Fundamentals of MIMO Wireless Communications
Part V

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Antenna selection and spatial modulation

- Multiple-input multiple-output (MIMO) systems
  - increases capacity and
  - minimize bit error rate
- as compared to single-input single-output (SISO) system
- But, they have
  - higher fabrication cost and
  - energy consumption
- due to multiple radio frequency (RF) chains
Antenna selection and spatial modulation

• An RF chain usually has
  • low noise amplifier,
  • frequency converters,
  • analog-digital and
  • digital-analog converters,
  • filters,
  • etc.
Antenna selection and spatial modulation

• A dedicated RF chain is needed for every antenna
  • which makes implementation cost and complexity higher
• Antenna selection minimizes this
  • by using lesser number of RF chains and
  • by connecting selected antennas with RF chains
• with the help of switches
Antenna selection and spatial modulation

• Select the best set of antennas at the transmitter or receiver
  • so as to maximize the
  • channel capacity or
  • received signal-to-noise ratio (SNR)
Antenna selection and spatial modulation

- Fig. Transmit antenna selection in MIMO systems
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• A single transmitting antenna which
  • maximizes the SNR at the receiver is selected
• to transmit the message bits as shown by dashed lines in Fig.
• We have shown antenna 2 as selected antenna for illustration purpose only
  • but it could be any one the transmit antennas
  • which maximizes the received SNR
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- Basically, there will be a RF switch
  - which will connect to the selected transmitting antenna and
  - sends signals from that particular transmitting antenna only
- All other antennas at the transmitter are sitting idle
- Hence there are no issue for inter-antenna interference
- this MIMO technique,
  - after single antenna selection at the transmitter,
  - the whole system looks like a SIMO system
Antenna selection and spatial modulation

• one can apply suitable diversity combining scheme like  
  • maximal ratio combining (MRC) or  
  • selection combining (SC) at the receiver  
• Based on the type of the diversity scheme employed at the receiver,  
• there are two types of TAS as follows  
• (a) Transmit Antenna Selection (TAS)/ Selection Combining (SC)  
• (b) Transmit Antenna Selection (TAS)/ Maximal ratio combining (MRC)
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• Let the fading coefficients from \( i^{th} \) transmitting antenna to \( j^{th} \) receiving antenna is denoted by 

\[
    h_{ij}, i \in \left[ 1, N_T \right], j \in \left[ 1, N_R \right]
\]

• Two types of Transmit Antenna Selection (TAS) is described below

• (a) Transmit Antenna Selection (TAS)/ Selection Combining (SC)

• We select only one antenna at the transmitter and receiver
  • that gives the highest SNR

• The link which gives the highest received SNR is determined by
Antenna selection and spatial modulation

\[ I_{ij} = \arg \max_{1 \leq i \leq N_T, 1 \leq j \leq N_R} \left\{ h_{t,ij} = |h_{i,j}|^2 \right\} \]

- where \( i,j \) represent the antennas corresponding to the best link at transmitter and receiver respectively and
- \( I_{ij} \) denote the best link
- Only the antennas that correspond to the best link are active at a time in this case
- So we have single RF chain at the transmitter as well as the receiver
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- (b) Transmit Antenna Selection (TAS)/ Maximal ratio combining (MRC)
- It is assumed that the number of RF chains at receiver is same as the number of receiver antennas
- The resulting received SNR for Maximal ratio combining (MRC) combining scheme is given by

\[ \gamma_t = \sum_{n=1}^{N_r} \gamma_n \]
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- where \( \gamma_n \) is the instantaneous SNR of the \( n^{th} \) branch
- The transmitting antenna that maximizes SNR at the receiver can be determined by

\[
I_i = \arg \max_{1 \leq i \leq N_T} \left\{ h_{t,i} = \sum_{j=1}^{N_R} |h_{i,j}|^2 \right\}
\]

- where \( I_i \) is the transmitting antenna that maximizes the received SNR
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• Example

• Find the probability of error for Transmit Antenna Selection (TAS)/Maximal ratio combining (MRC) over i.i.d. Rayleigh fading MIMO channel

• Solution:

• We may select the antenna which maximize the receive SNR

\[ I_i = \arg \max_{1 \leq i \leq N_T} \left\{ h_{t,i} = \sum_{j=1}^{N_R} |h_{i,j}|^2 \right\} \]
Antenna selection and spatial modulation

• For Rayleigh fading channel,
  • $I_i$ are i.i.d. Chi square distributed with the
  • probability density function (PDF) and
    \[
p(x) = \frac{x^{N_r-1}e^{-x}}{\Gamma(N_r)}, \quad x \geq 0
    \]
  • cumulative distribution function (CDF)
    \[
P(x) = 1 - e^{-x} \sum_{i=0}^{N_r-1} \frac{x^i}{i!}, \quad x \geq 0
    \]
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• Using order statistics, PDF of $I_{(N_r)}$ such that $I_1 \leq I_2 \leq \cdots \leq I_{(N_r)}$ can be given as

$$p_{(N_t)}(x) = N_t [P(x)]^{N_t-1} p(x) = N_t \left[ 1 - e^{-x} \sum_{i=0}^{N_r-1} \frac{x^i}{i!} \right]^{N_t-1} \frac{x^{N_r-1} e^{-x}}{\Gamma(N_r)}$$

• Assume binary phase shift keying (BPSK) for TAS/MRC MIMO system, then instantaneous SNR at the MRC receiver
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\[ \gamma_b = \gamma \sum_{j=1}^{N_r} \left| h_{N_t, j} \right|^2 = \gamma (N_t) \]

- The average bit error rate (BER) can be obtained from conditional error probability (CEP) for BPSK

\[ P_e = \int_0^\infty Q\left(\sqrt{2\gamma_b}\right) p_{\gamma_b}(\gamma_b) d\gamma_b \]
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• Thus, closed form expression for BER is

\[ P_e = \frac{N_t}{\Gamma(N_r)} \sum_{k=0}^{N_t-1} \left[ \frac{(-1)^k}{[2(k+1)]^{N_r}} \left( \frac{N_t-1}{k} \right)^{k(N_r-1)} \sum_{t=0}^{N_r} \sum_{j=0}^{N_r-t-1} f_4 f_5 \right] \]

• where the expression for functions are

\[ f_4 = a_t(N_r, k)(N_r + t - 1)! \left( 1 - \sqrt{\frac{\gamma}{\gamma + k + 1}} \right)^{N_r + t} \]

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• where \( a_t(N_r, k) \) is the coefficient of \( z^{2t} \) in the expansion of

\[
\left( \sum_{i=0}^{N_r-1} \frac{z^2}{2(k+1)} \right)^k
\]

\[
\left( \sum_{i=0}^{N_r-1} \frac{z^2}{2(k+1)} \right)^k
\]
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\[ f_5 = 2^{-j} \left( N_r + t - 1 + j \right) \left( 1 + \sqrt{\frac{\gamma}{\gamma + k + 1}} \right)^j \]
Antenna selection and spatial modulation

• Soft antenna selection for closely spaced antennas
• Hard antenna selection (HAS) does not have good performance for practical situation
  • like closely spaced antennas
  • which results in high antenna correlation

Antenna selection and spatial modulation

- This problem can be overcome by using soft antenna selection (SAS)
- In this, all the antennas are active and
  - a transformation is performed in RF domain upon the received signals across all the antennas
  - and select antennas after the transformation
- Some SAS schemes are:
  - (a) Fast Fourier Transform (FFT) based selection
  - (b) Phase shift based selection
Antenna selection and spatial modulation

• Suppose we have $N_T \times N_R$ MIMO system
  • Let L be the number of antennas to be selected at the receiver
  • At the receiver, the best L antenna elements are selected

• In HAS, we choose the best subset of antenna elements
  • for which the capacity of the system is highest among all capacity values
  • achieved by any other possible antenna subset
Antenna selection and spatial modulation

- The instantaneous capacity expression for a MIMO system with uniform power allocation is given by

\[
C = \log_2 \det \left( \mathbf{I}_{N_T} + \frac{\rho}{N_T} \mathbf{H}^H \mathbf{H} \right)
\]

- With antenna selection under capacity maximization, the effective capacity now becomes

\[
C = \log_2 \det \left( \mathbf{I}_{N_T} + \frac{\rho}{N_T} \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \right)
\]
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• To perform receive antenna selection a matrix $\tilde{\mathbf{H}}$ of $L \times N_T$ is selected from the full channel matrix $\mathbf{H}$,
• such that the chosen subset created by striking $N_R - L$ rows from $\mathbf{H}$ results in maximum capacity.
Antenna selection and spatial modulation

• For spatial multiplexing systems,
  • capacity due to antenna selection at the receiver has been shown to be
  • comparable to the full complexity system
  • as long as the number of selected chains at the receiver
  • is greater than or equal to the number of transmit antenna elements i.e.,

\[ L \geq N_T \]
Antenna selection and spatial modulation

• For correlated MIMO channels,
  • HAS schemes perform considerably worse than a full complexity system
  • SAS schemes have been proved to be better for correlated channel
• Let us assume correlated Rayleigh fading MIMO channel (Kronecker model, see chapter 3)
Antenna selection and spatial modulation

- The \((i, j)^{th}\) entry of \(\mathbf{R}_{Tx}\) and \(\mathbf{R}_{Rx}\) is given by

\[
J_0 \left( \frac{2\pi d_{ij}}{\lambda} \right)
\]

- where \(J_0\) is the zeroth order Bessel function of the first kind,
- and \(d_{ij}\) denotes the distance between the \((i, j)^{th}\) antenna elements.
Antenna selection and spatial modulation

- Hybrid antenna selection:
  - $n_R$ antennas are selected out of $N_R$ receiving antennas like in conventional hard antenna selection.
  - Note that HAS is to maximize the capacity but hard antenna selection is to maximize the SNR.
  - The capacity for this scheme is

$$C_{Hybrid} = \max_{S \in S_{col}} \log_2 \det \left( I_{N_T} + \frac{\rho}{N_T} (SH)^H SH \right)$$
Antenna selection and spatial modulation

- where $S$ is an $n_R \times N_R$ matrix, defined as selection matrix
- that extracts $n_R$ rows from $H$ that are associated with the selected subset of antennas,
- whose cardinality $|S_{col}| = \binom{N_R}{n_R}$ is given by
- where $S_{col}$ is the collection of all possible selection matrices
Antenna selection and spatial modulation

- **FFT-based Selection:**
- For this SAS scheme, the antenna selection is performed on the virtual channel $\mathbf{F}_H$
- where $\mathbf{F}$ is $N \times N$ FFT matrix
- All the received observation streams are sent through a Fourier transform before selection
- The $(k,l)^{th}$ entry of $\mathbf{F}$ is given by:

$$F(k,l) = \frac{1}{\sqrt{N}} \exp \left\{ -j2\pi \frac{(k-1)(l-1)}{N} \right\}$$
Antenna selection and spatial modulation

- \( \forall k, l \in [1, N] \)
- This system capacity can be expressed as

\[
C_{\text{FFT}} = \max_{S \in \mathcal{S}_{\text{col}}} \log_2 \det \left( \mathbf{I}_{N_T} + \frac{\rho}{N_T} \mathbf{S}_{\text{FH}}^H \mathbf{S}_{\text{FH}} \right)
\]
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- **Phase-Shift Based Selection:**
- This is another type of soft selection scheme in which all the received observation streams are passed through a phase shift matrix $\Theta$
- The matrix $\Theta$ is a $n_R \times N_R$ which performs phase shift implementation in the RF domain
- It serves as a $N_R - to - n_R$ switch with $n_R$ output streams
Antenna selection and spatial modulation

- The SVD of $\mathbf{H}$ as $\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H$
- Let the largest singular value of $\mathbf{H}$ be denoted as $\lambda^1_H$
- $\mathbf{u}^1_H$ as the left singular vector of $\mathbf{H}$ associated with $\lambda^1_H$
- second largest singular value of $\mathbf{H}$ be denoted as $\lambda^2_H$
- $\mathbf{u}^2_H$ as the left singular vector of $\mathbf{H}$ associated with $\lambda^2_H$
- The phase shift matrix $\mathbf{\Theta}$ can be expressed as
  $$\mathbf{\Theta} = \exp\{j \times \text{angle}\{\mathbf{u}^1_H, \mathbf{u}^2_H, \ldots, \mathbf{u}^n_R\}\}$$
Antenna selection and spatial modulation

• where $\text{angle}\{\cdot\}$ gives the phase angles, in radians, of a matrix with complex elements,

• $\exp\{\cdot\}$ denotes the element-by-element exponential of a matrix

• The capacity expression for this system can be written as

$$C_{PS} = \log_2 \det \left( I_{N_T} + \frac{\rho}{N_T} (\Theta H)^H (\Theta \Theta^H)^{-1} \Theta H \right)$$

Antenna selection and spatial modulation

• What is Spatial Modulation?
• In Spatial modulation, we do modulation over space
• Assume $k$ bit information blocks are to be sent from the spatial modulation (SM) based transmitter
• First it makes sub-blocks of $n$ bits and $m$ bits
  • where $n$ bits are spatially modulated ($n$ bits decides which antenna will be active and transmitting the $m$ bits) and
  • $m$ bits are modulated using digital modulation schemes like M-ary modulation
Antenna selection and spatial modulation

• In other words, \( m \) bits are transmitted physically,
  • but effectively similar to transmitting \( k=m+n \) bits
• There is a restriction on the number of transmit antennas and
• it should be a integer exponent of 2
Antenna selection and spatial modulation

Fig. SM MIMO system model
Antenna selection and spatial modulation

- Example
- Explain the transmission of 011 message bit using 2×2 and 4×4 SM MIMO systems
Antenna selection and spatial modulation

Fig. 2×2 SM MIMO system’s transmission of input message bit 011
Antenna selection and spatial modulation

- SM mapping table for $N_T=2$ and $M=4$ (QAM)

<table>
<thead>
<tr>
<th>Input bits</th>
<th>Antenna index</th>
<th>Transmit symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>1</td>
<td>1+j</td>
</tr>
<tr>
<td>001</td>
<td>1</td>
<td>1-j</td>
</tr>
<tr>
<td>010</td>
<td>1</td>
<td>-1-j</td>
</tr>
<tr>
<td>011</td>
<td>1</td>
<td>-1+j</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>1+j</td>
</tr>
<tr>
<td>101</td>
<td>2</td>
<td>1-j</td>
</tr>
<tr>
<td>110</td>
<td>2</td>
<td>-1-j</td>
</tr>
<tr>
<td>111</td>
<td>2</td>
<td>-1+j</td>
</tr>
</tbody>
</table>
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SM mapping table for $N=4$ and $M=2$ (BPSK)

<table>
<thead>
<tr>
<th>Input bits</th>
<th>Antenna index</th>
<th>Transmit symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>001</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>010</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>011</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>101</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>110</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>111</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
Antenna selection and spatial modulation

• Example
• Explain the reception of 011 message bit using 2×2 and 4×4 SM MIMO systems

• Solution:
• The first effort of the receiver is to estimate from which antenna the symbol has been sent
  • This will give some part of the message bit
• Then it estimates the symbol
  • This will recover all the message bits
Antenna selection and spatial modulation

- **Performance analysis of Spatial modulation**
- At the receiver there are two steps for detection of the transmitted message bits
- Assume that the transmit antenna \( j \) is active at a particular instant of time and
- the corresponding channel vector is \( \mathbf{h}_j \)
- Hence the received signal \( \mathbf{y} \) can be represented as

\[
\mathbf{y} = \mathbf{Hx}_{jq} + \mathbf{n}
\]
Antenna selection and spatial modulation

• which can be further simplified as

\[ y = h_j x_q + n \]

• where

\[
x_{jq} = \begin{bmatrix}
 j^{th} \text{ position} \\
 \rightarrow \\
 0 \\
 0 \\
 0 \\
 \vdots \\
 x_q \\
 \vdots \\
 0 \\
 0 
\end{bmatrix}
\]
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• First part of the receiver detection is the transmit antenna index \((j)\) estimation

\[
\hat{j} = \arg \max_j \frac{|h_j^H y|}{\|h_j\|^2}
\]

• Second part of the detection process detects the symbol which has been transmitted from the \(j^{th}\) transmit antenna

\[
x_q = \arg \min_q \left\| h_j^H x_q \right\|^2 - 2 \text{Re}\left\{ h_j^H y x_q^* \right\}
\]

Antenna selection and spatial modulation

• Assume two detection processes are independent i.e.,
• transmit antenna index estimate and estimation of the transmit symbol
• Let us denote $P_a$ is probability that the antenna index estimation is incorrect and
• $P_s$ is probability is that the transmitted symbol estimation is incorrect
• Then the probability of correct estimation can be represented as

$$P_c = (1 - P_a)(1 - P_s)$$
Antenna selection and spatial modulation

• Hence the probability of error is given as

\[ P_e = 1 - P_c = 1 - (1 - P_a)(1 - P_s) = P_a + P_s - P_a P_s \]

• If we assume that the M-ary modulation employed is QAM,

• then the conditional error of probability (CEP) is represented as

\[ P_s(E / \gamma) = aQ\left(\sqrt{b\gamma}\right) - cQ^2\left(\sqrt{d\gamma}\right) \]
Antenna selection and spatial modulation

• where \( a=2, \ b=1, \ c=1 \) and \( d=1 \) for 4-QAM

• Q-function can be approximated as a sum of two exponentials as follows

\[
Q(x) \approx \frac{1}{12} e^{-\frac{x^2}{2}} + \frac{1}{4} e^{-\frac{2x^2}{3}}
\]

• Then the above CEP can be approximated as

\[
P_s(E / \gamma) \approx \frac{a}{12} e^{-\frac{b\gamma}{2}} + \frac{a}{4} e^{-\frac{2b\gamma}{3}} - \frac{c}{144} e^{-b\lambda} - \frac{c}{16} e^{-\frac{4b\gamma}{3}} - \frac{c}{24} e^{-\frac{7b\gamma}{6}}
\]
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- In order to find the average probability of error, we need to integrate the CEP over the pdf of the received SNR, hence

\[
P_s(E) = \int_0^\infty P_e(E / \gamma)p_\gamma(\gamma)d\gamma = \int_0^\infty \left\{ aQ(\sqrt{b\gamma}) - cQ^2(\sqrt{d\gamma}) \right\}p_\gamma(\gamma)d\gamma
\]

\[
\Rightarrow P_s(E) \equiv \int_0^\infty \left( \frac{a}{12} e^{-\frac{b\gamma}{3}} + \frac{a}{4} e^{-\frac{2b\gamma}{3}} - \frac{c}{144} e^{-b\gamma} - \frac{c}{16} e^{-\frac{4b\gamma}{3}} - \frac{c}{24} e^{-\frac{7b\gamma}{6}} \right)p_\gamma(\gamma)d\gamma
\]
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• For 4-QAM, putting the values $a$, $b$, $c$ and $d$, we have,

\[ P_s(E) \approx \int_0^\infty \left( \frac{1}{6} e^{-\frac{\gamma}{2}} + \frac{1}{2} e^{-\frac{2\gamma}{3}} - \frac{1}{144} e^{-\gamma} - \frac{1}{16} e^{-\frac{4\gamma}{3}} - \frac{1}{24} e^{-\frac{7\gamma}{6}} \right) p_\gamma(\gamma) d\gamma \]

• The above integrals fit the definition of MGF of the received SNR and hence can be expressed in the form

\[ P_s(E) \equiv \sum_i \zeta_i \int_0^\infty e^{-\nu_i \gamma} p_\gamma(\gamma) d\gamma = \sum_i \zeta_i \text{MGF}_\gamma(\nu_i) \]
Antenna selection and spatial modulation

• Therefore,

\[
P_s(E) = \frac{1}{6} MGF_\gamma \left( \frac{1}{2} \right) + \frac{1}{2} MGF_\gamma \left( \frac{2}{3} \right) - \frac{1}{144} MGF_\gamma \left( \frac{1}{3} \right) - \frac{1}{16} MGF_\gamma \left( \frac{4}{3} \right) - \frac{1}{24} MGF_\gamma \left( \frac{7}{6} \right)
\]

• The probability of error in transmit antenna index estimation can be obtained as

\[
P_a = Q\left( \sqrt{\gamma_{\text{eff}}} \right) = \frac{1}{12} MGF_{\gamma_{\text{eff}}} \left( \frac{1}{4} \right) + \frac{1}{4} MGF_{\gamma_{\text{eff}}} \left( \frac{1}{3} \right)
\]
Antenna selection and spatial modulation

- where \( \gamma_{\text{eff}} = \frac{\gamma}{2} \| \mathbf{h}_j - \mathbf{h}_j \| \)

- The PDF of \( \eta-\mu \) fading distribution is given in

\[
p_{\eta-\mu}(x) = \frac{2\sqrt{\pi} h^\mu}{\Gamma(\mu)} \left( \frac{\mu}{\Omega_h} \right)^\mu \left( \frac{x}{H} \right)^{\mu-\frac{1}{2}} e^{-x} I_{\mu-\frac{1}{2}}(x'), x' = \frac{2\mu h x}{\Omega_h}
\]

- The MGF of \( \eta-\mu \) fading distribution (refer to chapter 2) can be obtained as

\[
\text{MGF}_{\eta-\mu}(s) = \left( \frac{4\mu^2 h}{(2\mu(h - H) + s)(2\mu(h + H) + s)} \right)^\mu
\]

Antenna selection and spatial modulation

• Hence, we can calculate the $P_s$ and $P_a$ and find the error probability of SM MIMO system over $\eta$-$\mu$ fading channel.
• Fig. depicts the SER vs SNR of $2\times2$ SM MIMO system over Nakagami-$q$ fading channel.
• which is a particular case of $\eta$-$\mu$ fading channel.
• It has been observed that for fixed $\mu=0.5$,
• as we increase $\nu$ from 0.1, 0.2, 0.3, 0.5 and 0.9,
• the SER improves consistently.
• Fig. SER vs SNR (dB) for 2×2 SM MIMO system considering Nakagami-q fading as a special case of η-μ fading channel for η=ν² and μ=0.5
Antenna selection and spatial modulation

• **Performance analysis of SM with antenna selection**

• The main problem with SM is that some of the antenna from which we are transmitting symbols may be experiencing the worst performance or dead link.

• Then we will have a very bad performance.

• In order to overcome this issue, we may do antenna selection at the beginning and apply SM on those antennas having the best links.

Antenna selection and spatial modulation

- SM systems combined with antenna selection can be divided into two phases:
  - (a) transmit antenna selection
  - (b) SM applied over selected antennas
- In order to do this, we need CSI available at transmitter
- $A_j$ is the received SNR due to transmission from antenna $j$ at the transmitter

\[
A_j = \sum_{i=1}^{N_r} |h_{i,j}|^2
\]
Antenna selection and spatial modulation

- Assuming i.i.d. MIMO Rayleigh fading channel,
- then $A_j$ has Chi-square distribution with the PDF

$$f_{A_j}(x) = \frac{x^{N_R-1} e^{-x}}{\Gamma(N_R)}, x \geq 0$$

- where $\Gamma(.)$ is the Gamma function and
- CDF

$$F_{A_j}(x) = 1 - e^{-x} \sum_{i=0}^{N_R-1} \frac{x^i}{i!}, x \geq 0$$
Antenna selection and spatial modulation

• From order statistics,
• we may select the best subset ($S$ out of $N_T$) of antennas at the transmitter
• It may be summarized as:
• (a) received SNR $A_j$s
• when only one transmit antenna ($j^{th}$ antenna) is active at the transmitter
  • are arranged in ascending order

\[
A_{(1)} \leq A_{(2)} \leq \cdots \leq A_{(r)} \leq \cdots \leq A_{(N_T-1)} \leq A_{(N_T)}
\]
Antenna selection and spatial modulation

- (b) $S$ antennas out of $N_T$
  - corresponding to highest $A_j$s (received SNR) are selected

\[
A_1 \leq A_2 \leq \cdots \leq A_r \leq \cdots \leq A_{N_T-1} \leq A_{N_T}
\]

- where $r = N_T - S + 1$
Antenna selection and spatial modulation

• PDF of $A_{(r)}$ can be given as

$$f_{A_{(r)}}(x) = \frac{1}{B\left(r, N_T - r + 1\right)} \left\{F_{A_j}(x)\right\}^{r-1} \left\{1 - F_{A_j}(x)\right\}^{N_T - r} f_{A_j}(x)$$

• where $B(\bullet, \bullet)$ is the Beta function

$$B(m, n) = \frac{(m-1)! (n-1)!}{(m+n-1)!}$$

Antenna selection and spatial modulation

- The PDF of received SNR $A_{(r)}$ can be given as

$$p_{A_{(r)}}(x) = \frac{1}{(N_T - r + 1) \Gamma(N_R)} \sum_{i=r}^{N_T} \sum_{j=0}^{i-1} \sum_{k=0}^{K} \frac{1}{B(i, N_T - i + 1)} \left( \begin{array}{c} i-1 \\ j \end{array} \right) (-1)^j C_k(j, N_R) x^{N_R + k - 1} e^{-x(N_T - i + j + 1)}$$
Antenna selection and spatial modulation

- where \( C_k(j, N_R) \) is the coefficient of \( x^k \) in the expansion of

\[
\left( \sum_{l=0}^{N_R-1} \frac{x^l}{l!} \right)^{N_T-i+j}
\]

- where \( K = (N_R - 1)(N_T - i + j) \)
Antenna selection and spatial modulation

- The transmission over a channel is in outage whenever the data rate for the transmission exceeds the capacity of the channel

\[
P_{\text{out}}(A_r, R) = P_r\left( A_r < \frac{2^R - 1}{\bar{A}(r)} = A_{th} \right)
\]

\[
P_{\text{out}}(A_r, R) = \frac{1}{(N_T - r + 1) \Gamma(N_R)} \sum_{i=r}^{N_T} \frac{1}{B(i, N_T - i + 1)} \sum_{j=0}^{i-1} (-1)^j \sum_{k=0}^{K} C_k(j, N_R) \gamma_{\text{inc}}(N_R + k, A_{th} (N_T - i + j + 1)) \left( N_T - i + j + 1 \right)^{N_R + k}
\]
Antenna selection and spatial modulation

• where \( C_k(j,N_R) \) is the coefficient of \( x^k \) in the expansion of

\[
\left( \sum_{l=0}^{N_R-1} \frac{x^l}{l!} \right)^{N_T-i+j}
\]

• where \( K = (N_R - 1)(N_T - i + j) \)
Fig. Outage probability vs. SNR curve for SM MIMO systems with antenna selection (R=2 bits/s/Hz)
Antenna selection and spatial modulation

• 2×2 SM MIMO is the SM system without any antenna selection
• For the 2×2 SM MIMO system, we want to send two bits at a time
• We use 1 bit for antenna selection,
  • this bit will decide where to send the symbol from antenna 1 or 2 and
• the second bit will decide
  • which BPSK symbol will be sent from the selected antenna
Antenna selection and spatial modulation

- 4/2×2 SM MIMO means it is a 4×2 MIMO system
  - select the 2 transmit antennas with the best links
  - out of the 4 transmit antennas and
  - apply SM MIMO system on the corresponding 2×2 SM MIMO system

- 6/2×2 SM MIMO means
  - it is a 6×2 MIMO system in which
Antenna selection and spatial modulation

- select the 2 transmit antennas with the best links
- out of the 6 transmit antennas and
- apply SM MIMO system on the corresponding 2×2 SM MIMO system

Observation:
- 6/2×2 SM MIMO outperforms 4/2×2 SM MIMO
- whereas 4/2×2 SM MIMO has better performance
- than the traditional 2×2 SM MIMO system
Advanced topics in MIMO wireless communications

• SISO based cooperative communication

• In this communication, we assume that there are three nodes viz.
  • source (S),
  • destination (D) and
  • relay (R)

(a) Amplify and Forward (AF) Protocol:

• Every relay (R) node, amplifies and re-transmits to the destination (D) the signal
  • it received from the source (S)
Advanced topics in MIMO wireless communications

• (b) Decode and Forward (DF) Protocol:
  • Every relay (R) decodes transmitted symbol from the source (S) and
    • if the decoding is successful,
    • the relay (R) sends the re-encoded symbol to destination (D)
      • otherwise relay (R) sits idle
Advanced topics in MIMO wireless communications

- Fig. Single relay based cooperative communication system
Advanced topics in MIMO wireless communications

• In cooperative communication
  • signal reaches from S to D in two phases

• In first phase
  • signal is transmitted from the S to R

• In the second phase,
  • the signal is transmitted from R to D,
  • if R decides to re-transmit the received signal
Advanced topics in MIMO wireless communications

- Consider a single relay based cooperative communication system with:
  - one source (S) communicating with
  - one destination (D) and
  - one relay (R)
- as depicted in Fig.
- In the first phase, the S broadcasts message data to R and D
Advanced topics in MIMO wireless communications

- The signals received at D and R are expressed as

\[ y_{s,d} = \sqrt{p_1} f x + n_1 \]
\[ y_{s,r} = \sqrt{p_1} g x + n_2 \]

- \( p_1 \) is the transmitted signal power from S to R and S to D
- \( x \) symbol is the transmitted from S
- \( n_1 \) and \( n_2 \) are ZMCSCG RVs with variance \( N_0 \)
Advanced topics in MIMO wireless communications

- Channel gain coefficients $f$ and $g$ are assumed to be
  - $k$-$\mu$ or $\eta$-$\mu$ distributed fading coefficients
  - between source (S) and destination (D) and/or
  - between source (S) and relay (R) respectively
Advanced topics in MIMO wireless communications

• In second phase, for DF protocol,
  • if the relay (R) is able to decode the transmitted symbol correctly,
  • then it forwards the decoded symbol with transmission power $p_2$ to the destination (D)
  • Otherwise the relay (R) sits idle
• The signal received at the destination (D) can be expressed as

$$y_{r,d} = \sqrt{\tilde{p}_2} hx + n_3$$
Advanced topics in MIMO wireless communications

• The transmission power $\tilde{p}_2 = p_2$
  • if the relay (R) decodes the transmitted symbol correctly,
  • otherwise $\tilde{p}_2 = 0$

• $n_3$ is ZMCSCG with variance $N_0$

• The channel gain coefficient $h$ is assumed to be
  • $k-\mu$ or $\eta-\mu$ distributed fading coefficients between relay (R) and destination (D)
Advanced topics in MIMO wireless communications

- Assume CSIR is available, no CSIT
- The destination (D) jointly combines the phase 1 and 2 received signal from the source (S)
- The detector used at the destination (D) is maximal ratio combining (MRC)
- The total transmitted power has to satisfy

\[ p_1 + p_2 = p \]
• The combined signal at the destination (D) can be expressed as

\[ y = \frac{\sqrt{p_1} f^*}{N_0} y_{s,d} + \frac{\sqrt{\tilde{p}_2} h^*}{N_0} y_{r,d} \]

• where \( f^* \) and \( h^* \) are the complex conjugates of \( f \) and \( h \) respectively

• The received SNR at destination (D) is expressed as

\[ \gamma = \frac{p_1 |f|^2 + \tilde{p}_2 |h|^2}{N_0} \]
Advanced topics in MIMO wireless communications

• Note that function is defined and approximated as

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{y^2}{2}\right) dy \equiv \frac{1}{12} \exp\left(-\frac{x^2}{2}\right) + \frac{1}{4} \exp\left(-\frac{2x^2}{3}\right)
\]

• Hence, for BPSK

\[
\psi_{\text{BPSK}}(\gamma) = Q(\sqrt{2\gamma}) \equiv \frac{1}{12} \exp(-\gamma) + \frac{1}{4} \exp\left(-\frac{4\gamma}{3}\right)
\]
Advanced topics in MIMO wireless communications

- Where $\gamma$ is SNR,

- $\psi_{BPSK} (\gamma) = Q(\sqrt{2\gamma})$ is conditional probability of error for BPSK

- Hence, the conditional BER of cooperative communication over the fading channel employing BPSK
  - Can be calculated as sum of the conditional BER for the two phases
Advanced topics in MIMO wireless communications

• In first case, assume that there may be no successful decoding at the relay (R) \( \tilde{p}_2 = 0 \)
• hence MRC receiver combines two signals one from the relay (R) and the other from source (S)

\[
\begin{align*}
\psi_{BPSK} \left( \frac{p_1 |g|^2}{N_0} \right) \text{ is the chance of incorrect decoding at the relay (R)}
\end{align*}
\]
Advanced topics in MIMO wireless communications

- In second case, assume that there is successful decoding at the relay (R)
- and MRC receiver combines two signals one from the relay (R) and the other from source (S)

\[ \tilde{p}_2 = p_2 \]

- In case 2 for BPSK, the chance of correct decoding at the relay (R) is

\[ \left( 1 - \psi_{BPSK} \left( \frac{p_1 |g|^2}{N_0} \right) \right) \]
Advanced topics in MIMO wireless communications

- Hence,  
\[ P_{BPSK}^{\text{cond}} = P_{BPSK}^{\text{cond,1st}} + P_{BPSK}^{\text{cond,2nd}} \]

- where  
\[ P_{BPSK}^{\text{cond,1st}} \] and  
\[ P_{BPSK}^{\text{cond,2nd}} \] are respectively for first and second cases mentioned above.

- Therefore,
\[
P_{BPSK}^{\text{cond}} = \psi_{BPSK} \left( \frac{p_1 |g|^2}{N_0} \right) \times \psi_{BPSK} (\gamma)_{\bar{p}_2=0} + \left( 1 - \psi_{BPSK} \left( \frac{p_1 |g|^2}{N_0} \right) \right) \times \psi_{BPSK} (\gamma)_{\bar{p}_2=p_2}
\]
Advanced topics in MIMO wireless communications

\[ \gamma = \frac{p_1 |f|^2 + \tilde{p}_2 |h|^2}{N_0}; \tilde{p}_2 = \begin{cases} 0 & \text{case 1} \\ p_2 & \text{case 2} \end{cases} \]

\[ \therefore P_{\text{cond}}^{\text{BPSK}} = \psi_{\text{BPSK}} \left( \frac{p_1 |g|^2}{N_0} \right) \times \psi_{\text{BPSK}} \left( \frac{p_1 |f|^2}{N_0} \right) + \left( 1 - \psi_{\text{BPSK}} \left( \frac{p_1 |g|^2}{N_0} \right) \right) \times \psi_{\text{BPSK}} \left( \frac{p_1 |f|^2 + p_2 |h|^2}{N_0} \right) \]
Advanced topics in MIMO wireless communications

• Assume that the fading channels \( f, g \) and \( h \) are
  • independent of each other and are identically distributed
• We can calculate the approximate average BER by averaging
  • the conditional BER over the channel gain coefficients \( f, g \) and \( h \) as

\[
P_{BPSK}(e) = \zeta_{1st}^{s,r}(p_1) \zeta_{1st}^{s,d}(p_1) + \left(1 - \zeta_{1st}^{s,r}(p_1)\right) \zeta_{2nd}^{s,r,d}(p_1 + p_2)
\]
Advanced topics in MIMO wireless communications

• Where

\[
\xi_{1st}^{s,r}(p_1) = \frac{1}{12} \text{MGF}_{p_1}(1) + \frac{1}{4} \text{MGF}_{p_1}\left(\frac{4}{3}\right); \xi_{1st}^{s,d}(p_1) = \frac{1}{12} \text{MGF}_{p_1}(1) + \frac{1}{4} \text{MGF}_{p_1}\left(\frac{4}{3}\right);
\]

\[
\xi_{2nd}^{s,r,d}(p_1 + p_2) = \frac{1}{12} \text{MGF}_{p_1}(1) \times \text{MGF}_{p_2}(1) + \frac{1}{4} \text{MGF}_{p_1}\left(\frac{4}{3}\right) \times \text{MGF}_{p_2}\left(\frac{4}{3}\right)
\]

• The MGF of \(k-\mu\) or \(\eta-\mu\) distributed fading are given in chapter 2
Advanced topics in MIMO wireless communications

- Fig. SER vs SNR for SISO based cooperative communication system employing
- (a) BPSK modulation over $k-\mu$ fading channel
- (b) QSPK modulation scheme over $\eta-\mu$ fading channel
Advanced topics in MIMO wireless communications

- **MIMO based cooperative communication over $\alpha$-$\mu$ fading channels**
- Let us consider a dual hop relaying system (see Fig.) over i.i.d. $\alpha$-$\mu$ fading channel
- where a S node communicate to the D with help of $\{R_1, R_2, \ldots, R_K\}$
- using DF protocol
Fig. Two hop multiple relays model
Advanced topics in MIMO wireless communications

• Assume S node is equipped with $N_T$ transmit antennas,
• $k^{th}$ relay node equipped with $N_K$ receive antennas and $N_T$ transmit antennas and
• D node equipped with $N_D$ receive antennas
• The OSTBC technique is applied at every transmit and receive nodes
• We assume $S_D$ data symbols are transmitted in $T$ time slots
• which are encoded with OSTBC codeword
Advanced topics in MIMO wireless communications

• We also assume that relays are half duplex,
• which means Rs either receive or transmit the information at any time
• The instantaneous SNR output at the $k^{th}$ R can be expressed as in

$$\gamma = \frac{TE_S}{S_D \sigma_n^2} \sum_{i=1}^{N_K} \sum_{j=1}^{N_T} |h_{i,j}^k|^2$$
Advanced topics in MIMO wireless communications

• with $E_s$ is power of transmitted signal per antenna and
• $\sigma_n^2$ is noise power,
• $h_{i,j}^k$ is the channel gain between the $i^{th}$ receive antenna and $j^{th}$ transmit antenna of the $S$-$k^{th}$ R
• We consider orthogonal transmission approach to transmit information from R nodes to D nodes
• We can transmit signal through time-division technique
Advanced topics in MIMO wireless communications

• or through frequency-division technique

• In time-division technique, total time slots \((\bar{K} + 1)T\) will be required to transmit \(S_D\) data symbols information from S to D,

• where \(\bar{K}\) is number of Rs participating in R nodes to D nodes transmission
Advanced topics in MIMO wireless communications

- In first $T$ time slots, $S$ nodes transmit the information to all $R$ nodes.
- Only those $R$s will participate for decoding sets that have good enough $S$-$R$ link.
- To provide successful decoding of all $S_D$ data symbols.
- In time-division technique, we also assume that channel condition do not change for $(\bar{K}+1)T$ time slots.
Advanced topics in MIMO wireless communications

• At the D nodes, for MRC receiver, the instantaneous SNR output can be expressed as

\[
\gamma = \frac{TE_S}{S_D \sigma_n^2} \sum_{k=0}^{\bar{K}} \sum_{i=1}^{N_K} \sum_{j=1}^{N_T} |\bar{h}_{i,j}^k|^2
\]
Advanced topics in MIMO wireless communications

• where $h_{i,j}^k (k \in [1, K])$ represents channel gain between the $i^{th}$ receive antenna and $j^{th}$ transmit antenna of $k^{th}$ R and D node link and

• $h_{i,j}^0$ represents channel gain between the $i^{th}$ receive antenna and $j^{th}$ transmit antenna of S-D link

• By using the theorem on total probability, we have,

$$P_{SER} = \sum_{K=1}^{K} \Pr \left( \frac{SER}{K} \right) \Pr \left( K \right)$$
Advanced topics in MIMO wireless communications

• where, \( P_{SER} \) denotes total symbol-error probability,

• \( K \) is total number of relays that participate for source-relays transmission,

• \( \overline{K} \) is total number of relays that participate for relays-destination transmission,

• \( \Pr(\overline{K}) \) denotes probability that only \( \overline{K} \in [1, K] \) relays participate for relays-destination transmission and

• \( \Pr(SER / \overline{K}) \) is the conditional SER for given \( \overline{K} \)
Advanced topics in MIMO wireless communications

- Assuming identical fading conditions for all the relays,
- the probability of any relay being able to decode the symbol correctly becomes $A$
- Then, it can be shown that

$$P(\bar{K}) = \bar{K} C_K A^K (1 - A)^{K - \bar{K}}$$
Advanced topics in MIMO wireless communications

- Since \( \overline{K} \) relays are used, the mgf of the iid \( \alpha-\mu \) wireless channels between the source to destination and relay to the destination can be written as

\[
M_\gamma(s) = \left( \frac{\alpha \mu^\mu \sqrt{k}}{l} \frac{\alpha \mu}{\Gamma(\mu)} \left( \frac{\gamma s}{l} \right)^2 \left( 2\pi \right)^{l+k-2} \right)^{NTND(\overline{K}+1)} \left( \frac{\mu}{\alpha} \frac{l}{s} \right)^{G_{l,k}^{k,l}} \left( I(l,1-\frac{\alpha \mu}{2}) \right)^{I(l,k,0)}
\]
The probability of error when $K$ relays are used for transmission can be calculated as follows:

$$P(SER|K) = \int_{0}^{\infty} P(SER|K, \gamma) p_{\gamma}(\gamma) d\gamma$$

where $P(SER|K, \gamma)$ is the conditional probability of error (CPE) for different modulation schemes used.
Advanced topics in MIMO wireless communications

• For BPSK,

\[ P(\text{SER}| \overline{K}) = \int_0^\infty Q(\sqrt{2\gamma}) p_\gamma(\gamma) d\gamma \approx \int_0^\infty \left( \frac{1}{12} e^{-\gamma} + \frac{1}{4} e^{-\frac{4\gamma}{3}} \right) p_\gamma(\gamma) d\gamma = \frac{1}{12} M_\gamma(1) + \frac{1}{4} M_\gamma\left(\frac{4}{3}\right) \]

• For 4-QAM,

\[ P(\text{SER}| \overline{K}) = \int_0^\infty \left( 2Q(\sqrt{\gamma}) - Q^2(\sqrt{\gamma}) \right) p_\gamma(\gamma) d\gamma \approx \int_0^\infty \left( \frac{1}{6} e^{-\frac{\gamma}{2}} + \frac{1}{2} e^{-\frac{2\gamma}{3}} - \frac{1}{144} e^{-\gamma} - \frac{1}{16} e^{-\frac{4\gamma}{3}} - \frac{1}{24} e^{-\frac{7\gamma}{6}} \right) p_\gamma(\gamma) d\gamma \]

\[ = \frac{1}{6} M_\gamma\left(\frac{1}{2}\right) + \frac{1}{2} M_\gamma\left(\frac{2}{3}\right) - \frac{1}{144} M_\gamma(1) - \frac{1}{16} M_\gamma\left(\frac{4}{3}\right) - \frac{1}{24} M_\gamma\left(\frac{7}{6}\right) \]
Fig. Approximate & Monte Carlo simulations of SER performance of MIMO based cooperative communication employing BPSK modulation over the Rayleigh fading channel.
• Fig. Approximate and Monte Carlo simulations of SER performance of MIMO based cooperative communication employing 4-QAM modulation over the fading channel.
Advanced topics in MIMO wireless communications

• **Large-scale MIMO systems**

• Large-scale (LS) MIMO systems are aiming to employ hundreds or thousands of antennas at the transmitter and receiver

• What are the advantages?

• We can have large diversity and rate gains

• For example, for a $100 \times 100$ MIMO system,

• the achievable diversity gain is 10,000 and rate gain is 100
Advanced topics in MIMO wireless communications

• Single User LS-MIMO: capacity analysis

• The received signal vector for a point-to-point or single user (SU) MIMO, we have considered till now, can be expressed as

\[ y = Hx + n \]

• The instantaneous capacity for MIMO channel is given by

\[ C = \log_2 \left| I_{RH} + \frac{P}{\sigma_n^2 N_T} Q \right| \text{ bits / s / Hz} \]
Advanced topics in MIMO wireless communications

• Assume full rank MIMO channel matrix, then

\[ R_H = \min\{N_R, N_T\} = m \]

• Hence the above instantaneous capacity can be expressed as

\[
C = \sum_{i=1}^{m} \log_2 \left( 1 + \frac{\lambda_i P}{N_T \sigma_n^2} \right) \quad \text{bits / s / Hz}
\]
Advanced topics in MIMO wireless communications

• Since the trace of a square matrix is equal to sum of its eigenvalues, i.e.,

\[
    \sum_{i=1}^{m} \lambda_i = \sum_{i=1}^{m} \sigma_i^2 = \text{trace}(Q)
\]

• The worst case for capacity is when only one singular value of the channel matrix H is not zero, i.e.,

\[
    \lambda_1 = \sigma_1^2 = \text{trace}(Q)
\]
Advanced topics in MIMO wireless communications

• Such cases are appropriate for line-of-sight (LOS) propagation
• Hence, the instantaneous capacity for MIMO channel is lower bounded by

\[
C \geq \log_2 \left( 1 + \frac{P(\text{trace}(Q))}{N_T \sigma_n^2} \right) \quad \text{bits / s / Hz}
\]
The best case is when all the singular values are equal, i.e.,

\[
m\lambda_e = \text{trace}(Q)
\]

This is suitable for iid channel matrix

Hence, the instantaneous capacity for MIMO channel is upper bounded by

\[
C \leq m \log_2 \left( 1 + \frac{P(\text{trace}(Q))}{mN_T \sigma_n^2} \right) \quad \text{bits / s / Hz}
\]
Advanced topics in MIMO wireless communications

• If we normalize the magnitude of the channel gain coefficients equal to one, then,

$$\text{trace}(Q) \approx N_T N_R$$

• Hence, the instantaneous capacity for MIMO channel is bounded as follows

$$\log_2 \left( 1 + \frac{PN_R}{\sigma_n^2} \right) \quad \text{bits/s/Hz} \leq C \leq m \log_2 \left( 1 + \frac{nP}{N_T \sigma_n^2} \right) \quad \text{bits/s/Hz} ; n = \max\{N_R, N_T\}$$
Advanced topics in MIMO wireless communications

- **Large-scale asymptotic analysis:**
- Case 1: Let $N_T \to \infty$, keeping $N_R$ fixed
- Assume the favourable condition of channel orthogonalization
- where the rows of the channel matrix are asymptotically orthogonal, i.e.,

$$\left( \frac{HH^H}{N_T} \right)_{N_T \gg N_R} \approx I_{N_R}$$

- Note that $m = N_R$ for this case
Advanced topics in MIMO wireless communications

• Hence

\[ C_{NT} \gg N_R \approx \log_2 \left| I_{N_R} + \frac{PL_{NR}}{\sigma_n^2} \right| = N_R \log_2 \left( 1 + \frac{P}{\sigma_n^2} \right) \]

• Case 2: Let \( N_R \to \infty \), keeping \( N_T \) fixed

• Assume the favourable condition of channel orthogonalization

• where the columns of the channel matrix are asymptotically orthogonal, i.e.,

\[
\left( \frac{H^H H}{N_R} \right) \approx I_{N_T}
\]
Advanced topics in MIMO wireless communications

• Note that $m = N_T$ for this case

• Hence,

$$C_{N_R \gg N_T} \approx \log_2 \left| I_{N_T} + \frac{PN_R I_{N_T}}{N_T \sigma_n^2} \right| = N_T \log_2 \left(1 + \frac{PN_R}{N_T \sigma_n^2} \right)$$

• Note that $C_{N_T \gg N_R}$ and $C_{N_R \gg N_T}$ are both highly favourable scenarios

• since they achieve the capacity upper bound mentioned above
Advanced topics in MIMO wireless communications

- Multiuser LS-MIMO: capacity analysis
- Multi-user MIMO scenario is most applicable for cellular communications
- where multiple users transmit and receive signals from a base station
- there are two types of MU-MIMO channel models
- Multiple-access channel (MAC)
- Broadcast channel (BC)
Fig. Illustration of Uplink (Multiple Access Channel and Downlink (Broadcast channel) for MU-MIMO system

\[ N_{BS} = 14, \quad N_{MS} = 1, \quad N_{U} = 3 \]
Advanced topics in MIMO wireless communications

- **Multiple-access channel:**
- In MAC, we assume that a single base station (BS) is receiving signals from multiple mobile station (MS) or users
- We will denote the number of users $N_U$ and
- each MS or user is equipped with $N_{MS}$ antennas
- Signals from multiple users are up linked to a BS equipped with $N_{BS}$ antennas
Advanced topics in MIMO wireless communications

• Assume each user is sending transmit signal vector
  \[ \mathbf{x}_{i}^{UL} \in \mathbb{C}^{N_{MS} \times 1}, i = 1,2, \ldots, N_{U} \]
• to the BS
• The corresponding channel for each user with the BS is
  \[ \mathbf{H}_{i}^{MAC} \in \mathbb{C}^{N_{BS} \times N_{MS}}, i = 1,2, \ldots, N_{U} \]
• The received signal vector at BS
  \[ \mathbf{y}^{UL} \in \mathbb{C}^{N_{BS} \times 1} \]
Advanced topics in MIMO wireless communications

• can be expressed as

\[ y_{UL} = H_1^{MAC} x_{UL} + H_2^{MAC} x_{UL} + \cdots + H_{N_U}^{MAC} x_{UL} + n_{UL} = \sum_{i=1}^{N_U} H_i^{MAC} x_i^{UL} + n_{UL} \]

• where \( n_{UL} \in C^{N_{BS} \times 1} \) is the additive white Gaussian noise vector with zero mean and covariance matrix of \( \sigma_n^2 I_{N_{BS}} \)

• The above equation may be expressed as

\[ y_{UL} = H^{MAC} x_{UL} + n_{UL} \]
Advanced topics in MIMO wireless communications

• Where

\[ H^{MAC} = \begin{bmatrix} H_1^{MAC} & H_2^{MAC} & \cdots & H_{N_U}^{MAC} \end{bmatrix} x^{UL} = \begin{bmatrix} x_{UL}^1 \\ x_{UL}^2 \\ \vdots \\ x_{UL}^{N_U} \end{bmatrix} \]

• Broadcast channel:

• The BC is a single BS sending signals to multiple MSs or users in the downlink

• Note that each MS will receive independent signal vector

\[ x^{DL} \in \mathbb{C}^{N_{BS} \times 1} \]
Advanced topics in MIMO wireless communications

• The corresponding channel vector for BS to each user will be a matrix

\[ H_{i}^{BC} \in \mathbb{C}^{N_{MS} \times N_{BS}}, i = 1, 2, \ldots, N_{U} \]

• The received signal vector at the \( i^{th} \) MS \( y_{i}^{DL} \in \mathbb{C}^{N_{MS} \times 1}, i = 1, 2, \ldots, N_{U} \) can be expressed as

\[ y_{i}^{DL} = H_{i}^{BC} x^{DL} + n_{i}^{DL} \]
Advanced topics in MIMO wireless communications

• where $\mathbf{n}_{i}^{DL} \in \mathbb{C}^{N_{MS} \times 1}$ is the additive white noise vector with zero mean and covariance matrix $\sigma_{n}^{2} \mathbf{I}_{N_{MS}}$

• If we assume time-division duplex (TDD) transmission as depicted in Fig. then

$$\mathbf{H}_{i}^{BC} = \left(\mathbf{H}_{i}^{MAC}\right)^{T}$$

$$\mathbf{y}_{i}^{DL} = \left(\mathbf{H}_{i}^{MAC}\right)^{T} \mathbf{x}^{DL} + \mathbf{n}_{i}^{DL}$$
Advanced topics in MIMO wireless communications

• For all users,

\[ y^{DL} = (H^{MAC})^T x^{DL} + n^{DL} \]

• where \( H^{MAC} = \begin{bmatrix} H_1^{MAC} & H_2^{MAC} & \cdots & H_N^{MAC} \end{bmatrix} \)

\[ y^{DL} = \begin{bmatrix} y_1^{DL} \\ y_2^{DL} \\ \vdots \\ y_{NU}^{DL} \end{bmatrix} \quad \quad n^{DL} = \begin{bmatrix} n_1^{DL} \\ n_2^{DL} \\ \vdots \\ n_{NU}^{DL} \end{bmatrix} \]
Advanced topics in MIMO wireless communications

• **Capacity and Matched Filter Precoding (Massive MIMO)**

• Consider BS equipped $N_{BS}$ with antennas serving $N_U$ single antenna users

• Let us denote the $i^{th}$ user to the $j^{th}$ antenna of the BS

\[ h_{ji} = g_{ji} \sqrt{d_i} \]

• where $g_{ji}$ and $d_i$ represent the complex small–scale fading and large-scale fading coefficients respectively

Advanced topics in MIMO wireless communications

- Therefore, 
  \[ H^{MAC} = GD^2 \]

- where
  \[
  D = \begin{pmatrix}
  d_1 \\
  d_2 \\
  \vdots \\
  d_{NU}
  \end{pmatrix}
  \quad G = \begin{pmatrix}
  g_{11} & g_{12} & \cdots & g_{1NU} \\
  g_{21} & g_{22} & \cdots & g_{2NU} \\
  \vdots & \vdots & \ddots & \vdots \\
  g_{NBS1} & g_{NBS2} & \cdots & g_{NBSNU}
  \end{pmatrix}
  \]
Advanced topics in MIMO wireless communications

• Assume favourable condition where all column vectors of the channel are orthogonal, i.e.,

$$\begin{pmatrix} G^H G \\ N_{BS} \end{pmatrix}_{N_{BS} \gg N_U} \approx \mathbf{I}_{N_U}$$

$$\begin{pmatrix} \left( H^{MAC} \right)^H H^{MAC} \end{pmatrix}_{N_{BS} \gg N_U} \approx \frac{1}{D^2} G^H G D \frac{1}{D^2} = N_{BS} D$$
Advanced topics in MIMO wireless communications

• Let us assume uplink transmit power for each user is \( \frac{P}{N_U} \)

• Hence the capacity of the virtual MIMO

• (BS equipped with \( N_{\text{BS}} \) antennas acting as receiver and \( N_U \) single antenna users as transmitter) for uplink (MAC) is given as

\[
C_{N_{\text{BS}}\gg N_U}^{\text{MAC}} \approx \log_2 \left| I_{N_U} + \frac{PN_{\text{BS}}D}{N_U \sigma_n^2} \right| = \sum_{i=1}^{N_U} \log_2 \left( 1 + \frac{PN_{\text{BS}} d_i}{N_U \sigma_n^2} \right)
\]

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• Let us consider a simple linear processing matched-filter (MF) at the BS

\[
\left( H_{\text{MAC}} \right)^H y_{UL} = \left( H_{\text{MAC}} \right)^H \left( H_{\text{MAC}} x_{UL} + n_{UL} \right) \approx N_{BS} \mathbf{D} x_{UL} + \left( H_{\text{MAC}} \right)^H n_{UL}
\]

• Note that \( \mathbf{D} \) is a diagonal matrix, therefore,

• MF processing at the BS decouples the signals from each user
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• The SNR for each user can be evaluated as

\[ \frac{P}{\sigma_n^2 N_U} \]

• Since we have parallel independent Gaussian channels,
• the capacity of this will be the same with that of
Advanced topics in MIMO wireless communications

- Hence MF is an optimal processing at the BS
- when the number of antennas at the BS grows large
- BS has full CSI, so adaptive power allocation could be carried out
- Hence, the capacity for downlink (BC) may be expressed as

\[
C^{BC} = \max \log_2 \left| \frac{P}{\sigma_n^2} \mathbf{H}^{BC} \mathbf{D}_p \left( \mathbf{H}^{BC} \right)^H \right|
\]

\[
I_{NS} + \frac{P}{\sigma_n^2} \mathbf{H}^{BC} \mathbf{D}_p \left( \mathbf{H}^{BC} \right)^H
\]
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• where \( \mathbf{D}_p \) is a positive diagonal matrix with power allocation for \( N_U \) users as

\[
\mathbf{D}_p = \begin{pmatrix}
p_1 \\
p_2 \\
\vdots \\
p_{N_U}
\end{pmatrix}, \quad \sum_{i=1}^{N_U} p_i = 1
\]

• Assume favourable condition

• where all row vectors of the channel are orthogonal,
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- we have the capacity for downlink (BC) as
  \[
  C_{N_{BS} \gg N_U}^{BC} \approx D_p
  \]

- Like MAC case, we may use MF precoder and send the transmitted signal vector as
  \[
  x_{pre}^{DL} = \left( H^{MAC} \right)^* D^{-\frac{1}{2}} D_2^{-\frac{1}{2}} x^{DL}
  \]
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• Hence,

\[ y^{DL} = \left( H^{MAC} \right)^T x^{DL}_{pre} + n^{DL} = \left( H^{MAC} \right)^T \left( H^{MAC} \right)^* D^{-1/2} D_P^{1/2} x^{DL} + n^{DL} = N_{BS} D^{1/2} D_P^{1/2} x^{DL} + n^{DL} \]

• Since both \( D_P \) and \( D \) are diagonal matrices,

• hence, all the signals transmitted from BS to each user are decoupled totally
Advanced topics in MIMO wireless communications

• Multi-cell LS-MIMO: precoders
• It is well known that there are many cells in a cellular network and users in each cell are served by a BS
• Fig. illustrates a multi-cell MIMO based cellular networks
• Only seven cells are shown for illustration purpose
• The UL, DL and interference signals are also shown for two neighbouring cells for four mobile users
Fig. Multi-cell MIMO based cellular network (BS equipped with 14 antennas and single antenna MS or user, each cell has 2 users for illustration purpose)
Advanced topics in MIMO wireless communications

• In TDD based LS-MIMO system,
• pilot sequences are transmitted from the users to the BS
• to help BS in estimating the channels
• Usually pilot sequences are orthogonal and
• they are limited in number for a given period and bandwidth
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• Generally same set of pilot sequences are assigned in all cells
• So users in the neighbouring cells with the same set of pilot sequences
• will have high level of interference also popularly known
• as *pilot contamination* in the literature
• As the number of cells or BSs increases this interference will increase
Advanced topics in MIMO wireless communications

• As we have seen for MU-MIMO in which we have considered a single BS,
• MF processing completely decouple the signals from each user completely
• But for multicell MIMO, the estimated channel vector in each cell is a linear combination of channel vectors of users in other cells that use the same pilot
• Hence, MF processing will not work well
Advanced topics in MIMO wireless communications

• Other than MF processing, other precoders are reported in the literature

• (a) ZF precoding

• In ZF beamforming we premultiply the transmit signal vector by

\[
W_{ZF} = \frac{1}{\sqrt{\gamma_l}} \left( \hat{H}_l \right)^H \left( \hat{H}_l \hat{H}_l^H \right)^{-1}
\]
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• where $\hat{G}_l$ is the estimated CSI matrix of users in the $l^{th}$ BS

$$\hat{H}_l = d_{ll}^{-1} \hat{G}_l, d_{ll} = \begin{pmatrix} \sqrt{d_{1l}} \\ \sqrt{d_{2l}} \\ \vdots \\ \sqrt{d_{lN_u l}} \end{pmatrix}$$

$$\gamma_l = \frac{\text{trace}(\left(\hat{H}_l\right)^H \hat{H}_l)}{N_U}$$
Advanced topics in MIMO wireless communications

• (b) Regularized ZF precoding
• In RZF beamforming we premultiply the transmit signal vector by

\[ \mathbf{W}_{RZF} = \frac{1}{\sqrt{\gamma_l}} \left( \mathbf{\hat{H}}_l \right)^H \left( \mathbf{\hat{H}}_l \left( \mathbf{\hat{H}}_l \right)^H + \delta \mathbf{I}_{N_U} \right)^{-1} \]

• where \( \delta \) is a parameter that balances the interference suppression and SNR decrease
• Usually \( \delta \) is chosen as \( \frac{N_{BS}}{20} \)

Advanced topics in MIMO wireless communications

• Users at the cell edge problem

• The instantaneous capacity of \( i \)\textsuperscript{th} user at time slot \( t \) in a multicell MIMO network can be calculated as

\[
C_i(t) = \log_2 \left( 1 + \text{SINR}_i(t) \right)
\]

• where signal-to-interference-plus-noise ratio (SINR) is defined for \( i \)\textsuperscript{th} user as

\[
\text{SINR}_i = \frac{P_{ds}(t)}{P_{\text{noise}}(t) + P_{\text{IUI}}(t) + P_{\text{ICI}}(t)}
\]
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- $P_{ds}(t)$ is the power of the desired signal at time slot $t$
- $P_{IUI}(t)$ is the power of the inter-user interference (intracell) at time slot $t$
- $P_{ICI}(t)$ is the power of the intercell interference at time slot $t$
- $P_{noise}(t)$ is the power of the noise at time slot $t$
As illustrated in Fig., users at the cell edge will have very high level of $P_{ICI}(t)$ (quite near to the BS of neighbouring cells) and very low level of $P_{ds}(t)$ (far away from the serving BS). Consequently they have a lower value of SINR and capacity.
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- **Heterogenous networks:**
- Another solution to the problem of cell edge users is to have a small cell networks like pico cells within the macro cell networks
  - Macro cell radius (1-30 km)
  - Micro cell radius (200m -2 km)
  - Pico cell radius (4m – 200m)

+ [http://www.wirelesscommunication.nl/reference/chaptr04/cellplan/cellsize.htm](http://www.wirelesscommunication.nl/reference/chaptr04/cellplan/cellsize.htm)
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• In such heterogeneous networks
  • (networks of macro cells and pico cells),
• there is a high level of co-channel interference as shown in Fig.
• Interference suppression in such networks could be carried out in three steps:
  • (a) Triangular decomposition of joint channel matrix $H$ and
  • extraction of the equivalent interference channel model
  • which reduces the inter-cell interference to half

Fig. DL and UL interference model in multi-cell MIMO heterogeneous networks
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• (b) From equivalent interference channel model,
  • use Signal-to-leakage-plus-noise-ratio
  • to compute the equivalent pre-coding matrices
  • to suppress rest of the inter-cell interference

• (c) Compute the intra-cell interference suppressing
  • precoding matrices for each user
Advanced topics in MIMO wireless communications

• MIMO Cognitive Radios

• What is Cognitive Radio?

• Cognitive radio (CR) is an intelligent wireless communication system
  • that is aware of its surrounding environment

• It is a smart and flexible radio (Secondary User ((unlicensed)) device)
  • that monitors and senses its radio environment
  • for potential spectrum opportunities