Performance analysis comparison of transmit antenna selection with maximal ratio combining and orthogonal space time block codes in equicorrelated Rayleigh fading multiple input multiple output channels

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Abstract: In this study, bit error rate (BER) performance of multiple input multiple output (MIMO) systems employing transmit antenna selection with maximal ratio combining (TAS/MRC) and orthogonal space time block codes (OSTBC) is analysed and compared. Analysis has been done for several modulation schemes in equicorrelated Rayleigh fading channels. Novel infinite series expressions for BER of TAS/MRC are proposed in this work. The authors observe that the existing literature on performance analysis of TAS/MRC in correlated channels implicitly or explicitly assume no correlation on the transmitter side. In this work the authors overcome this shortcoming. An alternate closed-form expression for BER performance of OSTBC MIMO systems is derived. This closed-form expression is computationally efficient than existing expressions. Monte Carlo simulations were performed to validate the analytical results. It has been observed that TAS/MRC outperforms OSTBC in terms of BER performance in equicorrelated Rayleigh fading channel. As a case study, performance comparison of cognitive radio (CR) links employing OSTBC and TAS/MRC MIMO systems is investigated in order to show the usefulness of the present analytical study. Even though TAS is the optimum transmit antenna technique in terms of BER performance, OSTBC seems to be a promising alternative for more realistic scenarios in CR systems.

1 Introduction

Multiple input multiple output (MIMO) systems are introduced to cope up with the growing need of high data rate communication and better quality of service. Today, it is almost axiomatic that future wireless applications including cellular communication, device-to-device communication etc. will be multiple antenna systems. Two particular cases of MIMO systems are (a) transmit antenna selection with maximal ratio combining (TAS/MRC) [1] and (b) orthogonal space time block coding (OSTBC) [2]. Both these MIMO schemes have been studied extensively over the past decade. In [1], bit error rate (BER) and outage for TAS/MRC systems are analysed assuming i.i.d. Rayleigh fading channels whereas Yang [3] reports the BER analysis in correlated fast fading channels for TAS/MRC. BER performance analysis with arbitrary correlation at the receiver antennas of TAS/MRC MIMO systems in Nakagami fading environment is also investigated [4]. It is worth noting that though the authors of [3–5] report performance analysis of TAS/MRC in correlated channel fading conditions, they implicitly assume no correlation on the transmitter side to simplify the analysis. In this work, we overcome this shortcoming by analysing performance of TAS/MRC scheme considering equicorrelation both at the transmitter and the receiver sides. This will be more clear in Section 2.1.

TAS/MRC is an uncoded MIMO technique. A coded MIMO technique for two antennas is reported in [6], also known as Alamouti scheme. This scheme was then generalised to arbitrary number of transmit antennas in [2] also popularly called as orthogonal space time block codes (OSTBC). Performance analysis of MIMO systems employing OSTBC in correlated channel fading conditions has been done in [7]. Recently, there has been a growing interest to study performance analysis of these MIMO schemes for cognitive radios [8–11]. For MIMO-based cognitive radio systems, the popular antenna selection scheme discussed in [1] is no longer optimal in terms of minimum achievable BER for the secondary users. Recent works including [8–10] propose several different ways to optimally or sub-optimally select secondary user antenna.
A summary of several antenna selection schemes has been done in [12]. It is important to note that uncorrelated channel fading conditions have been assumed in all these works except in [9]. The utility of the formulae we derive in this work is demonstrated by taking an example of a MIMO-based cognitive radio system. This example opens avenues for future research as it shows that the performance gain of TAS/MRC over OSTBC becomes less significant with stricter interference constraints in cognitive radio networks.

In this work, we analyse the BER performance of TAS/MRC and OSTBC in equi-correlated channel fading conditions. The analytical expressions have been derived for several popular modulation schemes including MPAM, MQAM, DBPSK, MSK, BFSK and BPSK. We derive infinite series expressions for TAS/MRC MIMO systems in correlated Rayleigh fading channels assuming the same correlation at transmitter and the receiver. For BER performance of OSTBC MIMO, closed-form expressions are derived assuming correlated Rayleigh fading conditions. Although the closed-form expression for OSTBC MIMO is reported in [7], this alternate expression is more compact and computationally efficient when applied for equi-correlated channels. The accuracy of expressions derived in this paper is validated with the help of Monte Carlo simulations. It has been reported in [1, 13] that TAS/MRC scheme outperforms the STBC of the same diversity order for i.i.d. Rayleigh fading channels. It has been further shown in [14] that TAS/MRC outperforms all quasi-OSTBC schemes for i.i.d. Rayleigh fading channel. But i.i.d. Rayleigh fading channels is a very ideal scenario. Practical MIMO channels are rarely i.i.d. Rayleigh fading. We will consider correlated Rayleigh fading which is more appropriate for practical MIMO channels.

The rest of the paper is organised as follows. In Section 2, the analytical formulae of BER for MIMO systems employing TAS/MRC or OSTBC with BPSK modulation are derived. In Section 3, we generalise our work done in Section 2 to other modulation schemes like BFSK, DBPSK, MPAM and MQAM. The numerical and simulation results are given in Section 4. This is then followed by conclusion in Section 5.

**Notations:**
- $P_e$ is the probability of bit error,
- $P_s(x)$ is the conditional error probability (CEP),
- $f_X()$ denotes probability,
- $X^T$ denotes transpose of vector $x$,
- $f_{X|Y}( )$ and $F_{X|Y}( )$ are the probability distribution function and cumulative distribution function of random variable $X$, respectively,
- $\psi_{X}( )$ is the characteristic function of random variable $X$,
- $\gamma$ is the average received signal-to-noise ratio (SNR) per user antenna and $\rho$ is the power correlation factor.

## 2 BER analysis of a MIMO system employing TAS/MRC or OSTBC

We consider an $L_t \times L_r$ MIMO system, with the total transmit power of a user with $L_t$ antennas as $P_t$. Let $h_{ij}$ be the equi-correlated Rayleigh fading channel from $i$th transmit antenna to $j$th receive antenna. The power correlation coefficient is assumed to be $\rho$. Let the variance of the real and imaginary components of $h_{ij}$ be $\sigma^2$ each. The average SNR per antenna of the received signal is $2P_t\sigma^2$. Let $\gamma_i = P_s|h_{ij}|^2$. It is equi-correlated exponentially distributed. Then $\gamma_i = \sum_{j=1}^{L_r} \gamma_j$. The noise is considered to be zero mean and unit variance complex Gaussian distributed.

### 2.1 MIMO system employing TAS/MRC with BPSK modulation

A system is said to use TAS scheme if a subset of $L_t$ transmit antennas is selected for actual transmission. Generally, the TAS scheme is accompanied with the use of MRC on the receiver side. The resultant scheme is commonly denoted as TAS/MRC. The system model of TAS/MRC is taken same as equation (2) of [1]. We will consider equi-correlated Rayleigh fading channels. In this paper, we will focus on selection of single transmit antenna which maximises the total received power. The selection decision is made at the receiver and then it is communicated back to the transmitter through a feedback channel. The decision is made according to the following criterion [1]

$$I = \arg\left(\max_{i} \sum_{j=1}^{L_r} |h_{ij}|^2 \right)$$

where $I$ is the antenna index selected for transmission.

Average probability of error, $P_e$ can be given by

$$P_e = \int_{0}^{\infty} P_s(x)f_X(x)dx$$

$$= -\int_{0}^{\infty} P'_s(x)F_X(x)dx$$

where $P'_s(x)$ is the derivative of $P_s(x)$, which is the CEP as a function of instantaneous SNR. For BPSK, $P_s(x) = Q(\sqrt{2x})$ and thus we obtain

$$P_e = \frac{1}{2\sqrt{\pi}} \int_{0}^{\infty} e^{-\frac{x}{2}}F_X(x)dx$$

The CDF, $F_{\gamma,TAS}(\gamma)$, can be calculated as below

$$F_{\gamma,TAS}(\gamma) = \int_{0}^{\infty} \left(\max_{i} \sum_{j=1}^{L_r} P_s|h_{ij}|^2 < x \right)$$

$$= P(\gamma_1 < x, \gamma_2 < x, \ldots, \gamma_{L_r} < x)$$

$$= P(\hat{\gamma}(x))$$

where $x$ is a $L_t \times 1$ vector given by $x = (x_1 x_2 \cdots x_{L_r})^T$ and $\gamma = (\gamma_1 \gamma_2 \cdots \gamma_{L_r})^T$. In [1, 3, 4], (6) is implicitly assumed to be equal to $[F_{\gamma}(x)]^{L_t}$, thus assuming no correlation on transmitter side.

In order to find the CDF of $\gamma$, taking into account the correlation at both transmitter and receiver sides, we first find the characteristic function $\psi_{\gamma(x)}$, where $\psi_{\gamma(x)} = E[\exp(j \gamma^T \gamma)]$.

$$E[\exp\left(j \sum_{i=1}^{L_t} w_i \gamma_i\right)]$$

$$E[\exp\left(j \sum_{i=1}^{L_t} w_i \sum_{k=1}^{L_r} \gamma_{ik}\right)]$$
where $\mathbf{u}$ and $\mathbf{x}$ are $1 \times L_t L_r$ vectors given by $\mathbf{u} = (w_1 \ldots w_1 \ldots w_{L_t} \ldots w_{L_t})^T$ and $\mathbf{x} = (\gamma_{11} \ldots \gamma_{L_t L_r} \ldots \gamma_{L_t L_r})^T$.

The RHS of the above equation is nothing but equation (72) of [15]. Thus, using an equivalent expression in equicorrelated Rayleigh fading case [15, equation (86)] followed by some mathematical manipulations, we obtain (see (11))

$$F_{\gamma T A S}(x) = \frac{1 - \sqrt{\rho}}{1 + \sqrt{\rho}(L_r L_t - 1)} \sum_{n=0}^{\infty} \left( \frac{\sqrt{\rho} L_r}{1 + \sqrt{\rho}(L_r L_t - 1)} \right)^n \times \sum_{\substack{(l_1, l_2, \ldots, l_{L_t}) \leq n \leq (l_1 + l_2 + \cdots + l_{L_t})}} \frac{n!}{l_1! l_2! \cdots l_{L_t}!} \int_0^\infty e^{-\frac{x}{2 \sqrt{\rho}}} \frac{\Gamma(l_1 + L_r, x/2P_c \sigma^2 (1 - \sqrt{\rho}))}{\Gamma(l_1 + L_r)} \, dx$$

Thus, using (3) and (13), we obtain

$$P_e = \frac{1 - \sqrt{\rho}}{1 + \sqrt{\rho}(L_r L_t - 1)} \sum_{n=0}^{\infty} \left( \frac{\sqrt{\rho} L_r}{1 + \sqrt{\rho}(L_r L_t - 1)} \right)^n \times \sum_{\substack{(l_1, l_2, \ldots, l_{L_t}) \leq n \leq (l_1 + l_2 + \cdots + l_{L_t})}} \frac{n!}{l_1! l_2! \cdots l_{L_t}!} \int_0^\infty \frac{e^{-\frac{x}{2 \sqrt{\rho}}} \Gamma(l_1 + L_r, x/2P_c \sigma^2 (1 - \sqrt{\rho}))}{\Gamma(l_1 + L_r)} \, dx$$

The average received SNR per antenna, when transmit power is 1, is given by $\gamma = 2 \sigma^2$. Henceforth, we will use the notation $\zeta = (1/P_c \gamma (1 - \sqrt{\rho}))$. The integral in (14) can be evaluated by linearising the product of incomplete gamma functions using the method given in [16]. Using the same method the linearised product is calculated in Appendix 1. It can be given as

$$P = \prod_{k=1}^{L_t} \frac{\gamma(v_k, \zeta)}{\Gamma(v_k)} = \sum_{i=0}^{\infty} (-1)^i \sum_{\substack{0 \leq l_1, l_2, \ldots, l_{L_t} \leq i \leq l_1 + l_2 + \cdots + l_{L_t}}} L_t \, e^{-\zeta} \sum_{m=0}^{U} C_{m L_t} x^m$$

where $v_k = L_r + i_k$, $U = \sum_{i=1}^{L_t} (v_k - 1)$ and $C_{m L_t} = \sum_{\substack{m_1, m_2, \ldots, m_{L_t} \geq 1}} \prod_{k=1}^{L_t} \frac{\zeta}{m_k}$. Using this result, $P_e$ can be expressed as

$$P_e = \frac{K(1 - \sqrt{\rho})}{2 \sqrt{\pi}} \tilde{u}(K, \rho, L_r, L_t, I_m)$$

where $K = (1/1 + \sqrt{\rho}(L_r L_t - 1))$ and $\tilde{u}$ is a new operator defined as (see (17))

$$I_m = \int_0^{\infty} e^{-q \sqrt{x}} x^m e^{-\tilde{u} x} \, dx$$

Using equation (3.326) in [17], we can reduce this integral to

$$I_m = \frac{\Gamma(m + 0.5)}{(\tilde{u} + 1)^{m + 0.5}}$$

2.2 MIMO system employing OSTBC with BPSK modulation

Following [18, section 4.3.2], it can be shown that the BER performance of a $L_t \times L_r$ MIMO system employing OSTBC is equivalent to that of a $1 \times L_r L_t$ SIMO system using MRC scheme. We are using the similar MIMO system model for OSTBC as equation (1) of [19]. We will assume equicorrelated Rayleigh fading channels. Using equation (2)

$$\psi_{\gamma(w)} = \frac{1 - \sqrt{\rho}}{1 + \sqrt{\rho}(L_r L_t - 1)} \sum_{n=0}^{\infty} \left( \frac{\sqrt{\rho} L_r}{1 + \sqrt{\rho}(L_r L_t - 1)} \right)^n \times \sum_{\substack{(l_1, l_2, \ldots, l_{L_t}) \leq n \leq (l_1 + l_2 + \cdots + l_{L_t})}} \frac{n!}{l_1! l_2! \cdots l_{L_t}!} \prod_{i=1}^{L_t} \frac{1}{(1 - 2jw_i P_c \sigma^2 (1 - \sqrt{\rho}))^{l_i + 1}}$$

$$\tilde{u}(K, \rho, L_t, L_r, I_m) = \sum_{m=0}^{\infty} (KL_t \sqrt{\rho})^m \sum_{\substack{0 \leq l_1, l_2, \ldots, l_{L_t} \leq i \leq l_1 + l_2 + \cdots + l_{L_t}}} \beta_{m L_t} \sum_{i=1}^{L_t} (-1)^i \sum_{m=0}^{U} C_{m L_t} I_m$$
of [19], we can write the combined signal at the receiver

\[ r_n = \|H\|_F^2 s_n + \hat{n}_n \]  \hspace{1cm} (20)  

where information bits are mapped as symbols \( s_1, s_2, \ldots, s_L \), and \( \hat{n} \) is the noise after combining with distribution \( CN(0, \|H\|_F^2) \) since the noise before combining was of unit variance. This equation can be re-written as

\[ r_n = \sum_{i=1}^{L_1} \sum_{j=1}^{L_1} |h_{ij}|^2 s_n + \hat{n}_n \]  \hspace{1cm} (21)  

where \( h_{ij} \) are the components of the channel matrix \( \|H\|_F \). If we vectorise \( \|H\|_F \) by appending one column below another, the received signal can be written as

\[ r_n = \sum_{i=1}^{L_1} h'_i s_n + \hat{n}_n \]  \hspace{1cm} (22)  

where \( h'_i \) are the components of the new channel matrix after vectorisation. Hence, BER analysis of \( L_1 \times L_1 \) OSTBC MIMO will be same as that of \( 1 \times L_t \) SIMO system with MRC scheme. The CDF of received SNR is given as [20]

\[ F_\gamma(x) = 1 + ab^{L-1} \left( -\frac{e^{-ax}}{a(b-a)x^2} \right) + e^{-hx} \sum_{k=1}^{L-1} \frac{1}{b^k(b-a)x^k} \sum_{n=0}^{k-1} \frac{(bx)^n}{n!} \]  \hspace{1cm} (23)  

where \( L \) is the number of receive antennas, \( a = \sqrt{\frac{\rho}{1+(L-1)\sqrt{\rho}}} \) and \( b = \frac{1}{\sqrt{\rho}} \), where \( \rho \) is the average SNR per receiver antenna and \( \sqrt{\rho} \) is the power correlation coefficient.

As we are analysing the OSTBC system as a \( 1 \times L_t \) MRC system, \( L = L_t \), and \( \bar{Y} = \bar{P}_r \bar{\gamma} \) where \( \bar{P}_r \bar{\gamma} \) is the average SNR per receiver antenna of the \( L_t \times L_t \) system, when net transmit power is \( P_t \). Then

\[ \zeta = \frac{1}{a(b-a)x^2}, \quad \beta_k = \frac{1}{b^k(b-a)x^k}, \quad a = \frac{1}{\sqrt{\rho}}, \quad b = \frac{1}{\sqrt{\rho}} \]  

Thus, substituting (23) in (3), we obtain

\[ P_e = \frac{1}{2\sqrt{\pi}} \int_0^{\infty} e^{-\frac{x^2}{2}} \left( 1 + ab^{L-1}(\zeta - e^{-ax}) \right) dx \]  \hspace{1cm} (24)  

Substituting \( x = t^2 \) and using the equality \( \int_0^{\infty} e^{-t^2} dt = \sqrt{\pi} \) followed by simple mathematical simplifications, we obtain the closed-form expression of BER as

\[ P_e = \frac{1}{2} - \frac{\zeta ab^{L-1}}{2 + 1} + \frac{ab^{L-1} - 1}{2 + 1} b^n \Gamma(n + (1/2)) \]  \hspace{1cm} (25)  

### 3 Generalisation to other modulation schemes

We have also generalised the BER formulae for MIMO systems employing TAS/MRC or OSTBC for several popular modulation schemes like MPAM, MQAM, QPSK, BFSK, MSK and DBPSK. They are listed in Table 1. In order to map the three cases listed in the table to their respective modulation schemes, refer to Table 2. Here

\[ I_{O_1} = \frac{\sqrt{\tan^{-1}(2a/q) + 1}}{\sqrt{\pi a(q + 1)}} \]  \hspace{1cm} (26)  

\[ I_{O_2} = \frac{(2a/q)^{n+1/2}}{2^{n+1}} \]  \hspace{1cm} (27)  

\[ I_{T_1} = \frac{\Gamma(m + 0.5)}{(2a/q + \xi)^{m+1/2}}, \quad I_{T_2} = \frac{\Gamma(m + 1)}{(q + \xi)^{m+1/2}} \]  \hspace{1cm} (28)  

\[ I_{T_3} = \frac{2m}{\sqrt{\pi}} \left( 1 + \frac{2m}{q} \right)^{-m+1/2} \]  \hspace{1cm} (29)  

<table>
<thead>
<tr>
<th>Case</th>
<th>( P_e ) for OSTBC</th>
<th>( P_e ) for TAS/MRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( P_e^{(1,0)} = \frac{P}{2} - \frac{\zeta p a b^{L-1}}{2\sqrt{1 + 2a/q}} + \frac{p a b^{L-1} - 1}{2\pi} \sum_{k=1}^{L-1} \frac{b^n}{n!} \sum_{m=0}^{n} \frac{(2b)^m}{m!} \frac{\Gamma(m + 0.5)}{\sqrt{m + 1/2} \sqrt{b^{m+0.5}}} )</td>
<td>( P_e^{(1,1)} = \frac{P}{2} \cdot \frac{\sqrt{\zeta}}{\sqrt{\rho}} K(1 - \sqrt{\rho}) \sqrt{\rho} )</td>
</tr>
<tr>
<td>II</td>
<td>( P_e^{(2,0)} = P - \frac{\zeta p a b^{L-1}}{2\sqrt{1 + 2a/q}} + \frac{p a b^{L-1} - 1}{2\pi} \sum_{k=1}^{L-1} \frac{b^n}{n!} \sum_{m=0}^{n} \frac{(2b)^m}{m!} \frac{\Gamma(m + 1)}{\sqrt{m + 1/2} \sqrt{b^{m+1}}} )</td>
<td>( P_e^{(2,1)} = \frac{P a q}{2 \sqrt{1 - \sqrt{\rho}} \sqrt{\rho}} K(1 - \sqrt{\rho}) \sqrt{\rho} )</td>
</tr>
<tr>
<td>III</td>
<td>( P_e^{(3,0)} = P^{(1,0)} - \frac{P}{4} + r a b^{L-1} \left( \frac{q}{2\pi} \sum_{k=1}^{L-1} \frac{b^n}{n!} \sum_{m=0}^{n} \frac{(2b)^m}{m!} \frac{\Gamma(m + 1)}{\sqrt{m + 1/2} \sqrt{b^{m+1}}} \right) )</td>
<td>( P_e^{(3,1)} = P^{(1,1)} - \frac{\zeta p a b^{L-1}}{2\sqrt{1 - \sqrt{\rho}} \sqrt{\rho}} K(1 - \sqrt{\rho}) \sqrt{\rho} )</td>
</tr>
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</table>
The definitions of the various parameters used here (as well as the operator) are the same as given in Sections 2.1 and 2.2. The proofs of the various formulae mentioned in this section are given in Appendix 2. We have done extensive Monte Carlo simulations to verify these closed-form/infinite series formulae and we have observed a close agreement in the simulation and analytical results for all cases as shown in the next section.

4 Results and discussions

4.1 Performance comparison of TAS/MRC and OSTBC

Simulations were performed assuming transmit power of MIMO system, $P_t = 1$ for $10^5$ iterations. Simulations were carried out to validate the derived expressions of the CDF of received instantaneous SNR for TAS/MRC MIMO first and then the BER performances of TAS/MRC MIMO and OSTBC MIMO schemes. Fig. 1 shows the CDF of received instantaneous SNR for TAS/MRC MIMO systems for different values of average SNR (0 and 5 dB) and correlation coefficient (0.1 and 0.5). The analytical CDF plot is obtained using (13). It is observed that the analytical and simulated CDF of received SNR are in close agreement. This validates the correctness of expression (13) in Section 2.1. The two obvious observations from Fig. 1 are: (i) the slope of CDF curve decreases with increasing average received SNR and (ii) for the same average received SNR there exist a crossing of the CDF curves for different values of $\rho$.

The BER performances of TAS/MRC and OSTBC MIMO systems with different modulation techniques are plotted in Figs. 2 and 3 respectively. It is observed that the analytical and simulated values of BER are almost equal. It is observed that the BER performance degrades with increase in the constellation size. Fig. 3 also compares the exactness of the closed-form expression (25) with the BER values obtained from the closed-form expression reported in [7]. From Figs. 2 and 3, we can infer that TAS/MRC outperforms OSTBC for equicorrelated Rayleigh fading MIMO channels.

![Graph showing BER comparison](image)

We have also analysed the effect of correlation on BER for different values of average received SNR per receiver antenna, as shown in Fig. 4. From this plot, it can be seen that when $\gamma = 0$ dB, the BER of OSTBC is about 23.9% higher than TAS/MRC for highly correlated channels ($\rho = 0.9$). Whereas when $\gamma = 5$ and 10 dB, this value is about

<table>
<thead>
<tr>
<th>Case</th>
<th>CEP</th>
<th>Modulation scheme</th>
</tr>
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<tbody>
<tr>
<td>I</td>
<td>$P_e(x) = pQ(\sqrt{q}/x)$ if $(p, q) = (1, 2)$, then BPSK modulation if $(p, q) = (1, 1)$, then BFSK modulation</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>$P_e(x) = \exp(-q/x)$ if $(p, q) = (2(M-1)/M, 6 \log_2 M/M^2 - 1)$, then MPAM modulation if $(p, q) = (0.5, 1)$, then DBPSK modulation</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>$P_e(x) = pQ(\sqrt{q}/x) - r^2(\sqrt{q}/x)$ if $(p, q, r) = (2, 2, 1)$, then QPSK or MSK modulation if $(p, q, r) = (2, 2, 2)$, then coherent DPSK modulation</td>
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Note that first 50 terms of the infinite summation have been used to plot the analytical results.

![Graph showing CDF comparison](image)
32.81 and 58.52%, respectively. We can also observe that the BER plots for OSTBC and TAS/MRC tends to converge as the correlation increases.

4.2 Comparing run-time complexity of BER for OSTBC

Fig. 5 shows that (25) is significantly faster in computations than the existing closed-form expressions for BER analysis of OSTBC systems as given in [7]. The figure shows that our formula for computing BER is about 39.1 times faster when number of transmit and receive antennas is 1, and 4.5 times faster when the number of antennas is equal to 6. This shows that our code runs significantly faster on an average. However, it should be noted that the exact time to run the code on different systems will be different.

Now-a-days, researchers are investigating the possibility of using millimetre wave spectrum for cellular communication [22]. The use of millimetre wave frequencies allows us to squeeze in a large number of antennas in the same area. Such systems are said to implement massive MIMO. A typical massive MIMO system may have something like

64 × 16 antenna array [23]. Recall that \( L = L_t L_r \) is the product of number of antennas at the transmitter and receiver. The time complexity of the BER expressions is dependent on \( L \). For massive MIMO systems the value of \( L \) is of the order of \( 10^3 \) or it may even go up to \( 10^4 \). That would make the improved run-time complexity in analytical expressions for BER much more significant.

4.3 MIMO-based cognitive radio system: a case study

Consider a cognitive radio system with one primary user (PU) and two secondary users (SU1 and SU2), as shown in Fig. 6. The channels are assumed to be equicorrelated Rayleigh block fading channels. The noise is considered to be zero mean and unit variance complex Gaussian distributed. It is assumed that the SU1 will transmit only when it senses the PU to be idle (SU2 is only going to listen to transmissions of other users). Spectrum sensing is done by SU1 using energy detection with MRC scheme. Let \( B \) be the time bandwidth product for the signal to be detected. Thus, 2\( B \) samples are collected in an observation interval for energy detection.

The two interference constraints are \( \eta \), the maximum allowable interference power when the secondary user correctly detects the absence of PU transmission and \( \xi \), the probability that the interference level exceeds \( \eta \). The primary base station conveys a set of these interference constraints \( \{ \eta, \xi \} \) to the secondary users. The secondary user transmission is based on the fixed power control scheme [11].
Fig. 7 summarises a step-by-step method to analyse the BER performance of cognitive radio systems based on the work done in [11]. We use the formulae for probability of correct detection and false alarm for secondary user implementing MRC in equicorrelated Rayleigh fading channel as derived in [24].

Table 3 shows the effect of PU interference constraint on the percentage decrease in BER when TAS/MRC is used instead of OSTBC. The parameter values used for the simulations are $\xi = 0.1$, $B = 6$, false alarm probability is set to 0.01 and the average SNR of the sensing channel is set to 0 dB. The numerical results indicate that when the maximum allowable interference to the PU, $\eta$, is low, the percentage decrease in BER when TAS/MRC is used instead of OSTBC is also significantly lower as compared to a case with less stringent interference constraints. Table 3 indicates that for MIMO-based cognitive radio systems with strict interference constraints, the performance gain by using TAS/MRC over OSTBC becomes significantly less. OSTBC is simpler to implement than TAS/MRC as it does not require feedback channel. Both these observations prompt us to conclude that for cognitive radio systems OSTBC seems to be an attractive option over TAS/MRC.

5 Conclusion

A novel infinite series expressions for analysing BER performance of TAS/MRC MIMO systems are derived for several popular modulation schemes. The convergence and validity of the derived infinite series expressions are verified by comparison with the simulation results. An alternate computationally efficient closed-form expression for BER analysis of OSTBC MIMO systems is also derived. We observed that, in general, TAS/MRC outperforms OSTBC for equicorrelated Rayleigh fading channels. But, numerical results show that OSTBC is still an attractive choice for MIMO-based cognitive radio systems with strict interference constraints as the gain in performance by using TAS/MRC as compared to OSTBC becomes less significant in this case. In future, we would be extending this work to consider different correlations at both transmitter and receiver sides.

6 References

7 Appendix

7.1 Appendix 1: evaluation of the product of incomplete gamma functions

Let us call \( P = \prod_{i=1}^{\infty} (\gamma(v_k, \zeta) / \Gamma(v_k)) \), where the symbols \( \zeta, v_k \) and \( L_k \) have the same meaning as given in (15). \( v_k \) can take only integer values. Thus, we can use equation (8.352) in [17] to represent incomplete gamma function as

\[
\gamma(v_k, \zeta) = (v_k - 1)! \left[ 1 - e^{-\zeta} \sum_{m=0}^{v_k-1} \frac{\zeta^m}{m!} \right] 
\]

(31)

Thus, using \( \Gamma(v_k) = (v_k - 1)! \), we can rewrite \( P \) as

\[
P = \prod_{k=1}^{L_k} \left[ 1 - e^{-\zeta_k} \sum_{m=0}^{v_k-1} \frac{\zeta_k^m}{m!} \right] 
\]

(32)

Using the lemmas given in (7) and (8) of reference paper [16], it can be derived that

\[
P = \sum_{i=0}^{L_k} (-1)^i \sum_{0\leq l_1,\ldots,l_{i-1}\leq 1} e^{-i\zeta_k} \prod_{i=1}^{i} \sum_{m=0}^{v_k-1} \frac{\zeta_k^m}{m!} \] \[= \sum_{i=0}^{L_k} (-1)^i \sum_{0\leq l_1,\ldots,l_{i-1}\leq 1} e^{-i\zeta_k} \sum_{m=0}^{U_k} C_{m,i} \zeta_k^m \]

(33)

(34)

Here \( L_k = (v_k - 1)L_k \), \( U = \sum_{i=1}^{L_k} L_k \) and \( C_{m,i} = \sum_{m_1+m_2+\ldots+m_i=v_k-1} \prod_{i=1}^{i} \frac{\zeta_k^{m_i}}{m!} \) where \( 0\leq m_1\leq \ldots\leq m_i \leq L_k \).

7.2 Appendix 2: derivation of the BER formulae in Table 1

7.2.1 Some important integrals:

\[
H_{n,m}(b) = \int_0^\infty x^n \text{erfc}^m(x) e^{-b x^2} dx 
\]

(35)

where \( \text{erfc}(x) \) is the complementary error function. A recursive relation to solve this integral is given in [25].

\[
H_{n,m}(b) = H_{n,0}(b) - \frac{2}{\sqrt{\pi}} \int_0^1 f_{n+1,m-1}(y, b + y^2) dy 
\]

(36)

where

\[
f_{n,m}(y, b) = \int_0^\infty x^n \left( \int_0^\infty e^{-r^2} dr \right)^m e^{-b x^2} dx 
\]

and \( H_{n,0}(b) = \frac{2}{\sqrt{\pi}} \int_0^1 f_{n+1,m-1}(y, b + y^2) dy \).

(a) Evaluation of \( H_{n,1}(b) \): Using the recursion relation of (36), we obtain

\[
H_{n,1}(b) = H_{n,0}(b) - \frac{2}{\sqrt{\pi}} \int_0^1 f_{n+1,0}(y, b + y^2) dy 
\]

(37)

where

\[
f_{n+1,0}(y, b + y^2) = \int_0^\infty x^{n+1} e^{-b x^2 y^2} dx 
\]

(38)

\[
= H_{n+1,0}(b + y^2) 
\]

\[
= \frac{1}{2}(b + y^2)^{-(n+2)/2} \Gamma\left( \frac{n + 2}{2} \right) 
\]

It can be written as

\[
H_{n,1}(b) = \frac{1}{2} b^{-n+1} \Gamma\left( \frac{n + 1}{2} \right) 
\]

(39)

\[
= \frac{1}{(n + 2)/2} \int_0^1 \frac{1}{\sqrt{\pi}} \cos^n \theta d \theta 
\]

(40)

where

\[
\int_0^\cos^n \theta d \theta = \frac{1}{2^n} \int_0^{n/2} x + \frac{1}{2^{n-1}} \sum_{k=0}^{n-2/2} \frac{\zeta_k^m}{m!} \] \[= \frac{\sin(n-2k)x}{n-2k} 
\]

(41)

if \( n \) is even

\[
= \frac{1}{2^{n-1}} \sum_{k=0}^{(n-1)/2} \frac{n}{k} \sin(n-2k)x
\]

(42)

if \( n \) is odd

\[
= x 
\]

(43)

(b) Evaluation of \( G_n(b, q) \):

\[
G_n(b, q) = \int_0^\infty \frac{x^n e^{-bx}}{\sqrt{x}} Q(\sqrt{q x}) dx 
\]

(44)

Substituting \( x = 2y^2 q \) and \( Q(x) = (1/2) \text{erfc}(x/\sqrt{2}) \) in (44),

\[
G_n(b, q) = \left( \frac{q}{b} \right)^{1/2} \int_0^\infty y^2 \text{erfc}(y) e^{-(2b/q) y^2} dy 
\]

(45)

Using (35) in above expression, we obtain

\[
G_n(b, q) = \left( \frac{q}{b} \right)^{1/2} H_{n,1}(\frac{2b}{q}) 
\]

(46)
Thus

\[ G_n(b, q) = \left( \frac{2}{qb} \right)^{n+0.5} \left[ \frac{\Gamma(n + 0.5)}{2} - \Gamma(n + 1) \int_0^{\pi} \cos^{2n} \theta d\theta \right] \]

(47)

where

\[ \int_0^\theta \cos^{2n} \theta d\theta = \frac{1}{2\pi n} \left( \frac{2n}{n} \right) \theta + \frac{1}{2^{2n-1}} \sum_{k=0}^{n-1} \left( \frac{2n}{k} \right) \frac{\sin(2n-2k)\theta}{2n-2k} \]

if \( n \neq 0 \)

(48)

\[ = \theta \quad \text{if} \quad n = 0 \]

(49)

7.2.2 Derivation of the BER formulae in Table 1: The expression for \( F_\gamma(x) \) is given in (23). Substituting \( F_\gamma(x) \) and \( P'_e(x) \) in (2), it is easy to derive the BER expression in the first two cases. In this section, we will therefore only look at Case III for OSTBC in detail. The expressions for all other cases can be derived in a similar way.

(a) Case III for OSTBC: Solving (2), it can be shown that

\[ P_e^{(3,0)} = P_e^{(1,0)} - \sqrt{\frac{q}{2\pi}} \int_0^\infty e^{-qx/2} F_\gamma(x) Q(\sqrt{qx}) dx \]

\[ = P_e^{(1,0)} - \sqrt{\frac{q}{2\pi}} I_1 + \sqrt{\frac{q}{2\pi}} \frac{q}{rabL} I_2 \]

\[ - \sqrt{\frac{q}{2\pi}} \frac{q}{rabL} \sum_{k=1}^{L-1} \beta_k \sum_{n=0}^{b^n} I_3 \]

where \( P_e^{(1,0)} \) is the BER in Case I for OSTBC and the integrals \( I_1, I_2 \) and \( I_3 \) are given as

\[ I_1 = \int_0^\infty e^{-qx/2} Q(\sqrt{qx}) dx = G_0(q/2, q) = \frac{\sqrt{\pi}}{2q} \]

\[ I_2 = \int_0^\infty e^{-(a+q/2)x} Q(\sqrt{qx}) dx = G_0(a + q/2, q) = I_{O_1} \]

\[ I_3 = \int_0^\infty e^{-(b+q/2)x} Q(\sqrt{qx}) dx = G_n(b + q/2, q) = I_{O_2} \]

where \( I_{O_1} \) and \( I_{O_2} \) are given in Section 3.