Outage Probability Bound of Decode and Forward Two Way Relay employing Optical Spatial Modulation over Gamma-Gamma Channels

Aniran Bhowal, Rakesh Singh Kshetrimayum

Abstract: In this paper, free space optical (FSO) communication has been studied for different atmospheric conditions. Spatial modulation (SM) has been used to improve the performance of two way relay cooperative communication. The proposed system employs decode and forward (DF) relaying technique along with physical layer network coding (PLNC), hence it has been named as optical spatial modulation (OSM) based decode and forward two way relay (DFTWR). It enables full duplex communication thereby enhancing the system efficiency. The transmit optical source index as well as the information are exchanged between the relay the symbol is decoded using a mapping scheme, which will complete the same operation by pairwise XOR-ing of the symbols. At the receiving side, the symbol is decoded using a mapping scheme, which will reduce the inter-laser optical interference and also exploit the natural mixing of signals due to superposition of optical waves coming from two different sources.

1 Introduction

Free space optical communication is an area of open research which has attracted the attention of researchers because of higher bandwidth, inherent security and ease of installation. It provides much higher data rates than conventional radio frequency (RF) communication [1–4]. However the drawback of FSO is that it suffers from the randomness of the atmospheric conditions. It is mainly a line-of-sight (LOS) communication which is ineffective for long distance communication. To enable transmission over larger distances, relays must be used. Bidirectional relays increase the efficiency of the system as simultaneous data transfer can take place between the nodes.

In this paper, spatial modulation (SM) along with physical layer network coding (PLNC) [5, 6] has been used for full duplex communication. This will result in higher data rate and bandwidth efficiency. In normal non-network coding scheme, 4 time slots are required for exchange of two packets, one in each direction, to avoid collision. In analog network coding scheme (ANC), 3 time slots are necessary to complete the same operation by pairwise XOR-ing of the symbols over the two sources. In PLNC, time slots required to achieve the same operation is 2 only. This is due to the network coding operation performed automatically in the superimposed optical waves. At the relay the symbol is decoded using a mapping scheme, which will reduce the inter-laser optical interference and also exploit the natural mixing of signals due to superposition of optical waves coming from two different sources.

An incoherent FSO system employs the technique of intensity modulation of optical sources like light emitting diode (LED)/Laser at the transmitting side and direct detection by photo-detector at the receiving side. The modulation scheme generally used in optical communication is on-off keying (OOK) but it requires adaptive threshold to adjust according to the atmospheric turbulence. So we are using binary phase shift keying (BPSK) as the modulation scheme. The disadvantage of using single input single output (SISO) FSO systems is that it suffers from inferior bit error rate (BER) performance. So we have opted for multiple input multiple output (MIMO) FSO systems [7, 8]. The idea of SM stems from the fact that multiple transmit and receive optical chains used in PLNC system leads to unnecessary power consumption and escalates the system cost and complexity considerably. Also there will be inter-laser optical interference due to use of multiple optical chains for several laser sources simultaneously. This can be overcome by SM technique where a single optical chain with a single laser source will be active for the selected transmit laser index. In SM, information is split into two parts one is used for transmit laser source activation and the other is mapped to a symbol using a particular modulation scheme.

The channel offers the biggest challenge in FSO communication systems. The unpredictable nature of the outdoor environment poses serious difficulties as the environment possesses strong atmospheric turbulence (AT) [9]. There are several channel models available in literature which takes into account the fading caused by AT. A few of the models are negative exponential, log-normal, K distribution, Gamma-Gamma (G-G) etc. It has been reported in literature that cascaded N-γ model with N=2 (also known as G-G model) is a suitable channel model for FSO communication systems as it satisfies the experimental results under weak to strong turbulence conditions [2].

According to modified Rytov theory, the small scale turbulence eddies caused by scattering are modulated by the large scale turbulence eddies caused by refraction. These turbulence effects can be modelled as two independent random processes. So irradiance can be written as the product of two independent random processes [10]. G-G channel model satisfies this condition as it is the product of two Gamma functions. Log-normal distribution is suitable only for weak turbulence condition as it has to satisfy the assumption that the magnitude of the scattered optical field should be less than the unperturbed phase gradient. Such scenarios occur only in case of single scattering phenomenon. In case of strong turbulence, multiple scattering occurs, hence log normal channel model cannot be used for such cases. Generalized Malaga channel model [11] exists in literature. But the G-G model can satisfy all the atmospheric turbulence conditions, so we are considering only the G-G model in our work. Pointing errors generally occur in case of satellite FSO communication where laser misalignment can cause deviations of the received signal. In our work, we have considered FSO cooperative communication for terrestrial purpose where link distance is less. So pointing error can be ignored for line of sight cooperative FSO communication [10].
Previous works have analysed error rate performance of FSO links over G-G channels [12]. Error analysis has also been done for MIMO case as in [13]. Cooperative communication has been utilized for FSO links and PLNC has been performed both for half-duplex and full duplex relays [5, 6]. Outage probability and bit error rate for such systems have been evaluated [14–16]. Performance of serial and parallel FSO relaying under different atmospheric conditions has also been reported in [17–19]. Performance analysis of space shift keying for FSO links has been done in [20] by evaluating average pairwise error probability and bit error rate. Outage probability for secure broadcasting system, having simultaneous wireless information and power transfer, has been calculated in [21]. The notion of optical spatial modulation (OSM), the counterpart of SM in RF domain, has been put forth by the authors for FSO links [22, 23]. However the concept of transmit laser selection in OSM has not yet been utilized for cooperative FSO communication.

The lower and upper bound on outage probability for full duplex OSM based FSO relay system has been evaluated and compared with other techniques available in literature. Monte Carlo simulations have been performed under different channel conditions to confirm the correctness of the analytical technique. The effect of varying various parameters of the channel has been studied and the results have been reported in this paper.

The fading channel model has been presented in Section II. Section III explains the relay based full duplex OSM system model for FSO links, while Section IV presents the performance analysis in terms of outage probability and average data rate. The analytical and simulation results have been provided in Section V. Finally Section VI summarizes and concludes the paper.

2 Channel Model

Optical signals are affected by atmospheric turbulence induced fading, which results in irradiance deviations in the received optical intensity, also known as scintillation. This phenomenon of scintillation is caused due to small changes in atmospheric temperature, humidity, each source node exchanges bits of data with the other source and receive chains to mitigate self interference. In this proposed system, each source node exchanges bits of data with the other source node using PLNC at the relay employing DF technique. For transmitting purpose, \(N_L\) lasers are used and for receiving side, \(N_L\) photodetectors of responsivity \(R_o\) are used. The relative gains of the links as described in Section II of [29] and Eq. 3 of [30] are given as: \(G_{S_iR} = (\frac{d_{SD}}{d_{SR}})^\gamma\), \(G_{S_R} = (\frac{d_{SD}}{d_{SR}})^\gamma\) where \(G_{S_iR}\) and \(G_{S_R}\) are the relative gain of the \(S_i\) to RN link and RN to \(S_i\) links respectively, \(\tau\) is the path loss coefficient, \(d_{SD}, d_{SR}\) and \(d_{RD}\) are the distances between \(S_i\) and \(S_j\), \(S_i\) and RN, and RN and \(S_j\) links respectively. Relative geometrical gain factor \(\kappa = \frac{G_{S_iR}}{G_{S_R}}\) denotes the location of RN node with respect to \(S_i\) and \(S_j\). \(\kappa\) is less than 1 if distance between relay node and \(S_j\) is lesser whereas \(\kappa\) is more than 1 if distance between relay node and \(S_i\) is lesser and if both distances are equal then \(\kappa\) is 1.

![Fig. 1: System model for OSM based DFTWR.](image)

Table 1 presents the value of turbulence parameters under different atmospheric conditions while Table 2 lists the value of the turbulence parameters for different link distances.

<table>
<thead>
<tr>
<th>Turbulence region</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\sigma_f^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>4.2</td>
<td>1.4</td>
<td>3.5</td>
</tr>
<tr>
<td>Moderate</td>
<td>4.0</td>
<td>1.9</td>
<td>1.6</td>
</tr>
<tr>
<td>Weak</td>
<td>11.6</td>
<td>10.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Link distance (Km)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\sigma_f^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.2</td>
<td>1.4</td>
<td>3.5</td>
</tr>
<tr>
<td>6</td>
<td>8.24</td>
<td>1.03</td>
<td>26.57</td>
</tr>
<tr>
<td>8</td>
<td>10.19</td>
<td>1.01</td>
<td>45.02</td>
</tr>
</tbody>
</table>

3 Proposed System Model

The simplest form of 3 node bidirectional cooperation model, comprising of 2 source nodes- Source 1 (S1) and Source 2 (S2), and relay node (RN), has been considered, as depicted in Fig. 1. There is no direct link between S1 and S2. All the nodes are spatially modulated full duplex (SMFD) nodes with \(N_L\) lasers and \(N_L\) photodetectors. It is assumed that the nodes have independent transmit and receive chains to mitigate self interference. In this proposed system, each source node exchanges bits of data with the other source node using PLNC at the relay employing DF technique. For transmitting purpose, \(N_L\) LED/Lasers are used and for receiving side, \(N_L\) photodetectors of responsivity \(R_o\) are used. The relative gains of the links as described in Section II of [29] and Eq. 3 of [30] are given as: \(G_{S_iR} = (\frac{d_{SD}}{d_{SR}})^\gamma\), \(G_{S_R} = (\frac{d_{SD}}{d_{SR}})^\gamma\) where \(G_{S_iR}\) and \(G_{S_R}\) are the relative gain of the \(S_i\) to RN link and RN to \(S_i\) links respectively, \(\tau\) is the path loss coefficient, \(d_{SD}, d_{SR}\) and \(d_{RD}\) are the distances between \(S_i\) and \(S_j\), \(S_i\) and RN, and RN and \(S_j\) links respectively. Relative geometrical gain factor \(\kappa = \frac{G_{S_iR}}{G_{S_R}}\) denotes the location of RN node with respect to \(S_i\) and \(S_j\). \(\kappa\) is less than 1 if distance between relay node and \(S_j\) is lesser whereas \(\kappa\) is more than 1 if distance between relay node and \(S_i\) is lesser and if both distances are equal then \(\kappa\) is 1.
In all the nodes, a RF subcarrier signal is modulated by source data and the modulated signal is then used to modulate the intensity of optical source like Laser. The modulated signal in RF domain is sinusoidal, so a DC bias is added to make it positive in order to drive the optical source. The addition of DC bias would ensure that the negative part of the BPSK modulated signal is removed. The optical source transmit laser is used. The transmitted information is directly proportional to the intensity of the transmitted optical field. This property is used to recover the electrical signal which is demodulated by a RF coherent BPSK demodulation scheme to recover the original data [10].

Semiconductor optical amplifier (SOA) based Mach-Zehnder interferometer (MZI) [31] is used to design an all optical XOR gate. A symmetrical MZI is used with one SOA in each arm located at the same relative position. The input logic signals A and B are taken as inputs into the two arms of the MZI via two multiplexers. They act as control signals. A clock signal is splitted into two parts using a coupler, thus forming the probe signals. If both the control signals are identical, output is zero as the SOA-MZI is balanced. But if the optical source transmit data using BPSK (Binary Phase Shift Keying) modulation scheme to recover the original data [10].

For \( N_L = 2 \), the transmit laser source which is made active is chosen as

\[
j_k = \begin{cases} 
2, & \text{if } c_k^{LSB}[n] = 1 \\
1, & \text{if } c_k^{LSB}[n] = 0 
\end{cases}
\]  

where \( s \) represents the \( n^{th} \) time slot and \( k \) indicates the source node which may be 1 or 2 in this case. If \( N_L > 2 \), then \( c_k^{LSB}[n] \) number of bits (from LSB side) will be used for transmit laser source activation and \( \log_2 m \) message bit will be used for M-ary modulation scheme.

We will consider BPSK modulation scheme for our case. The table of laser source activation depending upon the message bits has been listed in Table 3 and Table 4 for \( N_L=4 \) and \( N_L=8 \) respectively.

**Table 3** Optical spatial modulation for \( N_L=4 \)

<table>
<thead>
<tr>
<th>Input bits</th>
<th>Laser index</th>
<th>BPSK symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>001</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>010</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>011</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>101</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>110</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>111</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

\( c_k^{LSB}[n] \) represents the bits which may be 0 or 1 in this case. Both the sources transmit data using BPSK (Binary Phase Shift Keying) at the relay node simultaneously. The received signal obtained at the \( j_R \) photodetector of relay node in \( n^{th} \) time slot is expressed as:

\[
y_{R}^{\hat{n}}[n] = \sum_{k=1}^{2} \sqrt{R_c P_k G_{S_k,R}} h_{k,j_R} x_k^{MSB}[n] + i_R[n] + w_R[n]
\]  

(5)

where \( h_{k,j_R} \) Represents \( N-\gamma \) channels between the selected laser \( j_k \) of source node and photodetector \( j_R \) of the relay node. The variables \( i_R[n] \) and \( w_R[n] \) are the background and ambient light noise respectively.

**Table 4** Optical spatial modulation for \( N_L=8 \)

<table>
<thead>
<tr>
<th>Input bits</th>
<th>Laser index</th>
<th>BPSK symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>0001</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>0010</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>0011</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>0100</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>0101</td>
<td>6</td>
<td>-1</td>
</tr>
<tr>
<td>0110</td>
<td>7</td>
<td>-1</td>
</tr>
<tr>
<td>0111</td>
<td>8</td>
<td>-1</td>
</tr>
</tbody>
</table>

with a bar indicate photodetector used at the receiver while those without bar indicate laser index at the transmitter. \( P_k \) indicates the transmit power at the source node \( S_k, k=1,2 \). \( w_R[n] \) represents the shot noise from ambient light at the relay node. Shot noise from ambient light is independent of the signal and can be represented as white Gaussian random noise with zero mean and variance \( N_0 \).

The self interference at the relay node is indicated by \( i_R[n] \). It is described as Gaussian random variable with zero mean and power \( P_k = N_0/[10^{\gamma/10} - 1], \omega \geq 0 \) where \( \omega \) denotes the signal to noise ratio (SNR) loss factor in dB. So the received signal at the relay node can be modelled as a complex Gaussian random variable. \( x_k^{MSB}[n] \) represents the message bit from the MSB side which is used for M-ary modulation scheme. The parameters \( j_1, j_2, x_1^{MSB}[n], x_2^{MSB}[n] \) are obtained using maximum likelihood (ML) detection method [32] as shown below:

\[
\hat{x}_1^{MSB}[n] = \frac{(j_1, j_2, \hat{x}_1^{MSB}[n], \hat{x}_2^{MSB}[n])}{(j_1, j_2, \hat{x}_1^{MSB}[n], \hat{x}_2^{MSB}[n])} \left[ y_{R}^{\hat{n}}[n] - \sum_{k=1}^{2} \sqrt{P_k} h_{k,j_R} x_k^{MSB}[n] \right]^2
\]  

(6)

Here \( j_1 \) is the estimated transmit laser index of \( S_1 \), \( j_2 \) is the estimated transmit laser index of \( S_2 \). The laser source selection at the relay for the next time slot is done according to the formula \( j_R = j_1 \oplus j_2 \). The encoded symbol \( x_k^{MSB}[n] \), forwarded by the relay to the source nodes is given by:

\[
x_k^{MSB}[n] = x_1^{MSB}[n] \oplus x_2^{MSB}[n]
\]  

(7)

where the estimated values of the message bits coming from the two sources, at the relay are \( x_1^{MSB}[n] \) and \( x_2^{MSB}[n] \) and \( \oplus \) is the bitwise XOR operator. The source nodes receive the signals in the next time slot, which is given by:

\[
y_{k}^{j_R}[n+1] = \sqrt{R_c P_k G_{S_k,R}} h_{j_k,j_R} x_k^{MSB}[n] + i_{k}[n] + w_k[n], \quad k = 1, 2
\]  

(8)

where \( j_k \) denotes the photodetector of the \( k^{th} \) source node. If channel state information (CSI) is available at the source nodes, the parameters \( j_R, x_k^{MSB}[n] \) are detected using ML method. The symbols estimated at the source node are \( j_R \) and \( x_k^{MSB} \). The symbols
detected at the $k^{th}$ source node are denoted by $\hat{x}_k^{MSB}$ and $\hat{x}_k^{LSB}$.

$$\hat{x}_k^{LSB}[n] = j_R \oplus j_k, \quad k = 1, 2$$

(9)

$$\hat{x}_k^{MSB}[n] = \hat{x}_k^{MSB}[n] \oplus \hat{x}_k^{MSB}[n], \quad k = 1, 2$$

(10)

4 Performance analysis

Analytical expressions for lower and upper bounds on outage probability have been derived and verified in this section.

4.1 Lower bound of outage probability

Assume $\eta_k = \frac{P_k}{N_0}$, $\eta_r = \frac{P_r}{N_0}$, $k = 1, 2$ be the signal to noise ratio (SNR) at source nodes and relay node respectively. Thus $\eta_k$ and $\eta_r$ are the average electrical SNRs at the corresponding nodes. The signal to interference plus noise ratio (SINR) at relay node and source node $S_k$ are denoted by $\gamma_{S_k,R}$ and $\gamma_{S_k,R,k}$ respectively. The signal to interference plus noise ratio (SINR) at relay node is given by:

$$\gamma_{S_k,R} = \frac{\eta_r R_c G_{S_k,R} (h_{k,j,k,j,n})^2}{\frac{L_s n_0}{N_0} + 1}, \quad k = 1, 2$$

(11)

The SINR at source node $S_k$ is given by

$$\gamma_{S_k,R,k} = \frac{\eta_k R_c G_{S_k,R} (h_{k,j,k,j,n})^2}{\frac{L_s n_0}{N_0} + 1}, \quad k = 1, 2$$

(12)

The lower bound on the outage probability of OSM based DFTWR for a data rate of $R_d$ bits/s can be defined as:

$$P_{out,min}^{LB}(R_d) = P_r[2 \ast \log_2(N_L) + \log_2\left(1 + \min(\gamma_{S_k,R,k}^1, \gamma_{S_k,R,k}^2, \ldots, \gamma_{S_k,R,k}^N)\right) \times \frac{4}{\gamma_{S_k,R,k}^\min} \geq R_d \frac{L_s n_0}{N_0} + 1]$$

(13)

It is assumed that there is no error in laser selection. The addition with the pre-log term is due to the fact that $\log_2(N_L)$ bits are used for activating a particular laser and since it is full duplex communication so $2 \ast \log_2(N_L)$ bits will be used. The detailed explanation for this equation is provided in Appendix. The max-min criteria has been used for selecting the optimum SNR for lower bound in this case [33, 34]. Assume $\psi = \min(\gamma_{S_k,R,k}^1, \gamma_{S_k,R,k}^2, \ldots, \gamma_{S_k,R,k}^N)$ and $\gamma_{th} = 2R_d - 2\log_2(N_L) - 1$. The CDF of $\psi$ is written as

$$P_{out,min}^{LB}(\gamma_{th}) = F_{\psi} (\gamma_{th})$$

$$= P\left\{ \min(\gamma_{S_k,R,k}^1, \gamma_{S_k,R,k}^2, \ldots, \gamma_{S_k,R,k}^N) \leq \gamma_{th}\right\}$$

$$= 1 - \prod_{k=1}^N \left\{1 - P\left(\gamma_{S_k,R,k} \leq \gamma_{th}\right)\right\}$$

$$= 1 - \prod_{k=1}^N \left\{1 - \left\{1 - P\left(\gamma_{S_k,R,k} \leq \gamma_{th}\right), \ldots, \gamma_{S_k,R,k} \leq \gamma_{th}\right\}\right\}$$

$$= 1 - \prod_{k=1}^N \left\{1 - \left\{1 - P\left(\gamma_{S_k,R,k} \leq \gamma_{th}\right) \right\} \times \left\{1 - P\left(\gamma_{S_k,R,k} \leq \gamma_{th}\right) \right\}\right\}$$

$$= 1 - \prod_{k=1}^N \left\{1 - F_{\gamma_{S_k,R,k}} (\gamma_{th}) \right\}$$

(14)

$|h_{k,j,k,j,n}|$ follows cascaded N-$\gamma$ distribution with N=2. Let $y = |h_{k,j,k,j,n}|^2$. The final CDF of SINR, following CDF given in Eq. 3, can be written as using Eq. 11:

$$F_{\gamma_{S_k,R,k}} (\gamma_{th}) = \left(\frac{\alpha \beta}{\alpha + \beta}\right) \left(\frac{\gamma_{th} \omega_{abs} \gamma_{\psi}}{\gamma_{\psi} \omega_{abs} \gamma_{S_k,R,k}} \right)^{\frac{\alpha + \beta}{\alpha \beta}}$$

(15)

where $\omega_{abs} = 10^{0.5/10}$. Depending upon the value of $k$, $G_{S_k,R}$ or $G_{S_k,R,k}$ and $\eta_k$ or $\eta_r$ are chosen. $\eta_k$ denotes the selected laser source of $k^{th}$ node and $j_R$ denotes the selected photodetector at the relay node. The terms $F_{\gamma_{S_k,R,k}} (\gamma_{th})$, $F_{\gamma_{S_k,R}} (\gamma_{th})$ etc. are obtained using Eq. 15. These values are put into Eq. 14 to obtain the final CDF of $\psi$. However there is no closed form expression available so the calculation has been solved in Mathematica.

Now the outage probability at the source node $S_k$, $k = 1, 2$ can be computed as [34]

$$R_p = P [2 \ast \log_2(N_L) + \log_2\left(1 + \min(\gamma_{S_k,R,k}^1, \gamma_{S_k,R,k}^2, \ldots, \gamma_{S_k,R,k}^N)\right) \leq R_d]$$

$$= 1 - \prod_{k=1}^N \left\{1 - (1 - F_{\gamma_{S_k,R,k}} (\gamma_{th}))\right\}, \quad k = 1, 2$$

(16)

The pre-log term is used due to the same reason as explained earlier in (13). Assume $\psi_{S_k} = \min(\gamma_{S_k,R,k}^1, \gamma_{S_k,R,k}^2, \ldots, \gamma_{S_k,R,k}^N)$. The CDF of $\psi_{S_k}$ is written as

$$F_{\psi_{S_k}} (\gamma_{th}) = F_{\psi} (\gamma_{th})$$

$$= \left(\frac{\alpha \beta}{\alpha + \beta}\right) \left(\frac{\gamma_{th} \omega_{abs} \gamma_{\psi}}{\gamma_{\psi} \omega_{abs} \gamma_{S_k,R,k}} \right)^{\frac{\alpha + \beta}{\alpha \beta}}$$

(18)

where depending upon the value of $k$, $G_{S_k,R}$ or $G_{S_k,R}$ is chosen. Here $j_R$ denotes the photodetector of the source node $S_k$. The term $F_{\psi_{S_k}} (\gamma_{th})$ is computed in Eq. 17 using Eq. 18.

The overall outage probability of the system can be computed by combining the outage probabilities at the source and relay nodes. It is defined as:

$$P_{out} (\gamma_{th}) = P_{out,min}^{LB}(\gamma_{th}) + (1 - P_{out,min}^{LB}(\gamma_{th})) F_{\psi} (\gamma_{th})$$

$$= 1 - \prod_{k=1}^N \left\{1 - F_{\gamma_{S_k,R,k}} (\gamma_{th}) \right\}$$

$$+ \left[\prod_{k=1}^N (1 - F_{\gamma_{S_k,R,k}} (\gamma_{th})) \right] \times \left[\prod_{k=1}^N \left\{1 - F_{\gamma_{S_k,R,k}} (\gamma_{th})\right\}\right]$$

$$= 1 - \prod_{k=1}^N \left\{1 - (1 - F_{\gamma_{S_k,R,k}} (\gamma_{th}))\right\}$$

(19)

The total transmission consists of two phases. In the first phase, when the source nodes transmit message to the relay, the outage
probability can be defined by the term $P_{out}^{UB}$, obtained from Eq. 14. The second phase is the broadcasting stage from the relay. The second term in Eq. 19 i.e. $(1 - P_{out}^{UB}(\gamma_{th}))P_{out}^{S_{th}}(\gamma_{th})$ gives the outage probability for this phase. $P_{out}^{S_{th}}(\gamma_{th})$ is obtained by solving Eq. 17. Eq. 19 does not have closed form expression, so analysis is performed on the individual terms.

### 4.2 Upper bound of outage probability

Now the upper bound similarly can be derived for a data rate of $R_d$ bit/s,

$$P_{out}^{R_{UB}}(R_d) = P_{th}[2 \cdot \log_2(N_L)] + \log_2(1 + \min(\gamma_{S_1,R}, \frac{\gamma_{S_2,R}}{\gamma_{S_1,R}}, ..., \frac{\gamma_{S_{NL},R}}{\gamma_{S_{NL-1},R}}, \frac{\gamma_{S_{NL},R}}{\gamma_{S_{NL-1},R}})) < R_d$$

(20)

The other steps would remain the same as that of lower bound computation. The minimum SNR has to be chosen for the upper bound as explained in [34]. Assume $\psi = \min(\gamma_{S_1,R}, \frac{\gamma_{S_2,R}}{\gamma_{S_1,R}}, ..., \frac{\gamma_{S_{NL},R}}{\gamma_{S_{NL-1},R}}, \frac{\gamma_{S_{NL},R}}{\gamma_{S_{NL-1},R}})$ and $\gamma_{th} = 2^{R_d} - 2^{\log_2(N_L)} - 1$.

$$P_{out}^{R_{UB}}(\gamma_{th}) = F_{\psi_{th}}(\gamma_{th}) = 1 - \prod_{k=1}^{L}(\prod_{j=1}^{N_L}(1 - F_{\psi_{th}}(\gamma_{th})))$$

(21)

where $F_{\psi_{th}}(\gamma_{th})$ is defined in Eq. 15. So the overall upper bound on outage probability of the system can be defined as:

$$P_{out}^{UB}(\gamma_{th}) = P_{out}^{R_{UB}}(\gamma_{th}) + (1 - P_{out}^{R_{UB}}(\gamma_{th}))P_{out}^{S_{th}}(\gamma_{th})$$

= $1 - \prod_{k=1}^{L}(\prod_{j=1}^{N_L}(1 - F_{\psi_{th}}(\gamma_{th})))$

$$x 1 - \prod_{k=1}^{L}(\prod_{j=1}^{N_L}(1 - F_{\psi_{th}}(\gamma_{th})))$$

(22)

In the first phase, when the source nodes transmit message to the relay, the outage probability can be defined by the term $P_{out}^{ub}$. The second phase is the broadcasting stage from the relay. The second term in Eq. 22 i.e. $(1 - P_{out}^{ub}(\gamma_{th}))P_{out}^{S_{th}}(\gamma_{th})$ gives the outage probability for this phase. $P_{out}^{S_{th}}(\gamma_{th})$ is obtained by solving Eq. 17. Closed form expressions cannot be obtained due to the intractable nature of the terms. Hence they have been solved in Mathematica.

### 4.3 Average data rate

The average data rate for this proposed system can be computed for the proposed system and compared with the available systems. Amplify and forward two way relaying (AFTWR), amplify and forward one way relaying (AFOWR), ANC, and PLNC are the methods available in literature. Average data rate of the proposed system deploying DF relaying technique can be written as:

$$\bar{R} = E[2 \cdot \log_2(N_L)] + \log_2(1 + \min(\gamma_{S_1,R}, \frac{\gamma_{S_2,R}}{\gamma_{S_1,R}}, ..., \frac{\gamma_{S_{NL},R}}{\gamma_{S_{NL-1},R}}, \frac{\gamma_{S_{NL},R}}{\gamma_{S_{NL-1},R}}))$$

(23)

where $E[\cdot]$ denotes expectation. All the terms have been explained while deriving lower bound expression.

In AFTWR, the system suffers from more residual self-interference than AFOWR because all the nodes in AFTWR operate in full duplex mode, while only the relay operates in full duplex mode in AFOWR. The average data rates of the systems are defined in [35].

### 4.4 Outage probability analysis for direct system

A bidirectional system without relay (DFSO) has been considered and its outage probability has been derived for G-G channel. There are two devices $S_1$ and $S_2$ which are each equipped with a laser and a photodetector, so that they can exchange data with each other. The signal to interference plus noise ratio (SINR) at any of the nodes is given by:

$$\gamma_{S_1,S_2} = \frac{h_{S_1,S_2}}{\sqrt{\eta_1^2} + 1}$$

(24)

where $h_{S_1,S_2}$ denotes the channel coefficient between the two devices $S_1$ and $S_2$, following Gamma-Gamma distribution. 

$$P_{out} = P(\gamma_{S_1,S_2} < \gamma_{th}) = F_{\gamma_{S_1,S_2}}(\gamma_{th})$$

(25)

$$\frac{\Gamma(\alpha_2)\Gamma(\beta_2)}{\Gamma(\alpha_1)\Gamma(\beta_1)}(\frac{\gamma_{th}}{\eta_1})^{\frac{\alpha_2}{\alpha_1} - 1} (\frac{\gamma_{th}}{\eta_1})^{\beta_2 - 1} - (\frac{\gamma_{th}}{\eta_1})^{\beta_2 - 1}$$

where $\gamma_{th} = 2^{R_d} - 1$ and CDF can be written using Eq. 3 and Eq. 24. For this analysis, link distance will be 4 Km as no relay is used. Accordingly values of $\alpha$ and $\beta$ have been calculated.

### 5 Results and Discussion

In this section, bounds on outage probability have been plotted analytically for various system parameters and verified by means of Monte Carlo simulations. Strong AF, $N_{L}=2$ and $L$ (distance between source nodes and relay)=2 Km are considered unless explicitly stated.

The outage performance of OSM based DFTWR system is compared with PLNC evaluating all the lower and upper bounds. It is evident from Fig. 2 that outage probability value is least for OSM system. This result is consistent with the results in [15] also where the authors have used log normal channels. The outage probability for PLNC and log normal channel have been calculated using Eq. (14) of [15], and subsequently the results have been plotted in Fig. 2 of our paper with the labelling as PLNC_LOG. The outage probability for direct FSO have been calculated for our parameters using Eq. (15) of [15] and the results have been labelled as DFSO_LOG in Fig. 2 of our paper. So we have compared our proposed results with two methods available in literature and we can observe that our proposed methods performs better with the same parameters.

The different SNR values required to achieve certain target outage probability values for different methods have been tabulated in Table.
5. Three different target outage probability values are chosen as 0.1, 0.08, and 0.05. The SNR gain of OSM based DFTWR over PLNC is 6 dB or more which signifies the fact that the proposed system is beneficial. The SNR gain of OSM based DFTWR over conventional DFSO is more than 10 for all cases. As the outage value decreases, the gain in SNR for our proposed method is much more which is beneficial. This is due to the fact that extra bits are used for estimating the trans-remitting optical source, causing one optical RF chain to be active at a time, thereby reducing the inter-optical interference.

### Table 5: Comparison of lower bounds on outage probability

<table>
<thead>
<tr>
<th>$P_{out}$</th>
<th>OSM_DFTWR</th>
<th>PLNC</th>
<th>DFSO</th>
<th>PLNC_LOG</th>
<th>DFSO_LOG</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>19</td>
<td>26</td>
<td>30</td>
<td>33</td>
<td>35</td>
</tr>
<tr>
<td>0.08</td>
<td>21</td>
<td>28</td>
<td>32</td>
<td>35</td>
<td>37</td>
</tr>
<tr>
<td>0.05</td>
<td>25</td>
<td>31</td>
<td>36</td>
<td>36</td>
<td>38</td>
</tr>
</tbody>
</table>

![Fig. 2: Comparison of outage probability between different systems.](image1)

The upper bounds on outage performance of the proposed method is compared with that of PLNC in Fig. 3 analytically and by means of Monte Carlo simulations. The bounds show improvement in performance for OSM based DFTWR for both $\omega=0$ and $\omega=3$. It can be seen that the proposed system has 2 bps data rate more than that of PLNC. This is due to the bits which are required for laser selection which in our case is 1. Since the communication is full duplex, hence the increment factor in data rate will be 2. This can be explained with an example. For $\omega=3$ and outage probability value of 0.15, PLNC has a data rate of 3 bps while OSM based DFTWR has a data rate of 5 bps. It can also be observed that the outage probability increases with an increase in data rate leading to inferior system performance. For OSM based DFTWR and $\omega=0$, when SNR at the two nodes is 30 dB, outage probability is 0.03 for data rate (R)=3 while outage probability is 0.13 for R=5 and 0.75 for R=10. So we can conclude that lower target data rates are suitable for better system performance.

For different SNR loss factors, $\omega=0$ dB, $\omega=3$ dB and $\omega=6$ dB, the lower bounds on outage probabilities have been shown in Fig. 4. The target data rate is 3 bps and the SNR at relay node is equal to the SNR at the source nodes. It can be observed from the graphs that increase in SNR loss factor degrades the system performance due to increase in inter-laser optical interference. The increase in self-interference causes an increase in outage probability. For example, for SNR=20 dB, the outage probability value for $\omega=0$ dB is 0.08, outage probability value for $\omega=3$ dB is 0.09, while for $\omega=6$ dB, the value is 0.13. Thus we can observe that the outage probability value increases with increase in SNR loss factor.

In Fig. 5, the performance of OSM based DFTWR has been analysed in terms of link distance. It is evident from the graph, that as the link distance increases (as distance between source nodes and relay is increased), the outage probability increase leading to inferior system performance. As distance increases, there is an increase in fading caused by atmospheric turbulence leading to such results. The analytical results have been justified by means of simulation results.

The performance of OSM_DFTWR has been verified by Monte Carlo simulations for different atmospheric conditions in Fig. 6. The simulation results are in close agreement with the analytical results. This system gives reasonably good performance even under strong and moderate atmospheric turbulences where fluctuations in atmosphere is much more. The outage probability is least for weak condition while for strong condition, outage probability is maximum. So we can conclude that G-G channel for this proposed system is suitable as it gives optimum performance even in harsh atmospheric conditions.

The number of transmitting laser is varied and its effect on outage probability for OSM based DFTWR is studied analytically in Fig. 7 and verified by means of Monte Carlo simulations also. SNR of both the nodes and the relay is varied while $N_L$ is varied as 2, 4, and 8. We know that spectral efficiency will increase with increase in number
of transmit lasers but the outage probability will also increase with increase in \( N_L \). There will be an increase in bit rate by a factor of 2 if we use \( N_L=4 \) system rather than a \( N_L=2 \) system, but it also requires an additional 4 dB of SNR to maintain the same outage value as that of \( N_L=2 \) system. The coding gain is equal to the horizontal shift of the outage curves. Similarly to increase the bit rate by a factor of 3 for \( N_L=8 \) system, it requires additional \( 4 + 4 = 8 \) dB of SNR as compared to \( N_L=2 \) system. Thus there should be a compromise between spectral efficiency and error rate. The increase in transmit lasers will lead to more inter-laser optical interference thereby leading to a decrease in performance.

### 6 Conclusion

The analytical expressions for outage probability of OSM based DFTWR system over G-G channels has been derived and verified with Monte Carlo simulations. Laser source activation is done based on the message bits. This system will be beneficial for device to device cooperative communication. The proposed system shows improvement in terms of outage performance and average data rate performance over the systems available in literature. Also it takes only two time slots to perform the OSM based DFTWR operation thereby doubling the efficiency of a conventional AF/DF system.

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**Appendix**

Referring to Eq. (7) of [37], we can write:

\[
I(X, X_{ch}; Y) = I(X; Y|X_{ch}) + I(X_{ch}; Y)
\]  

(26)

where \( I(X; Y|X_{ch}) \) denotes the mutual information between the modulation domain space and the output space, and \( I(X_{ch}; Y) \) denotes the mutual information between the spatial domain space and the output space. Here one transmit source is activated which is determined by the source laser symbol \( x_{ch} \), \( x \) is the symbol emitted from the transmitted laser and modulated by a conventional modulation scheme and \( y \) is the output. So \( I(X; Y|X_{ch}) \) is determined by the channel model and modulation scheme which we are using and we have computed that in our paper. To calculate the mutual information between spatial domain and output space, we take the help of upper bound as closed form expression is not available.

\[
I(X_{ch}; Y) = H(Y) - H(Y|X_{ch})
\]  

(27)

and the conditional entropy of the received signal, \( H(Y|X_{ch}) \) can be written as:

\[
H(Y|X_{ch}) = -\frac{1}{N} \sum_{i=1}^{N} \int_{0}^{\infty} \frac{1}{\pi \sigma_i^2} e^{-\frac{|y|^2}{\sigma_i^2}} \log_2 \left[ \frac{1}{\pi \sigma_i^2} e^{-\frac{|y|^2}{\sigma_i^2}} \right] dy
\]

(28)

Putting Eq. (28) and Eq. (29) in Eq. (27), we obtain the upper bound of the spatial domain mutual information as:

\[
I(X_{ch}; Y) \leq \log_2 N
\]  

(30)

Since our system is bidirectional so \( 2 \log_2 (N_L) \) term will be added to the mutual information of modulation domain and output domain space. Capacity is the maximum amount of mutual information that
can be transmitted over a given channel bandwidth. Hence the extra term has to be added to the capacity expression.

Referring to Eq (3)- (6) of [38], the authors have clearly mentioned that the SM capacity is computed as the capacity of conventional modulation plus the capacity of the space domain. Referring to Eq. (5) of [38] which calculates the capacity for space domain, we can observe that no closed form expression is available, so $I(X;Y) = \log_2(N)$. Since our system is bidirectional so the term $2\log_2(N)$ is added. On observing Eq. (13) of [39] and the corresponding explanation for the factor $N$, we can notice that outage probability has been calculated for SM. The (R-2) factor is due to spatial modulation in this paper. Referring to Eq. (5) of [40], it can be observed that the authors have again considered the term $\log_2(N_v)$ for capacity calculation of SM. All these explanations are considering that no closed form expression is achievable for calculating capacity of spatial domain part. This same assumption has been considered in our work and hence we have added the same factor.

7 References