Local search based near optimal low complexity detection for large MIMO System

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Abstract—This paper presents a low complexity detection technique with near Maximum Likelihood (ML) performance for large multiple-input multiple-output (MIMO) systems. Large MIMO systems have gained popularity very soon because of high spectral efficiency and increased link reliability. ML based detection is known to give optimal result in terms of accuracy but due to extremely high computational complexity involved, detection time increases exponentially as the number of transmitter and receiver antennas increases. We propose an algorithm which gives near optimal performance along with much reduced computational complexity. Our results show that the proposed method outperforms linear detection technique named Zero Forcing (ZF) as well as heuristic based search algorithms named likelihood ascent search (LAS) and Reactive Tabu Search (RTS). Our algorithm finds the best solution restricted to a given Euclidean distance around initial solution. It searches all the neighbors of initial solution falling under dynamically calculated squared Euclidean distance based cost function value. As the number of antennas can vary in the range of tens to few thousands in large MIMO systems, this algorithm could be a substitute for ML based detection algorithm. We have considered Rayleigh fading channel for our simulations and assumed that perfect channel state information at the receiver (CSIR) is available.

Index Terms—Zero-forcing, large-MIMO system, low complexity detection, likelihood ascent search (LAS), neighborhood, reactive tabu search (RTS), near optimal solution, K-neighborhood search for ZF solution (K-NSF ZF).

I. INTRODUCTION

Our paper deals with large multiple-input multiple-output (MIMO) systems where term large MIMO refers to systems in which large number of antennas (in the range of tens to few thousands) are employed at transmitter as well as at receiver side. Multi antenna wireless systems, most commonly known as MIMO systems, have gained popularity at once because of its theoretical prediction of channel capacity gain over single-input single-output (SISO) channel capacity [9], [10]. With large number of antennas, we can achieve high spectral efficiency along with other advantages that MIMO system promises are increased link reliability and high power efficiency [11]. MIMO has become an essential element of major wireless communication standards including IEEE 802.11n (Wi-Fi), IEEE 802.11ac (Wi-Fi), HSPA+ (3G), WiMAX (4G) and Long Term Evolution (4G). MIMO systems will play a key role in further increasing the data rates in the range of few Gigabits for the upcoming fifth generation (5G) standards. The channel capacity of the MIMO channel is directly proportional to minimum of number of transmitter antenna ($N_T$) and receiver antenna ($N_R$) for a $N_T \times N_R$ MIMO system in rich Rayleigh scattering environment [1].

Large MIMO is still an emerging field and it can be a potential technology to meet the wireless communication's huge demand which is projected to grow rapidly in coming few years [2]. A key challenge faced in practically realizing large MIMO system is high computational complexity involved in detection of received signal on receiver side. Spatial interference coming from neighboring antennas becomes hindrance in MIMO signal detection. There are techniques available to mitigate spatial interference [8]. The increase in number of transmitter-receiver antennas in MIMO system not only increases spatial interference, furthermore it increases the detection complexity on receiver side significantly. Maximum Likelihood (ML) based detection has optimal performance but the ML based detection has got exponential time detection complexity [12]. Based on local neighborhood search, low complexity detection methods named reactive tabu search (RTS) and likelihood ascent search (LAS) have been reported in [3]. Soft-heuristic based detectors [13] performs considerably well for large MIMO systems with higher computational complexity.

In this work, we propose a local neighborhood search based algorithm named $k$-neighborhood search for ZF solution which gives near optimal performance with much reduced computational complexity. We need to initialize the search with an initial solution vector around which algorithm looks for optimal solution. We propose to use zero-forcing solution as the initial solution which in turn gives improved performance in term of relatively reduced detection complexity as well as ensures accuracy better than Zero-forcing solution. Our simulation results, using Binary phase shift keying (BPSK) modulation scheme, show that proposed algorithm outperforms conventional RTS and LAS in terms of both accuracy and detection complexity. Though ML gives optimal solution, it can’t be used in large MIMO systems because of its exponential rise in decoding complexity with increase in number of antennas. Considering accuracy and complexity, our proposed algorithm could be apt solution for deployment of...
MIMO technology in communication systems as it gives near ML performance with much reduced computational detection complexity.

The rest of the paper is structured as follows. Next, section II describes the system model used in our work. Section III describes state-of-the-art detection methods available for large-MIMO systems. In section IV, detailed working principle of our proposed algorithm is presented. Section V consists of simulation results using various MIMO detection techniques. Finally, Section VI concludes this work.

II. System Model

We consider a $N_T \times N_R$ MIMO system with $N_T$ transmit antennas and $N_R$ receive antennas, $H \in C^{N_R \times N_T}$ is $N_R \times N_T$ channel matrix with entries $h_{i,j} \in C$ denote the channel gain seen between $i^{th}$ receive antenna and $j^{th}$ transmit antenna. The real and imaginary part of path gain $h_{i,j} \in C$ are assumed to be independent identically distributed (i.i.d) zero mean Gaussian random variable, hence path gains exhibit Rayleigh fading. The MIMO communication system can be characterized by following linear channel model.

$$y = Hx + n$$  \hspace{1cm} (1)

where $x \in C^{N_T}$ is the transmitted signal vector consisting of $N_T$ BPSK modulated symbols transmitted from $N_T$ transmit antennas, $y \in C^{N_R}$ is the received signal vector and $n \in C^{N_R}$ is additive white Gaussian noise (ACWGN) with zero mean and $N_o$ variance. We have assumed that channel state information at receiver is available and channel gains may vary between two consecutive transmission. Fig. 1 shows a typical point-to-point MIMO system with $N_T$ transmit antennas and $N_R$ receive antennas.

![Fig. 1. $N_T \times N_R$ MIMO System.](image)

III. RELATED WORKS

A. Zero Forcing detection

In large-MIMO system, spatial interference from neighboring antennas becomes hindrance in detection, so in order to nullify the neighboring antenna’s interference, we project the received signal on to a subspace which is orthogonal to the interference part of received signal. ZF detector is linear detector which performs linear transformation on the received vector with the help of pseudo-inverse of the channel matrix $H$. Let $T \in C^{N_R \times N_T}$ be the pseudo-inverse of $H$ matrix, i.e.,

$$T = (H^H H)^{-1} H^H$$  \hspace{1cm} (2)

In addition to interference suppression, ZF detector enhances the noise variance and it will deteriorate the performance at low SNR [1]. ZF solution has been used as initial solution for $k$-neighborhood search for ZF solution algorithm. The initial solution could be chosen as Minimum Mean Squared Error [14] or Matched Filter [15] or Lattice reduced MIMO linear detector [16] or Element-Based Lattice Reduced [17] solutions.

B. Optimal detection

The detector, at the receiver, makes an estimate $\hat{x}$ of the transmitted signal $x$. Optimal solution is obtained by solving non-linear optimization problem of minimizing the squared Euclidean distance between actual received signal $y$ and theoretically received signal $Hx$ where $M$ is the set of equiprobable symbols and $x \in M^{N_T}$ [1]. The ML solution is estimated by,

$$\hat{x}_{ML} = \arg \min_{x \in M^{N_T}} |y - Hx|^2.$$  \hspace{1cm} (3)

Searching the optimal solution to the above optimization problem requires exhaustive search over $M^{N_T}$ solution vectors, i.e., exponential complexity in $N_T$. For large number of antenna (in the range of tens to thousands), it becomes infeasible to compute exact ML solution due to exponential computational complexity. Hence, ML based detection is not apt for large-MIMO systems. More recently, several low complexity detection algorithms based on local search techniques has been proposed which gives near optimal accuracy [4].

C. Likelihood Ascent Search (LAS)

Optimal signal detection is not practically realizable when signaling dimension becomes significantly large. Therefore, for large-MIMO system, local search based detection can be considered a good choice as they give near optimal solution within reasonable time. The key advantage of a local search algorithm is its neighborhood which keeps guiding the search to a better solution. LAS is a basic version of local search, where algorithm starts with an initial solution and keeps searching its neighborhood for a better solution which has lesser cost [5]. LAS algorithm might get trapped in the local minimum solution which it encounters for the first time and it may declare the local minimum to be the final solution. Thus, it fails to find potentially better minimum solution.

D. RTS

Tabu search is a global optimization algorithm [4], [7] based on local search which avoids cycles consisting of repeated visits of same node in the solution space. Reactive Tabu Search is a heuristic method which gives better performance to linear detectors when number of antennas are in the range of tens.
to few thousands. It iteratively moves from one solution to another solution and direction is decided by Tabu matrix. Solutions which has been visited earlier are marked as tabu in Tabu matrix with some positive non-zero entry for certain number of iterations called tabu period. Algorithm prohibits the visit to those solutions which are still in tabu period. The key advantage we get is that search can move to unexplored regions which might consist of global solution. In section V, we compare RTS algorithm with \textit{k-neighborhood search for ZF solution}.

IV. \textsc{K-distance neighborhood search for ZF solution}

A. \textit{K-distance neighborhood}

Given $N_R$ antennas at receiver side, all possible received vectors which varies at $k$ indices exactly out of $N_R$ indices from hypothesized vector $v = Hx$ are said to be $k$ distance away from the vector $v$. For instance, $v_1 = [-1 -1 -1 -1]$ and $v_2 = [-1 -1 -1 1]$, we can say $v_1$ is 2-distance away from $v_2$ and vice versa.

\textbf{Pseudo-code}

\textbf{Initialization} : sol\_vector $\leftarrow$ ZF solution vector; threshold\_cost\_value $\leftarrow$ average cost calculated at high SNR; mask $\leftarrow$ vector with $N_T$ entries all set to zero; cf\_val $\leftarrow$ threshold\_cost\_value; radius $\leftarrow$ 0; while (radius $\leq N_T$) do
\hspace{1em} mask $\leftarrow$ initialize with radius number of set bits; radius $\leftarrow$ radius+1; while (next unique permutation of mask possible) do
\hspace{2em} mask $\leftarrow$ Permute the set bits in the mask;
\hspace{2em} temp\_sol\_vector $\leftarrow$ Modify sol\_vector according to mask;
\hspace{2em} temp\_cost $\leftarrow$ calculate the cost of temp\_sol\_vector;
\hspace{2em} if (temp\_cost $<$ cf\_val) then
\hspace{3em} sol\_vector $\leftarrow$ temp\_sol\_vector;
\hspace{3em} break;
end
end
\textbf{Return}: sol\_vector;
\textbf{Algorithm 1}: K-neighborhood search for ZF solution

B. Algorithm

Let $S$ be the set of all possible vectors to be transmitted. Let \textbf{N}(x) represents the neighborhood of zero forcing solution $x$. \textbf{N}(x) $\subset$ S. Objective of this algorithm is to estimate the vector $x$ which is closest (in squared Euclidean distance sense) to the received signal $y$ in reasonable time. In our case, Cost function is defined as,

$$ C(z) = \| y - Hz \|^2 $$

(4)

where $z \in \textbf{N}(x)$. Cost function value associated with $z$ is calculated to be squared Euclidean distance of the hypothesized received signal $Hz$ from the actual received signal $y$. In this proposed algorithm, we define a stopping criteria to be a cost function threshold value (CFTV) which is calculated dynamically and algorithms stops searching if it finds any solution whose associated cost function value is lesser than CFTV. For a given SNR, the cost function threshold value is calculated by sending enough pilot symbols at higher SNR and average is taken of all the cost function values corresponding to those ML based best detected solutions in which at least one bit is in error. We initialize the search with zero-forcing solution as the search reaches CFTV faster when compared to random initialization from solution space. Algorithm starts searching in 1-distance away neighborhood of ZF solution, then 2-distance and so on until the minimum cost function value found till now is lesser than CFTV. As soon as CFTV is reached, visit to all remaining solutions falling under current squared Euclidean distance is prohibited thus reducing the computational complexity while ensuring reasonable accuracy as CFTV is calculated at relatively high SNR using ML detection. The \textit{k-neighborhood search for ZF solution} algorithm guides the search with the help of a mask $d = [d_1 \ d_2 \ d_3 \ldots \ d_{N_T}]$ where $d_i \in \{0,1\}$. Algorithm initializes the mask’s first bit to 1 and remaining all other entries set to zero. Then, it modifies the ZF solution at those indices where mask bits are set. All possible unique permutations of bits in the mask give different modified ZF solution and cost function associated with the modified ZF solution is compared with CFTV. If any of the modified solution’s cost is less than CFTV, corresponding modified ZF solution is declared as output. Otherwise, one more bit in the mask is set and algorithm keeps searching till it either reaches a mask with all bits set and all possible unique permutations covered or one of the modified solution gives cost function value less than CFTV. In the worst case, this algorithm behaves as ML algorithm (in running complexity sense) but in the average case, it gives near ML solutions in a very reasonable time.

V. \textsc{Simulation Results}

This section presents the simulation results evaluated using local search based low complexity detection method named \textit{K-neighborhood search for ZF solution} for 4 x 4, 8 x 8 and 16 x16 MIMO systems. To further clarify the importance of the proposed method, we compare the results with previous closely related state-of-the-art methods, i.e., ML, RTS, ZF and LAS. The underlying wireless communication system considered for simulation consists of same number of receive and transmit antennas. Information bits are sent over Rayleigh fading channel using BPSK modulation scheme and receiver extracts the information bits from the received signal using the above proposed method. Each entry of channel matrix $H$ is assumed to be independent identically distributed (i.i.d.) complex Gaussian random variable with zero mean and unit variance. The additive noise considered in this simulation is
also i.i.d. and has complex Gaussian distribution with zero mean and variance $N_0$, set according to the SNR specified in simulation. All simulations have been performed using a personal computer with Intel core i5-2450M processor at 2.5GHz with 4GB DDR3 RAM and 64 bit Windows 7 operating system.

Fig. 2. shows comparison between decoding accuracies of various detection techniques, i.e., ZF, RTS, ML and LAS with K-neighborhood search for ZF solution for 4 x 4 MIMO system. Assuming perfect CSIR is available, $4 \times 10^4$ BPSK modulated bits are transmitted by 4 transmit antennas in $10^4$ time slots and fraction of bits in error or bit error rate (BER) vs SNR is plotted showing the significance of K-NSF ZF over well known above mentioned detection techniques. At 1dB SNR, ZF shows 87.345% accuracy in a fraction of time, observed accuracy for ML based detection is 91.908% with $2^4 = 16$ cost function calculations involved, LAS and RTS shows 88.337% and 88.723% accuracy respectively both having maximum number of iteration for search set to be 4, hence 4 cost functions need to be calculated whereas K-NSF ZF gives an accuracy of 91.675% with 2.4 average no. of cost function calculations involved. Fig. 3 shows the average number of solutions visited by K-NSF ZF during estimation of transmitted signal for various types of MIMO system. The plot shows, as we increase the SNR of transmitted signal, number of visited solutions by K-NSF ZF decreases significantly when large number of antennas are present.

In the case of 16 x 16 MIMO system, at low SNR, both RTS (with tabu period of 8 iterations) and LAS with randomized search restricted over 20,000 solutions perform inferior to K-NSF ZF which at an average uses approximately 16,000 solution visits. In addition to this, K-NSF/ZF gives an accuracy which is comparable to ML’s accuracy along with much reduced computational complexity as ML uses $2^{16} = 65536$ cost function calculations to estimate the transmitted signal. Considering computational complexity and near optimal performance, it is reasonable to claim that K-NSF ZF provides a good alternative for large MIMO detection.

VI. Conclusion

To deal with computational complexity of large-MIMO systems, this work has put forward the use of local search
based algorithm giving near optimal solution. In this paper, the proposed algorithm aims at reducing the detection complexity at the receivers employed in these systems. It is initialized with ZF solution, hence ensures accuracy greater than or equal to ZF detection. In addition to this, proposed algorithm outperforms conventional local search based well known detection methods named LAS and RTS in terms of both accuracy and detection complexity. Furthermore, it gives near optimal performance with much reduced detection complexity when compared to exponential complexity of optimal detection. CFTV calculated during runtime helps the algorithm to escape from exponential complexity of optimal detection. A thorough analysis of the proposed algorithm including the computational complexity will be pursued in our future works.

REFERENCES