Analysis of UWB communication over IEEE 802.15.3a channel by superseding lognormal shadowing by Mixture of Gamma distributions

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ABSTRACT

In this paper, we calculate the bit error rate (BER) of the ultra-wide band (UWB) wireless communication system over IEEE 802.15.3a channel model by superseding the lognormal (LN) shadow fading distribution to the Mixture of Gamma (MG) distributions. In general, shadow fading is the effect of random fluctuation of received signal power around the mean path loss and it is modeled as LN distribution. In this work, we approximate the LN shadow fading distribution to the MG distributions, because of intractable, indefinite mathematical expression of the characteristic function (CF) and moment generating function (MGF) of the LN distribution, which are required to evaluate BER of the UWB communication system. To estimate the approximate MG distributions parameters, we use rth moment algorithm with least square non-linear curve fitting criteria. This work employs the L-fingers RAKE receiver which is needed to collect the total energy at the receiver in the UWB system. The results show that 5-MG distributions is the good fit for LN shadow fading distribution of the UWB communication system. The proposed analytical BER is closely matched to the simulated BER for all the four IEEE 802.15.3a channel models. It verifies the accuracy of the approximation considered in this BER analysis.

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1. Introduction

The low cost and high data rate features have drawn the considerable interest towards the Ultra-wide band (UWB) wireless indoor communication system. UWB offers frequency spectrum of 3.1–10.6 GHz as assigned by the Federal Communication Commission (FCC). FCC has put a constraint on maximum allowable transmitted power spectral density (PSD < –41.3 dBm) of the UWB signal to limit the interference with the existing wireless systems. The standardized channel model for short range high data rate UWB system is termed as IEEE 802.15.3a [1]. In the IEEE 802.15.3a channel model all the multipath components (MPCs) and shadow fading are lognormally distributed. It also offers the ability to resolve all the MPCs separately, makes it different from the conventional narrow band indoor channel model.

The mathematical expression of Lognormal (LN) distribution poses higher computational complexities and intractable integrals while conducting the performance analysis. Different approaches are available in the literature to overcome the computational complexities posed by the LN distribution during the performance analysis of the UWB system [2–4]. In Ref. [2], the author used Wilkinson’s method to approximate the sum of LN random variables (RVs) by another LN-RV. In Ref. [3], the author has approximated the sum of LN random variables (RVs) by the linear weighted sum of LN distributions via Pearson type IV distribution. In Ref. [4], the author has approximated the total received SNR by the different distributions depending upon the severeness of the fading. These distributions are namely Coxian, Mixture of Gamma (MG) and LN distributions. All these works have not considered the shadow fading effect during the error performance analysis of the UWB system. The shadow fading effect is a pivotal parameter to be taken into account during the error performance analysis as it is very much part of the indoor UWB wireless communication system.

The present work considers the shadow fading effect during the error performance analysis. In Ref. [5], the author suggested that LN distribution can be approximated by the Gamma distribution. In Ref. [6], the author has opined that MG distributions is the good approximation of the composite channels and many other RVs. It also offers a tractable mathematical expression of moment generating function (MGF), characteristic function (CF) and cumulative distribution function (CDF). Those remarkable properties of MG distributions motivate us to approximate LN shadow fading distribution by it in the present work. We also evaluate the CF based error performance analysis of the UWB system using the above approximation.

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The rest of this paper is organized as follows. The IEEE 802.15.3a UWB channel model is described in Section 2. MG distributions and estimation of its parameters is presented in Section 3. We derived an analytical BER expression of the UWB communication system over IEEE 802.15.3a channel model in Section 4. Section 5 discussed the estimation of the number of mixing coefficients of MG distributions. Section 6 presents the simulation and numerical results. Finally, conclusion is drawn in Section 7.

2. IEEE 802.15.3a UWB channel model

IEEE 802.15.3a is a standard channel model for high data rate, short distance UWB wireless communications systems. All the MPCs and shadow fading of the IEEE 802.15.3a channel model are LN distributed. The multi-cluster signals and MPCs in each cluster are also subjected to the independent LN fading. Due to this, it is modeled by the modified Saleh-Valenzuela (S-V) channel model. The channel impulse response (CIR) of IEEE 802.15.3a channel is given as [7]

$$h(t) = X \sum_{m=0}^{5} \sum_{r_0=0}^{R} \alpha_{r_0,m} \delta(t - T_m - \tau_{r,m})$$

(1)

where \(\alpha_{r_0,m}\) and \(X\) are the lognormally distributed representing the gain coefficient of the rth MPC in the mth cluster and shadow fading respectively. \(T_m\) and \(\tau_{r,m}\) are the cluster and MPCs arrival time. This cluster and MPCs arrival process are modeled by the Poisson process with their arrival rate (\(\Lambda\)) and (\(\lambda\)) respectively. On the basis of environmental measurements, the IEEE 802.15.3a has been categorized into four different channel models. The nomenclature of these four channel models are CM1, CM2, CM3 and CM4 whose parameters are given in Table 1.

On the basis of the total number of resolvable multipath component at the L-fingers RAKE receiver in the UWB system, Eq. (1) can be written as [2]:

$$h(t) = X \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l)$$

(2)

where \(L\) denotes the number of resolvable multipath components, \(\alpha_l\) and \(\tau_l\) are the channel fading coefficient and arrival time of the \(l\)th multipath respectively relative to the \(\tau_0 = 0\).

3. Mixture of Gamma (MG) distributions

Linear weighted sum of Gamma distribution offers tractable mathematical expression of MGF, CF and CDF. It also offers a good approximation of composite fading channel and many other RVs. The probability density function (PDF) of MG distributions is given as [8]

$$f_{MG}(x) = \sum_{i=1}^{n} p_i \frac{x^{\alpha_i-1}}{\Gamma(\alpha_i)} e^{-x/\beta_i}$$

(3)

where \(n\) denotes the number of mixing coefficients, \(p_i\) is the mixing coefficient of the \(i\)th Gamma distribution with \(\beta_i > 0\) and \(\sum_{i=1}^{n} \beta_i = 1\). \(\alpha_i\) and \(\beta_i\) are the shape and scale parameters of the \(i\)th Gamma distribution, for \(x > 0\).

MGF of MG distribution is given by

$$\eta_{MG}(s) = \sum_{i=1}^{n} p_i (1 - s/\beta_i)^{-\alpha_i}$$

(4)

In this work, we apply \(r\)th moment algorithm with the least square non-linear curve fitting criteria to estimate the parameters of the MG distributions. In the \(r\)th moment algorithm, we equate the first \(n\) moments of both the LN and MG distributions and estimate the shape and scale parameters of MG distributions. We then apply non-linear curve fitting criteria to estimate the mixing coefficients \(\hat{p}_i\) [4]. The estimation of the number of mixing coefficient \(n\) in the MG distributions is discussed in Section 5.

4. Performance analysis

In this paper, we evaluate the error performance of the UWB wireless communication system by approximating the LN shadow fading distribution by MG distributions. The conditional error probability (CEP) of the UWB system using L-fingers RAKE receiver is given as [9]

$$P_e(\gamma) = \frac{E_b}{N_0} \frac{e^{-1}}{\sqrt{(1 - \rho_t)\gamma}}$$

(5)

where \(\gamma = \frac{E_b}{N_0} \epsilon\) is the received SNR, \(E_b\) is the bit energy, \(N_0\) is the noise power spectral density and \(\epsilon\) is the total received energy. \(\rho_t = 0\) or 1 for orthogonal and antipodal signal respectively. The average bit error probability (ABER) of the UWB communication system over IEEE 802.15.3a channel can be calculated by averaging the CEP over the pdf of received energy. Now, the ABER of the orthogonal binary signal with L-fingers coherent RAKE receiver is given as

$$P_e = \int_0^\infty P_e(\gamma) f_{\gamma}(\gamma) dy$$

(6)

where \(f_{\gamma}(\gamma)\) denotes the PDF of the \(\gamma\).

In the UWB communication system, the total transmitted signal energy is dispersed over all the MPCs defined in (1). These MPCs are attenuated, delayed and eventually distorted replicas of the transmitted signals. UWB communication system offers that all the MPCs can be resolved separately thereby providing higher order of diversity. We used Rake receiver to utilize this diversity and to decode these MPCs independently. The Rake receiver is design to counter the effect of multipath fading occur due to the reflection from the obstacles present in the wireless communication channels. Rake receiver uses the sub receivers called as fingers of Rake receiver to decode the MPCs separately, provided the arrival time of the MPCs are more than the chip duration. After that it combine all the decoded MPCs to optimize the system performance. Using the L-fingers Rake receiver the collection of total received energy \(\epsilon\) of the UWB system in the observation time \((LT_c)\) is given as

$$\epsilon \approx X^2 \sum_{0 \leq T_m + \tau_{r,m} \leq LT_c} \alpha_{r,m}^2$$

(7)

$$\approx X^2 E_\epsilon$$

(7)
where \( \varepsilon_0 \) denotes the energy received at the L-fingers RAKE receiver without the shadow fading effect. It is assumed that both \( X \) and \( \varepsilon_0 \) are statistically independent from each other. Therefore, the pdf of \( \varepsilon \) can be computed as

\[
f_{\varepsilon}(x) = \int_{-\infty}^{\infty} \frac{1}{|y|} f_{x_0}(y) dy.
\]

Using the Gauss-Hermite formula (8) can be written as

\[
f_{\varepsilon}(x) = \sum_{a=1}^{N_H} \left( \frac{w_a^H}{|y|} \right) f_{x_0}(y) e^{\left( \frac{-y-x_0}{\sqrt{2\sigma^2}} \right)} \left| y-x_0 \right|_{x_0}^{t_{a}}.
\]

where \( \{w_a^H\}, \{x_0^H\} \) and \( N_H \), are the weight, absissa and number of points of the Gauss-Hermite polynomial. \( f_{x_0}(\cdot) \) and \( f_{x_0}(\cdot) \) are the PDFs of \( X \) and \( \varepsilon_0 \) respectively. As we know, the LN shadow fading is lognormally distributed with \( X \sim \text{ln} N(0, \sigma_x^2 \text{ dB}) \), therefore the square of LN shadow fading is also lognormally distributed with \( X^2 \sim \text{ln} N(0, 2\sigma_x^2 \text{ dB}) \). The distribution of \( X^2 \) after approximating it by MG distributions is given as

\[
f_{x_2}(\cdot) \approx f_{MG} (\cdot).
\]

From (10), CF of \( \varepsilon_0 \) is defined as

\[
\psi_{\varepsilon_0}(\tau) = C_{0,0}(\tau) e^{-\lambda \tau/10} \text{PDF} \left[ \frac{\text{ln} N(0, \sigma_x^2 \text{ dB})}{\Lambda_T(\tau)} \right],
\]

where

\[
C_{\tau,T}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left( -j \pi \left( \frac{\text{ln} N(0, \sigma_x^2 \text{ dB})}{\Lambda_T(\tau)} \right) \right) d\tau,
\]

\[
\psi_{\tau,T}(T, L) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left( -j \pi \left( \frac{\text{ln} N(0, \sigma_x^2 \text{ dB})}{\Lambda_T(\tau)} \right) \right) d\tau.
\]

The parameter \( \mu_{\tau,T} = \frac{10}{\Lambda_T(\tau)} \left[ \text{ln} 2 \right] - \frac{\text{ln} N(0, \sigma_x^2 \text{ dB})}{\sqrt{2}} \), \( \sigma = \sqrt{\frac{\sigma_x^2 + \sigma^2}{\Lambda_T(\tau)}} \), \( \sigma_0 = \frac{1}{\Lambda_T(\tau)} + \frac{1}{\Lambda_T(\tau)} \), \( \sigma_1 \) and \( \sigma_2 \) are the power delay factor and standard deviation for the cluster and MPCs respectively. The standard numerical values of these parameters are given in Table 1, \( |\{w_a^H\}|, |\{x_0^H\}| \) and \( N(H) \), are the weight, absissa and number of points of the Gauss-Legendre polynomial.

The PDF of \( \varepsilon_0 \) can be calculated as

\[
f_{\varepsilon_0}(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_{\varepsilon_0}(\tau) e^{-j\tau y} d\tau,
\]

where \( j = \sqrt{-1} \). Using (6), (9) and (10) and the Craig’s approximation of \( Q(x) \),

\[
\text{BER} = \frac{1}{\pi} \sqrt{2 \pi \sigma^2} \text{erf} \left( \frac{x}{\sqrt{2\pi \sigma}} \right)
\]

where \( F(x, y, \theta) = \int_{-\infty}^{\infty} \exp \left( -\frac{E_x y}{2\text{SNR}^2(\theta)} \right) f_{MG}(\frac{x}{y}) dx \).

Using (4),

\[
F(x, y, \theta) = y |\text{MC} | \left( \frac{-E_x y}{2\text{SNR}^2(\theta)} \right) \text{erf} \left( \frac{x}{\sqrt{2\pi \sigma}} \right).
\]

Combining (11)–(13), the final BER expression for the orthogonal binary signal employed UWB communication system is given in (14):
between simulated BER and ABER for MG distributions with \(n > 5\) and the difference between simulated BER and ABER for MG distribution with \(n = 5\) are very small. Even though \(n > 5\) makes the approximation closer to the exact one, it adds higher computational complexities in the calculation of ABER (15). Therefore, to maintain both the accuracy and less complexities we choose \(n = 5\) for MG distributions.

6. Numerical results and discussions

For the simulation purpose we set mean \(\mu = 0\) and standard deviation \(\sigma = 3\) dB for the LN shadow fading \((X = \ln(\lambda |0, 3\) dB)), therefore the mean and standard deviation of \(X^2\) are 0, 6 dB respectively \((X^2 = \ln(\lambda |0, 6\) dB)). To approximate the square of LN shadow fading by the MG distributions, we set \(n = 1, 3, 5\). The shape, scale and weight parameters of the MG distributions are given in Table 3. We plot PDF & CDF of the square of LN shadow fading distribution and its approximation (MG). It is shown in Figs. 2 and 3.

From these figures we observe that the approximation of the square of LN shadow fading distribution by the MG distributions for \(n = 5\) closely follows the exact one.

To calculate the BER of the UWB communication system, we set \(N^H = 40, N^L = 40, T_s = 1\) ns, \(L = 10\) and number of bits = 100,000. Fig. 4 shows the ABER comparison of the UWB communication system for four different cases over IEEE 802.15.3a CM1. The result shows that ABER (15) for Case II is closely match to the simulation because in Case II we approximate the LN shadow fading by 5-MG distributions. Fig. 5 shows the simulated and analytical ABER for Case II over the IEEE 802.15.3a CM1–4.

Case I: Analytical ABER of the UWB system for Ref. [9].
Case II: Analytical ABER (15) with 5-MG approximation.
Case III: Monte Carlo simulation of the UWB system.
Case IV: Analytical ABER (15) with 1-MG approximation.
7. Conclusions

In this paper, we evaluate the ABER of UWB wireless communication over the IEEE 802.15.3a channel model considering the shadow fading whose distribution is approximated by MG distributions. The proposed analytical results closely match with the simulation results. This verify the accuracy of the approximation used in ABER analysis of the UWB system. The proposed approach to calculate the ABER is accurate and faster than the existing approaches. Our computer (Intel(R) Core(TM) i5-3470 CPU 3.20 GHz with 4 GB RAM) takes 25 min for the proposed ABER calculation while the same computer takes nearly 33 h for [9] ABER calculations.

References


