# A game theory based land layout optimization of cities using genetic algorithm 

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#### Abstract

Layout optimization is carried out to achieve the true economic potential of cities by ensuring optimal land usage. This paper presents a novel framework to resolve conflicts between types of land usage while considering the effect of neighboring land plots to generate efficient city layouts. The optimization problem is modeled as a game between land-usage based players, each trying to maximize payoff based on expected layout modifications. It is worth noting that this framework aims to include multi-stakeholder competition, land patch shaping, and neighbor plot effects. A game-theory based approach is coupled with genetic algorithm in which swapping and resizing operators are used to generate a new solution. A parametric study on fitness evaluation of algorithmic output layouts is presented. The presented optimization framework has been implemented on land usage map of Guwahati, a city in north-east India.


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## 1. Introduction

Current city planners face challenges in deciding the land use layout to improve overall productivity and economic output. The economic value of each tract of land can be quantified via its usage and types of neighboring plots. Land use for a particular purpose in an area also sets a precedent for preferable usage in the neighboring areas. Considering these issues in planning land usage would result in better quality of life for residents. To address these issues, we develop this framework as a tool to allow regulators to design city layouts for economic preferences. The methodology is designed on the rationale to work across macro-scale land use allocation problems fit for changes seen over decades in cities. Types of land usage considered in this work are housing, industrial, recreation, services and agricultural, which makes land layout optimization a grid-based, discrete, intractable complexity problem with arbitrary definitions of fitness. This paper presents a new approach to solving layout optimization problem by integrating game theory into solution formulation via upgraded variation operators to consider the effects of relationships between neighboring land tracts.

Initially, a number of linear programming (LP) techniques were implemented with geographical information systems (GIS) on fixed size land tracts. Chuvieco (1993) employed technical, financial and ecological constraints with minimizing rural unemployment as the objective. Various conceptual and technical inconsistencies in the use of LP in (Chuvieco, 1993) were corrected by Arthur and Nalle (1997) by comparing the results from a widely used LINDO software system in (Schrage, 2016). The combination of this technology with GIS for landuse planning showed the potential for using modeling and optimization for land layout decisions while ensuring principled interpretation and application. However, LP based methods are applicable only to the problems having defined linear objectives and constraints, but converting semantic expressions of desirable objectives to linear equations remains a challenge.

To overcome the shortcomings in the LP based methods, heuristic-based algorithms such as cellular automata (CA) and simulated annealing (SA) were used. Li and Yeh (2000, 2002) extended CA and integrated with GIS information to allow planners to find better urban forms for sustainable development. The authors proposed a new method to simulate the evolution of multiple land uses based on the integration of neural networks and CA using GIS. Santé-Riveira, BoullónMagán, Crecente-Maseda, and Miranda-Barrós (2008) employed SA for allocation of land units to a set of possible uses on the basis of land use suitability and homogeneous usage compactness, which are fixed a priori. Aerts and Heuvelink (2002) demonstrated the use of SA to solve highdimensional non-linear optimization problems for multi-site land use allocation problems. This multi-objective optimization model minimizes development costs and maximizes spatial compactness simultaneously.

Li, Shi, He, and Liu (2011) integrated CA with ant colony optimization (ACO) to develop an integrated system named geographical simulation and optimization system (GeoSOS). In their case study, the CA component of the GeoSOS generated simulations of the industrial land use changes for some years in the following decade. The ACO component had also been revised from the conventional ACO to work on raster surfaces. Liu, Wang, Ji, Liu, and Zhao (2012) proposed a variant of ACO by incorporating multiple types of ants to solve multiple land use allocation problems. This multiple land allocation method has a spatial exchange mechanism which is used to deal with competition between different types of land use allocation.

Liu et al. (2012a, 2012b) proposed a particle swarm optimization (PSO) based model to maximize the attribute differences between land-use zones, spatial compactness, spatial harmony and the ecological benefits of the land-use zones. Constraints were designed as the quantity limitations for varying land-use zones, regulations assigning land units to a
certain land-use zone, and the stipulation of a minimum parcel area in a land-use zoning map.

Porta et al. (2013) used GAs to formulate and develop land use plans implementing two criteria, namely the land suitability and the shape regularity. The constraints and decision variables were selected based on legal rules and experts' criteria. In contrast, Stewart, Janssen, and Van Herwijnen (2004) focused on the development of goal programming and the associated GA for multi-objective land use planning. Brookes $(1997,2001)$ introduced GA for patch design, combining a region-growing algorithm, raster GIS functions, and multi-objective decision-making techniques into solving single and multiple patch problems. Li and Yeh (2005) demonstrated that GAs can be used with GIS to effectively solve the spatial decision problems for optimally deciding ' $n$ ' sites of a facility. Authors used detailed population and transportation data from GIS to calculate fitness functions and incorporate multiple objectives into the GA program. Holzkämper and Seppelt (2007) presented a flexible and easy to use GA-based library called LUPOlib (Land-Use Pattern Optimization-library) for optimizing the land use configurations. Morio, Schädler, and Finkel (2013) presented a framework for spatially explicit, integrated planning and assessment of brownfield (reuse of underused or abandoned contaminated land) redevelopment options. Balling, Taber, Brown, and Day (1999) also used a GA with a multi-objective fitness function to obtain a Pareto-optimal set of future land-use and corridor-upgrade plans for the city of Provo, Utah. Cao, Huang, Wang, and Lin (2012) proposed a spatial multi-objective land use optimization and solved using non-dominated sorting genetic algorithm-II. The case study presented in their paper has demonstrated the ability of the model to generate diversified land use planning scenarios which form the core of a land use planning support system.

Despite such an array of work using evolutionary algorithms (EA), the solutions to layout optimization are restricted to the definition of objective function and its characteristics. The objective function in referred publications does not consider the effect of land-use relations. In retrospect, a real-world problem such as land layout optimization has many more aspects such as conflicts between stakeholders, effects of market forces, land usage regulations, etc.. Such constraints need to be incorporated into mathematical models for generating optimal layouts. Since game theory (GT) is the study of strategic decision making, it is the ideal tool for modeling conflicts on quantified payoffs of stakeholders. We assume that decision making by economic groups (players) incorporates human interests via iterations in EA. Considering these possibilities, researchers have recently begun to incorporate GT into their models.

Several steps have been taken in the direction of integration of GT for resolving conflicts for land layout improvements. Liu et al. (2015) have created a land-use spatial optimization model through coupling GA and GT to augment land-use spatial optimization models for addressing local land-use competitions. Lin and Li (2016) integrated GT in layout optimization to resolve conflicts in eco-protected zoning areas. Hui and Bao (2013) developed a new analytic framework by studying the logic and strategy of conflicts of legal land acquisition from a behavioral perspective based on game theory. Liu et al. (2016) presented a
framework to study the implementation of concurrent algorithms on the fine sized real-world grid by use of augmented PSO. In comparison, we partake in this area by presenting an initial layout optimization framework that can be used by a regulating authority to obtain layouts with desired patch characteristics. Our approach modifies GT operators to include local laws and land conversion which imitate actual market transactions. This is a work towards enabling regulators to design policies that can lead to certain finalized layouts after transactions.

It can be observed from the literature that the land usage planning is an intractable problem with an associated social dimension. Solutions to this problem have been attempted with a range of optimization algorithms, from LP methods to more complex EAs with GT. In this paper, land distribution problem has been modeled by considering each economic sector as a player. The land layout optimization is formulated using game theory with a goal to maximize the social payoff while preserving land use preferences. Accordingly, the layout optimization algorithm has been designed and implemented as a game among the economic sectors where each player is maximizing its own payoff. The operators in EA implementation have been designed to represent land conversion and extension with respect to economic players, a practice mimicking real estate deals in markets. Following the concept of social good for assessing changes in layout, any changes in the distribution are considered valid only if all affected parties benefit via payoff values.

The paper is organized into six sections. Section 2 presents the problem formulation. Section 3 describes the optimization method. Section 4 presents results and discussion from a parametric study. A case study on Guwahati city, India is shown in section 5 . The paper concludes in section 6 with future work.

## 2. Problem formulation

The paper aims at solving the problem of layout optimization having multiple conflicting objectives for city planning by extending the work of Cao et al. (2011) in which minimizing land conversion costs, maximizing accessibility and maximizing compatibilities have been considered. We formulate the effect of optimal land zones and their distribution in the city layout by considering each zone as a player in the Game theoretic formulation. Landholding arrangements are represented using grid type schemata. Each land tract of certain usage type has desirable attributes such as size, shape, uniformity, closeness among other patches of the same type. Further, each land tract has a preference order for types of land usage in the neighboring patches. In this work, the objective of the problem has been designed to maximize patch desirability (size, shape, etc.), accessibility and compatibilities - each of which are conflicting objectives. The parameters are selected to sufficiently represent a player's choice for patch geometry and their distribution across the land map.

The problem has been formulated for land usage types of housing, industrial, recreation, services, and agricultural, where each land type is a player vying to maximize its payoff. The types of land usage considered here are selected to classify economic activities in an urban setting. For instance, the housing includes places of living; agriculture
refers to land-based food production; industrial zones include pollution generating establishments such as factories and manufacturing units. Recreational zones include social areas such as parks and shopping centers. The service sector would consist of information technology, design centers and other activities that are end-user facing. The payoff depends on the land tract attributes for each player. Thus, each player will specify an optimal value for each of the attributes of the land distribution. The payoff for the player should drop if the values achieved for attributes are different from the optimal value. The player based modeling is inspired by the GT based formulation in which it is desired that each change in land distribution would improve payoffs for all involved players.

The inputs to the given optimization problem as shown in Figure 1 are 1) total land area (in square units), 2) types of usage, 3) parameters characterizing optimal land type, 4) relationship definitions between players, and 5) expectations of each player in terms of characterizing parameters. The total land in a map is discretized based on available area and desired refinement. Types of usage are defined for considering optimal requirements for each stakeholder and further establishing relations among them. The requirements on physical attributes of patches for each stakeholder are represented via certain statistical parameters which are listed in Table 1. Relationships between players represent the compatibility objective of the problem. Each of the expectations of every stakeholder is given as certain values for each of the predefined parameters. The details act as input to GA based land optimization software presented in this paper. This method modifies the initial land map to obtain a final layout that maximizes the player objectives.

Figure 2 represents a graph-based model for evaluating the fitness of the land distribution. Each of the vertices represents a stakeholder and each edge represents the relationship between a pair of stakeholders. Thus, each vertex has statistical parameters representing properties of intraland usage. Each edge would represent compatibility relationships by considering inter-player parameters. The vertices in the model represent patch based payoff calculation factors, whereas the edges represent inter-player usage relation payoffs. The presented method is designed for obtaining preferred land patches for each player.

### 2.1. Statistical parameters for payoff calculation

The physical attributes and representative statistical parameters are presented in Table 1. Attributes such as size, shape, and uniformity are required for characterizing physical patch desirability for the stakeholder. The players denote individual payoffs with hat functions which are used to represent the existence of a unique optimal value

Table 1. Classification based on composition and configuration (McGarigal, 2016).

| 1 | Area | Patch area; radius of gyration, edge, largest patch index |
| :--- | :--- | :--- |
| 2 | Shape | Perimeter/Area, shape index, circumscribing circle index |
| 3 | Core area | Patch core area, core area index |
| 4 | Subdivision | Patch density, splitting density |
| 5 | Isolation | Euclidean nearest neighbor distance |
| 6 | Diversity | Similarity index |
| 7 | Contrast | Edge contrast index |
| 8 | Aggregation | Contagion, clumpiness, patch cohesion, landscape shape |



Figure 2. A model representing payoff calculation effects.
for payoffs with respect to each parameter. It assumes the existence of an optimal value for a player. For example, in a layout, 'residential' land player may encode the land size of 6 units as preferred value.

Attributes related to closeness and relationships represent accessibility and compatibility, both of which contribute to maximizing the economic output of the city. Each type of the properties can be represented by one of the corresponding parameters, which are most relevant to the problem at hand. Keeping in mind the square and discrete nature of land tracts, parameters in Table 2 are selected corresponding to each attribute in Table 1. Furthermore, the definition of each statistical parameter has been presented in Table 2.

Player relationships are encoded as an asymmetric matrix in Table 3. The final payoff is defined as a summation over both independent and relative parameters. To maintain the same sum of weight as individual payoffs, the parameters representing relations between various players are selected within a range of -5 to 5 . Here, -5 signifies low compatibility, and 5 signifies a high compatibility. The maximum and minimum values are selected to match the maximum weight of 6 attained in a player's evaluation of their parameters. Land use compatibility has


Figure 1. The function of layout optimization software.

Table 2. Statistical properties and description.

| Properties | Explanation |
| :--- | :--- |
| Mean Patch Area | Mean of four-point connected patch areas of a <br> distribution <br> Mean of the perimeter to area ratio. Represents <br> the regularity of the shape of the patches. <br> Perimeter/Area |
| Circumscribing square <br> diagonal <br> be covered by a patch |  |
| Patch Density | Represents the uniformity of patch <br> Euclidian Distance <br> Normalized Sum of the distances between two <br> member sets of patches |
| Patch Relation using <br> edge contrasts | This represents the contrast of a patch with the <br> neighboring patches |

Table 3. Land usage relationship definitions.

|  | Housing | Industrial | Recreational | Services | Agriculture |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Housing | 0 | -5 | 5 | 2 | -4 |
| Industrial | 3 | 0 | 3 | 3 | 3 |
| Recreational | 5 | -4 | 0 | 3 | -2 |
| Services | 5 | -4 | 3 | 0 | -4 |
| Agriculture | 3 | -5 | -2 | -2 | 0 |

been calculated by using the compatibility matrix approach in Cao et al. (2011). A formal decision-making framework such as analytical hierarchical process presented in Saaty (1987) and studied recently in De FSM Russo and Camanho (2015) can be employed for developing the compatibility matrix. To avoid the non-uniqueness encountered in this method, semi-definite programming as presented in Vandenberghe and Boyd (1996) can also be employed to derive such unique and optimal matrices on convex spaces. In this paper, the player relationships are encoded with an asymmetric matrix representing the nature of preferences for land areas. For instance, an industry land player would prefer being close to housing area for people to commute easily. In comparison, house owners would prefer recreation closer to their houses instead of polluting industries. Such relationships are expressed in Table 3 and can be tweaked for regulator objectives.

The rows in the relationship matrix belong to each player whose payoffs are stated on a cell of any patch. For each cell belonging to a player, the neighboring cell usage is used to determine the relationship payoffs by summing them up for 8 -point connectivity. The payoffs are further normalized to
obtain an equivalent representation for each patch. In the relationship matrix, a higher payoff represents preference towards the land type.

### 2.2. Playerwise payoffs

Each player has a unique optimal value for each statistical parameter. This optimal value is mapped to individual payoffs using hat functions represented in Figure 3. The calculated value of the statistical parameter in a distribution acts as input to hat function to obtain final payoff. It is noted that the maximum payoff is obtained at the optimal value for each parameter for each player. Any deviation from the optimal value leads to a payoff lower than optimum. Payoff obtained at the optimal value for a parameter can also be decided by the player himself. It is represented by the height of hat function in Figure 3, which acts as a map from statistical parameters to individual payoffs. The magnitude of the hat functions includes the influence of a player into the calculations for the social payoff.

## 3. Optimization method

In this paper, GA is used for layout optimization for a single layout. A general run of a proposed algorithm initializes the land layout as a grid with integers representing the land use type. The fitness evaluation of the layout is carried out using statistical parameters as defined in Tables 2 and 3. The relationship matrix values in Table 3 are based on edge connections between players as shown in Figure 2. This layout undergoes swapping and resizing consecutively, each of which returns a new land distribution only if a better distribution is obtained. Swapping and resizing operators have stochastic designs presented in the following sub-sections.

### 3.1. Population generation

A random population is generated for given edge length of square-shaped land distribution. In the upcoming sections, we present the results showing parametric effects on the

| Land usage statistical parameters |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Payoff Matrix | Mean <br> Patch <br> Area | Mean <br> Patch Edge length | Circumscribing circle | Patch <br> Density | Euclidian distance (isolation) |
| Housing |  |  |  |  |  |
| Industrial |  |  |  |  |  |
| Recreation |  |  |  |  |  |
| Services |  |  |  |  |  |
| Agriculture |  |  |  |  |  |
|  |  |  |  |  |  |

Figure 3. Hat function.

Table 4. Player definitions.

| Player 1 | Housing |
| :--- | :--- |
| Player 2 | Industrial |
| Player 3 | Recreation |
| Player 4 | Services |
| Player 5 | Agriculture |

evolution of single layout. Each land type is represented by a corresponding integer given in Table 4.

### 3.2. Payoff calculation

As shown in section 2.1, the parameters affecting payoff calculation are normalized, resulting in values 0 to 1 with methods presented in Table 5. The generalized scheme for calculation of players and eventually social payoffs are shown in Figure 4. The payoff is calculated based on two types of parameters, i.e. usage and relationship parameters. The usage parameters are calculated by converting the land matrix into the binary matrix for each player. The area is then segmented into patches whose properties are measured and a mean value for each property-player pair is obtained. Further, the neighborwise payoff is assigned as per relationship matrix on the edge of each patch and then normalized. For a given land distribution, a binary distribution of housing usage is obtained as has been shown in Figure 5. Such a distribution considers all other usage types as equivalent. This distribution is used to compute statistical parameters representing desirability of the land. For relationship parameters, the boundary of each patch is considered and the neighboring cells are computed based on relationship definition matrix given in Table 3. Each of the parameters is normalized and summed up to obtain the final payoff.

Color based labeled matrix showing a color-coded mapping of individual patches for a player-specific binary matrix is shown in Figure 6. Blocks of color other than

Table 5. Land usage statistical parameters.

| Parameters | Normalization Method |
| :---: | :---: |
| Mean patch area | Division by total land area |
| Mean patch edge length | Division by 4 times number of 4 -connected patches |
| Circumscribing square diagonal | Using diagonal length of the plot |
| Patch density | Already normalized |
| Mean euclidian distance (isolation) | Division by land diagonal length |
| Land usage relationship | $[-5,5] .(x+5) / 10$. Further division by patch area for every patch followed by averaging. |

blue in original land represent an individual patch. In Table 6, the player distribution specific parameters are calculated by averaging over contiguous patches in the binary matrix shown in Figure 5. Further, a relation parameter is also calculated and implemented for the entire layout. Let us define the variation operators used at each step of the presented EA method.

### 3.3. Swapping operator

Swapping operator is designed to exchange the land track between two land usage types or players. It has been implemented on a cell basis among pairs of players. Five pairs of players are taken ensuring equal representation for all players. For each pair of players, a cell is selected at random from a randomly selected patch and the two cells are swapped as shown in Figure 7(a,b). The outcome is considered acceptable if payoffs improve for both players. In case of multiple pairs achieving better payoffs, best net payoff distribution is selected for further stages of the algorithm.

### 3.4. Resizing operator

Resizing operator is the extension of occupied land by a player beyond its current expanse. Resizing operator has been implemented on a player owned cell basis. For each player, a random boundary cell in a random patch is selected. Further, a neighboring patch of foreign usage type is selected at random by the cell whose identity is then changed to the original cell as shown in Figure $7(\mathrm{a}, \mathrm{c})$. If the transaction leads to higher payoff for the parent and foreign players, then the change is accepted. In case of multiple transactions achieving better payoffs, best net payoff distribution is selected for further stages of the algorithm.

As shown in Figure 1, the layout optimization software inputs an initial land distribution with requisite data and outputs a final distribution. The details of the software have been represented in Figure 8. The flowchart begins with an initial land distribution and preferred values of statistical parameters for each player. The optimal values define the peak of hat functions for each parameter-player pair, i.e. the players obtain maximum payoff whenever the value of statistical parameters for a given layout match these values. The land distribution is passed on to 'swapping acceptance' function. This function creates five pairs of players such that each player is represented exactly twice. For each pair, cellbased swapping is carried out at random. When the


Figure 4. A flowchart depicting fitness evaluation.


Figure 5. Representational Binary matrix for a player.


Figure 6. Color-coded labels for each 4-connected patch in binary matrix corresponding to the individual player.

Table 6. Value of statistical parameter for layout in Figure 6.

| Statistical Parameters | Values |
| :--- | :--- |
| Mean Patch Area | 1.8 |
| Mean Perimeter to Area ratio | 3.4667 |
| Mean diagonal of circumscribing square | 2.0258 |
| Mean Patch Density | 0.95 |
| Mean Euclidian distance in Patches | 2.4015 |

swapping operator generates better payoff for both players, the swapped distribution is marked acceptable as it signifies an improvement. If multiple such solutions are accepted in the five swaps, then the one maximizing social payoff is chosen for replacing the original land distribution. The updated distribution is passed on for resizing operation. The operator resizes a random patch for each player and accepts the change if both players have improved payoffs. In case of multiple acceptances in the five resize operations, the distribution with maximum social payoff is selected for replacing original land distribution. Each of the payoff calculations is done by considering the hat curve mapping shown in Figure 3. This process is repeated for multiple iterations to ensure that relative error of social payoff has converged for each player.

## 4. Results and discussions

This section presents a parametric study of the proposed algorithm. The parametric effects of player-wise payoff weight, relational payoff weight, mean/median based fitness calculation and number of cells are studied. The controlled inputs in this optimization algorithm are land size, a number of iterations, player-wise optimal payoff values, and weights of relationship payoffs which are shown in Table 7. The complete land has been assumed to be a square layout divided into cells of uniform size. Thus, a layout of edge length 20 has a total of 400 units. Player-wise payoffs represent the weight of each player. In terms of Figure 3, each column represents the height of the hat function for the player. This aids the representation of a super-player who has greater weight in determining social payoff. It can be used to skew the final layout towards a particular sector. The payoffs for usage-based payoff include calculation of each parameter on a patch basis for each player. These parameters can then be used for representing a player by either taking their mean or their median. Medians are a more accurate representation of current status in a parameter for a player. In practice, use of mean based payoffs was found to ensure faster convergence.

The efficiency of the methods has been measured by a term representing percentage ratio of obtained payoff to net achievable payoff. Efficiency or Social Payoff percentage is calculated as the product of a number of players and sum of maximum achievable player payoffs.

The following initial payoff values have been obtained for initial layout.

The contribution of normalized relationship based parameter payoff is one-sixth of net player payoff. This has also


Figure 7. Swapped and Resized distribution.


Figure 8. Flowchart of GA based software.

Table 7. Input parameters

| S. No. | Variable parameters | Sample values |
| :--- | :--- | :---: |
| 1 | Land Edge, N | 20 |
| 2 | Number of iterations | 600 |
| 3 | Player wise payoffs | $\left[\begin{array}{cc}1 & 1\end{array} 111\right]$ |
| 4 | Statistical averaging parameter | Mean/Median |
| 5 | Relationship payoff | Unity (normalized) |

been varied and changed to a half. The results are documented in Case III in the upcoming section. Figure 9 shows the land distribution of edge length of 20 obtained at random. This distribution has been used as initial distribution for six case studies.

### 4.1. Case study I: general case

In this case study, the input parameters presented in Table 7 are considered. The percentage Social Payoff obtained in this case study is $72.92 \%$, as shown in Figure 10. In case of mean payoff, the variation of the social payoff is mostly smooth and levels off after 300 generations. Even with varied initial land distributions, the final payoffs obtained were close to this value. This trend has been observed for all variations studied in this report. Most of the units are integrated and patterns can be observed with respect to binary matrices of each player.

### 4.2. Case study II: player-wise payoff varied

This is continued parametric study from Case I where playerwise payoffs are varied to indicate relative preferences among players. This can be used to provide emphasis on a sector for

| Mean based initial payoff: | $[3.2122,3.3123,2.9040,2.7756,2.0954]$ |
| :--- | :--- |
| Median based initial payoff: | $[1.4208,1.3743,1.4576,1.4476,0.7805]$ |

Original Land Distribution


Figure 9. Original Land Distribution.
developing the city plan. Following player-wise payoffs is considered in this case study; $[1,1.2,0.95,1.05,1.07]$.

The percentage social payoff obtained in this case is $72.28 \%$ which is shown in Figure 11. The payoff ability can be used to represent super-user and dominated users in the society and payoffs can then be obtained. The percentage payoff obtained is almost the same as in the previous case.

### 4.3. Case study III: relational payoff weight varied

Continuing the case study I, the relationship payoffs are now increased to five times their original contribution. This ensures the equal contribution of usage and relationship payoffs in the calculation of individual player payoffs.


Figure 10. Case Study 1.


Figure 11. Case Study II.

Figure 12 shows that the percentage social payoff obtained is $75.18 \%$. The weight of relationship parameters in the calculations is multiplied by five units to ensure comparable payoff as that of land usage payoffs. Better efficiency is observed in this case. This can be attributed to the greater emphasis on patch relations as evident from Figure 12.

### 4.4. Case IV: player-wise and relational payoff weight varied

Compared to the case study I, both player-wise payoffs and relationship payoff weights are varied and the change in net social payoffs is recorded. The player-wise payoffs are considered as $[1,1.2,0.95,1.05,1.07]$ and the relationship
payoffs are increased to five times their original contribution. The mean-based payoff calculations are considered.

In this case, the percentage social payoff obtained is $75.23 \%$ which is shown in Figure 13. The net social payoff has risen compared to the case study I. The majority of the rise can be attributed to newly weighted relationship payoffs as can be observed in Case II and Case III.

### 4.5. Case V: payoff calculation method of mean followed by median

As a continued parametric case study I, the effect of variation of a statistical parameter in the calculation of player payoffs is varied. A simulation run is made on mean based payoff calculation followed by a median based one. Here the



Figure 13. Case study IV.
optimization was being run in two stages, $\mathrm{V}(1)$ and $\mathrm{V}(2)$, corresponding to mean and median-based payoff calculations.

For mean payoff based algorithm run, initial payoffs are [3.1550, 3.2537, 2.8106, 2.7638, 2.1329] and final payoffs are [4.9851, 4.0876, 4.8463, 3.7442, 4.3196]. In this case, Figure 14 shows that the percentage social payoff obtained is $74.04 \%$. This stage showed a rapid change in a payoff by increasing from 0.5 to 0.733 .

For median payoff based algorithm run, initial payoffs are $[3.7116,2.2954,3.7049,1.9616,3.1657]$ and final payoffs are [4.5305, 2.4981, 4.0643, 2.7901, 3.8188]. The percentage social payoff obtained is $59.02 \%$. This stage showed minor changes in payoff as shown in Figure 15 by improving from 0.52 to 0.59 . The runs have been made
in succession which implies a payoff of 0.74 in terms of mean based calculation, equivalent to 0.5 in terms of median based calculations.

In terms of mean based calculations, the percentage social payoff obtained is $71.61 \%$. Although the percentage payoff has diminished, this is a good method to bring variations into the layout which sometimes leads to layouts with lesser numbers of single unit land tracts. This change yielded better results as it means based payoffs record tiny improvements in layout whereas medians represent the actual situation and respond rarely with respect to any changes in layout. Thus, a run of mean followed by median allows for larger changes in the beginning followed by smaller changes in the later part.


Figure 14. Case Study V-1 based on the mean payoff.



Figure 15. Case Study V-2 based on the median payoff.

### 4.6. Case VI: payoff calculation method of median followed by mean

As a continued parametric case study I, the effect of variation of a statistical parameter in the calculation of player payoffs is varied. A run is made on median based payoff calculation followed by mean based one. The results obtained from mean based payoff variation run are presented in Case VI-2.

In this case, as shown in Figure 16, the payoff obtained and the corresponding layout is poor after a median payoff based run as the variations in the median are difficult to obtain with just a variation in the cell, although it is a more accurate representation parameter. In contrast, a secondary mean-based run as presented in Figure 17, Case VI-2 demonstrates significant improvements in social payoff and land distribution.

### 4.7. Case VII: larger land size

In this case, the effect of varying number of cells in the layout has been studied. The land edge size, N , is considered as 20,30 and 50 . A number of iterations are fixed 1000,2000 and 4000 for $\mathrm{N}=20,30$, and 50 , respectively. Equal player-wise payoffs are considered. Median based payoff calculations with normalized payoff relationship are used.

The simulation results for Case VII 1-3 show a net efficiency obtained as $74.01 \%, 72.82 \%$, and $69.12 \%$, respectively. From Figure 18, it can be observed that the percentage social payoff converges to $70 \%$ as the simulation progresses. This strengthens the hypothesis
that maximum efficiency achieved using the proposed algorithm has low sensitivity towards a variation of land size. In each case, the contiguous patches tend towards desired land sizes, shapes, and relative placement. This pattern is observed in Case VII 1-3. A similarity of aggregated land patch sizes, roundness and neighbor types is observed in each case in Figure 18.

## 5. Case study: Guwahati city, India

This section contains a case study on a selected land usage map of Guwahati city, India by Guwahati Metropolitan Development Authority (2013). Since the discussed algorithm can only input rectangular maps, we assume the white space to contain agricultural land situated on the outskirts of the city. The algorithm can consider non-participating land which will not be altered in the course of development. In this case, river water and other water bodies fall under this category.

The land map shown in Figure 19 was averaged and brought to smaller mesh to incorporate into appropriate run times for the algorithm, as depicted in Figure 20. The initial payoff corresponding to each land usage type is obtained as [4.3479, 2.3550, 4.3800, 2.1227, 5.1715].

As can be observed, the payoff of agricultural lands is much higher due to the greater presence of contiguous agricultural lands. The algorithm was implemented as shown in case study 1 and distribution obtained is shown in Figure 21. The final payoff corresponding to each land usage type is shown as [4.5467, 4.3096, 5.1880, 4.7489,


Figure 16. Case Study VI-1.


Figure 17. Case Study VI-2.
5.0748]. As can be observed, there have been no significant changes barring increase in the land of the services sector. The amount of land dedicated to agriculture has remained
almost unchanged to ensure no fall in the payoff at any change in its distribution. The regularity in shapes of industrial lands has improved.

Original land distribution


Optimal land distribution



Case VII-1


Case VII-2
Figure 18. Final land layout and payoff variations for case study VII.


Optimal land distribution



Case VII-3
Figure 18. Continued.


Figure 19. Land usage map of Guwahati city (https://gmda.assam.gov.in/ documents-detail/master-plan-guwahati-2025-maps).

## 6. Conclusions

This article proposed an algorithm towards tackling the problem of layout optimization with a novel framework based on genetic algorithm and game theory. This work is aimed at developing a methodology for developing economic zones in the cities over a time-period of decades. For this purpose, land usage has been classified as housing, industry, recreation, services, and agriculture. Each of the land usage types is classified as a player who aims to improve their payoff with variations over several iterations. Variation operators such as swapping and resizing are developed based on land
exchange and extension activities. In comparison to a GA, this method does not have a set of the population, but rather starts with a particular layout and improves it across iterations. The operators have been implemented on a cell basis and hold a promise of better performance if also applied on a patch basis.

Consideration of neighbor relationship based payoff is a major highlight of this work. It considers the effect of the proximity of one land type to another, such as that housing areas would prefer closeness to recreation and a distance from industry. The presence of self-sustaining units containing patches of each type of land unit can also be observed. Increasing the payoff of such relationship in payoff calculation of players has also led to improved layouts, as evident from social efficiency.

Player-wise payoff variation has helped capture the effect of super-user and dominated player. Such a variation can be used to capture dominance of a particular industry in a city and corresponding plans. In case of such variations, the social efficiency obtained is close to the general case.

It has been observed that median is a more accurate measure of the parameters. However, using medians causes the problem of slowing down of any variations in simulations. Thus, only median based payoff calculation leads to poor quality final layouts. A more reasonable method studied has been of applying a median based algorithm once a mean based algorithm has become saturated. This has led to lower ranges of patch sizes and improved layout despite slight depreciation in percentage social payoff. Also, this method shows the way towards improving payoffs after its stagnation.


Figure 20. A mesh representation of land usage map of Guwahati in Figure 19.


Figure 21. Optimal land usage map as output.

Other extensions such as patch based variations and inclusion of more statistical parameters can still be incorporated into this method to explore for better city layouts. The payoff functions can further be altered to include terrain and geological effects in the objective functions of each economic zone to improve the quality of predictions with this framework. The issue of defining parameters for neighboring matrix and optimal player parameters can be carried out with data-driven methods on desired city layouts. The current work displays the possibilities in layout design with a framework merging effects of current land patches, neighbors and a game theoretic algorithm design.

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