatial domain potential (vector and scalar) Green’s function in integral form have no analytical solution, such integrals are evaluated numerically. To evaluate the integrals, the integrand needs to be examined carefully in order to locate and avoid the pole(s). Roots of the functions \( D_{TE} \) and \( D_{TM} \) appear as poles in the integrand. In Figure 3, magnitude of \( D_{TE} \) and \( D_{TM} \) for HED on an ungrounded dielectric slab is plotted as a function of normalized wave number. From Figure 3, it can be observed that the term \( D_{TE} \) has one root between \( k_0 \) and 1.5\( k_0 \), i.e. there is one pole in the integrand of spatial domain magnetic vector potential Green’s function \( G^{xx}_A \).

The expression for spectral domain vector potential Green’s function for very thin substrate with moderate dielectric constant can be obtained by setting \( u_0 = u_1 \) for the term \( D_{TE} \) in Equation (17). In addition to this, it can be noted that \( u_0 \) and \( u_2 \) are equal since both the expressions indicate free space region. Then using integral in (21), we get the expression for spatial domain vector potential Green’s function as

\[ G^{xx}_A = \frac{\mu_0}{4\pi} \frac{e^{-jk_0 R_0}}{R_0} \] (23)

where \( R_0^2 = \rho^2 \) and \( \rho = \sqrt{(x - x_0)^2 + (y - y_0)^2} \) is the radial distance between source and the observer. It can be noted that for thin substrate case, the magnetic vector potential \( G^{xx}_A \) is free space Green’s function, and thus the expression become independent of the thickness and dielectric constant of the substrate. In addition to these, (23) is low-frequency approximation for vector potential Green’s function of a HED on an ungrounded dielectric slab.
Similarly, low-frequency approximation for spatial domain scalar potential Green’s function by setting $u_0 = u_1$ in (22) can be obtained, which can be expressed as

$$G_V = \left( K + \frac{1}{1 + \epsilon_r} \right) \left[ e^{-jk_0R_0} - (1 - K) \sum_{i=1}^{\infty} K^{2i-1} \frac{e^{-jk_0R_i}}{R_i} \right]$$

where $K = \frac{1 - \epsilon_t}{1 + \epsilon_r}$, $R_0^2 = \rho^2$ and $R_i^2 = \rho^2 + (2ih)^2$.

### 3.2. Evaluation of numerical integration for spatial domain potential Green’s function

In the previous section, it has been observed that the term $D_{TE}$ gives rise to a pole in the integrand of spatial domain vector potential Green’s function. Since for most of the practical cases PMA has a thin substrate, hence the expression in (23) can be directly used in MPIE for carrying out the analysis. Similar expression is also derived for thin and moderate dielectric constant material in case of scalar potential Green’s function in (24). Even though the substrate thickness is small, the expression in (24) depends both on thickness as well as dielectric constant. So, in order to analyze the physical effect of both thickness and...
dielectric constant of the substrate on PMA, the integrand of spatial domain scalar potential Green's function $G_V$ has been investigated in detail followed by numerical integration. Thus, instead of using Equation (24) directly in MPIE, numerical integration of $G_V$ is performed next. Figure 4 shows the plot of the integrand of spatial domain scalar potential Green's function $G_V$ of a HED on an ungrounded dielectric slab. For a given dielectric constant and substrate thickness, the branch cut and pole(s) are separate and distinct, which are denoted as Regions A and B respectively. Region A indicate the branch cut at $k_0$, whereas Region B comprises both of $TE$ and $TM$ poles. The poles which arises due to zeros of $D_{TE}$ and $D_{TM}$ in the denominator of the integrand $G_V$ are denoted as $TM$ and $TE$ surface wave pole. The number of pole increases with the increase in surface waves. For an HED over thin ungrounded lossless dielectric slab, the pole is real and it location can be theoretically estimated by

![Figure 4](image)

Figure 4. Plot of spatial domain scalar potential $G_V$ at different frequencies. (a) Variation of integrand of $G_V$ (real and imaginary) with normalized distance $k_\rho / k_0$ for the parameter $\epsilon_r = 4.4$, $h = 1.52$ mm and $k_0 \rho = 3$ at 10 GHz. (b) Variation of integrand of $G_V$ (real and imaginary) with normalized distance $k_\rho / k_0$ for the parameter $\epsilon_r = 4.4$, $h = 1.52$ mm and $k_0 \rho = 3$ at 5 GHz.
\[
\frac{k_\rho}{k_0} \approx 1 + \frac{(\epsilon_r - 1)^3 k_0 h}{4\epsilon_r} \quad \text{for } D_{TM}
\]
\[
\frac{k_\rho}{k_0} \approx 1 + \frac{(\epsilon_r - 1)^3 k_0 h}{4} \quad \text{for } D_{TE}
\]

Using (25), theoretically the location of \( TE \) and \( TM \) pole can be found out and they are located at \( 1.1k_0 \) and \( 1.49k_0 \), respectively, whereas the graphical location of \( TE \) and \( TM \) pole are at \( 1.01k_0 \) and \( 1.2k_0 \) as shown in Figure 4(a). The difference between theoretical and graphical location of both the poles depend on the choice of operating frequency. In Figure 4(a), the operating frequency is 10 GHz, where the difference is quite pronounced, whereas Figure 4(b) shows the location of \( TE \) and \( TM \) pole obtained analytically and graphically at lower frequency of 5 GHz. It can be seen that in Figure 4(b) the theoretical location of \( TE \) and \( TM \) pole are at \( 1.02k_0 \) and \( 1.05k_0 \), whereas graphically same is obtained at \( 1k_0 \) and \( 1.04k_0 \). Thus, we find that at lower operating frequencies the \( TE \) and \( TM \) pole can be found more accurately by using (25). Now, the branch cut for the present case occur at \( k_\rho = \pm k_0 \)

Figure 5. Plot of magnitude and phase of spatial domain scalar potential \( G_V \). (a) Variation of magnitude of scalar potential Green’s function \( G_V \) with radial distance \( k_0 \rho \) of HED on ungrounded dielectric slab of dielectric constant \( \epsilon_r = 4.4 \) and thickness \( h = 1.52 \text{ mm} \). (b) Variation of phase of scalar potential Green’s function \( G_V \) with radial distance \( k_0 \rho \) of HED on ungrounded dielectric slab of dielectric constant \( \epsilon_r = 4.4 \) and thickness \( h = 1.52 \text{ mm} \).
due to zeros of the function $u_0 = 0$. For the case of $u_1$, the terms in the integrand of $G_V$ are even functions of $u_1$. Hence branch cut is only considered for $u_0$. In addition to this, it has also been observed that lowering the substrate thickness the surface wave pole and branch cut coincides in the complex plane, which makes nearly omnidirectional radiation pattern.[14] Now to evaluate the spatial domain scalar potential Green’s function $G_V$, branch cut at $k_0$ and poles singularity at $1.01k_0$ and $1.2k_0$ must be avoided which are located on $k_{\rho}$ axis in order to integrate the function in a smooth manner. Now to overcome the branch cut, the integration is performed by substituting $k_{\rho}$ with $k_0\cos(ht)$ in the limit $[0, k_0]$, and to avoid the poles which are located in the limit $[k_0, \infty]$ the integration is performed using [6]. The magnitude and phase variation of spatial domain scalar potential Green’s function $G_V$ with distance is shown in Figure 5. The magnitude plot of $G_V$ in Figure 5 shows the formation of near field for the initial values of the distance of the nature $1/\rho$, whereas for the farthest value of distance far field become more dominant of behavior $1/\sqrt{\rho}$. The transition of near field region to far field is shown by rapid variation of phase in phase plot of Figure 5.
3.3. Effect of substrate and dielectric constant on scalar potential Green’s function

In this section, the effect of substrate material and its thickness on scalar potential Green’s function $G_V$ is considered.

It can be noted from Figure 6(a) that the normalized scalar potential Green’s function for a HED with ungrounded dielectric slab attains far field conditions more quickly in low thickness cases compared to high thickness cases. As surface waves are trapped more in thick substrate, the radiating power is reduced. Similarly from Figure 6(b), it can be concluded that though both lower and higher dielectric constant materials attain far field condition in similar fashion but for large dielectric material cases far field condition degrades rapidly with increased distance, because for large dielectric constant material, fields becomes highly confined within high dielectric region. Hence, it can be seen that for practical PMAs low thickness and low dielectric constant material are preferred to get maximum radiation characteristics. The issues discussed regarding the variation of scalar potential Green’s function are important, since for practical PMAs, selection of the thickness and dielectric constant of the substrate are important in getting proper radiation characteristics required for specific design.

4. Full wave analysis of PMA using MPIE–MoM

In this section, the MPIE which is discussed in [15–17] is solved for PMA, using the vector and scalar potential Green’s function derived in (21) and (22). The solution is based on solving the integral equation for unknown current and charge density using MoM. Now, the MPIE can be expressed as

![Figure 7. Geometry of a simple rectangular PMA on a substrate having thickness $h = 1.52$ mm and dielectric constant $\epsilon_r = 4.3$.](image)
The unknown current density $\vec{J}_s$ and charge density $q_s$ can be written as

$$\vec{J}_s = \sum_{i=1}^{N} \alpha_i \vec{T}_i (\rho)$$  \hspace{1cm} (27)$$

$$q_s = \sum_{i=1}^{N} \alpha_i \pi_i (\rho)$$  \hspace{1cm} (28)$$

where $\vec{T}_i (\rho)$, $\pi_i (\rho)$ are the basis function for current and charge density, and $\alpha_i$ are the unknown coefficients in (27) and (28).

The charge density basis functions can be represented in terms of current density basis function through the continuity equation which can be written as

$$\pi_i (\rho) = -\frac{\nabla \cdot \vec{T}_i (\rho)}{j\omega}$$  \hspace{1cm} (29)$$
Here, piecewise sinusoidal subdomain function is used as current density basis function for $\bar{T}_i(\rho)$.

Now, applying MoM to the integral equation in (26), gives us matrix equation in the form of

$$\begin{bmatrix} Z \end{bmatrix} [\alpha] = [b]$$

(30)

where $[Z]$ is the $2 \times 2$ impedance matrix and $[\alpha], [b]$ are the column matrix of unknown coefficients and known excitation voltage. Thus, the elements of the impedance matrix can be expressed as $Z_{ij} = a_{ij} + v_{ij}$, where $a_{ij}$ represents the contribution due to vector potential $\hat{A}$ is given by $a_{ij} = j \omega \int \bar{T}_i(\rho) \cdot \int G_A \cdot \bar{T}_j(\rho) dS' d\rho$, whereas for scalar potential $V$ can be written as $v_{ij} = \frac{1}{j \omega} \int \nabla \cdot \bar{T}_i(\rho) \cdot \int G_V \nabla \cdot \bar{T}_j(\rho) dS' d\rho$. The elements of the $2 \times 2$ impedance matrix can be explicitly written as
where $G_{xx}^A = G_{yy}^A$

Similarly, $Z_{xy}$ and $Z_{yx}$ can be evaluated. Hence, using the four elements of $2 \times 2$ impedance matrix $[Z]$, (30) can be solved for unknown coefficients of $\alpha$. Now as the current density becomes known, the parameter like input impedance and reflection coefficient for PMA of given dimension can be found out easily.

4.1. Results and discussion
The layout of a simple rectangular PMA is shown in Figure 7. The computed MPIE–MoM results for input impedance and reflection coefficient of printed rectangular monopole antenna are compared with HFSS results, which shows agreement as shown in Figures 8(a) and (b) and 9. However, results shows some deviation in Figures 8(a and b) and 9 which may be attributed to fact that in theory the feed gap between the patch and the ground is not being considered since in general Green’s function is evaluated for patch metallization only.

The detailed geometry of F-shaped PMA of [9] is depicted in Figures 10 and 11 shows the verification of reflection coefficient of F-shaped PMA using MPIE–MoM and compared with HFSS and experimental results addressed in [9].

5. Conclusion
Green’s potential functions (both scalar and vector) in spatial domain are derived for HED on a lossless dielectric slab which is not backed by conducting ground plane. Expressions for Green’s potential functions in spatial domain are derived here and are not available in existing literature. In addition to this, the variation of potential Green’s functions with
dielectric constant and substrate thickness have also been reported here. Applying the potential Green’s function to integral equation and solved using MoM based on Galerkin’s method to calculate input impedance and reflection coefficient of a printed rectangular monopole antenna. The computed results using MPIE–MoM are validated with simulated one using HFSS. Further, the reflection coefficient of a F-shaped PMA has also been evaluated using MPIE–MoM and the results are verified using both HFSS and experimental results.

Disclosure statement
No potential conflict of interest was reported by the authors.

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