An Improved Initial Population Strategy for Compliant Mechanism Designs Using Evolutionary Optimization

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***Abstract*— In this paper, an improved initial random population strategy using a binary (0-1) representation of continuum structures is developed for evolving the topologies of path generating complaint mechanism. It helps the evolutionary optimization procedure to start with the structures which are free from impracticalities such as 'checker-board' pattern and disconnected 'floating' material. For generating an improved initial population, intermediate points are created randomly and the support, loading and output regions of a structure are connected through these intermediate points by straight lines. Thereafter, a material is assigned to those grids only where these straight lines pass.**

In the present study, single and two-objective optimization problems are solved using a local search based evolutionary optimization (NSGA-II) procedure. The single objective optimization problem is formulated by minimizing the weight of structure and a two-objective optimization problem deals with the simultaneous minimization of weight and input energy supplied to the structure. In both cases, an optimization problem is subjected to constraints limiting the allowed deviation at each precision point of a prescribed path so that the task of generating a user-defined path is accomplished and limiting the maximum stress to be within the allowable strength of material. Non-dominated solutions obtained after NSGA-II run are further improved by a local search procedure. Motivation behind the two-objective study is to find the trade-off optimal solutions so that diverse non-dominated topologies of complaint mechanism can be evolved in one run of optimization procedure. The obtained results of two-objective optimization study is compared with an usual study in which material in each grid is assigned at random for creating an initial population of continuum structures. Due to the use of improved initial population, the obtained non-dominated solutions outperform that of the usual study. Different shapes and nature of connectivity of the members of support, loading and output regions of the non-dominated solutions are evolved which will allow the designers to understand the topological changes which made the trade-off and will be helpful in choosing a particular solution for practice.

I. INTRODUCTION

Compliant mechanisms are flexible elastic structures which can deform to transmit the force and/or generating some desired path on the application of applied load. Compliant mechanisms have shown many advantages over pseudo-rigid-body mechanisms as jointless and monolithic structures, involved less friction, wear and noise [1], ease of manufacturing without assembly, light weight devices [2] etc. Applications of compliant mechanisms are in the area of product design, off-shore structures, smart structures, MEMS [3] etc.

Based on continuum mechanics approach, studies have been made by considering homogenization method [4], [5] or material density approach [6] in which the discrete nature of problem is converted into continuous one with some threshold value. In an another approach, a discrete nature is preserved by using a binary (0-1) representation of material for defining a structure [7], [8], [9], [10], [11]. In this paper, a few important studies using binary representation of material for the synthesis of compliant mechanisms are discussed which are modeled using either truss/frame ground structures or two-dimensional continuum structures and are optimized using an evolutionary optimization. In truss/frame ground structures, presence of a truss/frame element depends on the value of a binary bit. With an additional approaches of flexible building blocks [12], spanning tree theory [13] and load path synthesis [14], topologies of compliant mechanisms are generated which are well-connected and free from gray scale and hence, results in an improved designs. Large displacement [15] compliant mechanisms and path generating [16], [17] compliant mechanisms are also designed using a binary approach and optimized using NSGA-II algorithm [18].

For representing a two-dimensional continuum structure using a Boolean variables, a design domain is discretized into quadrilateral elements and each element of a structure is either represented by material or void depending on corresponding to Boolean variable value. Using a modified evolutionary structural optimization (ESO) procedure [19], genetic programming [20] and genetic algorithms [21], [22], [23], [24], [25], [26], [27], [28], compliant mechanisms are designed with different objectives and tasks. Using a morphological technique of representing a structure, various problems of compliant mechanisms and structural optimization are solved in which Bezier curves are used to represent the structure [21], [22], [23], [24].

Using a Boolean representation of continuum structures, authors of this paper have introduced a new formulation for path generating compliant mechanisms [26], [27], [28]. With different sets of bi-objective problems in which constraints are imposed on the maximum stress and at the precision points of a user-defined path with some allowed deviation, topologies are

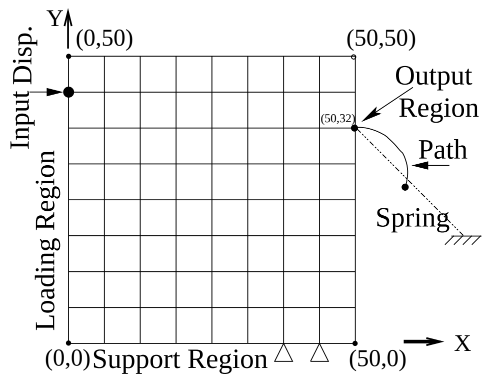


Fig. 1. Design space with loading, output and support regions.

generated using a local search based evolutionary optimization procedure. A local search method is used for further improving the designs but its performance is dependent on the non-dominated solutions of the applied NSGA-II algorithm [18]. In an usual way of generating an initial population, NSGA-II algorithm emphasizes non-dominated solutions iteratively. However, due to sparse placement of feasible solutions, a random initial population fails to find a diverse set of non-dominated solutions and prematurely converges close to a sub-optimal feasible solutions. In this paper, an improved initial random population strategy using a binary (0-1) representation of continuum structures is developed for evolving the topologies of path generating complaint mechanism. Single and two-objective optimization problems are solved using a local search based evolutionary optimization (NSGA-II) procedure and obtained results are also compared with a study in which initial population is created at random. The detailed descriptions about the problem formulation, local search based NSGA-II procedure with an improved initial population strategy and obtained results are reported in subsequent sections.

II. Problem Formulation

For designing the complaint mechanism, a design domain of 50 mm by 50 mm is divided into three regions of interest as shown in Figure 1. The first region is called support region where the structure is supported (restrained, with zero displacement) whereas, in the second region (loading region) some specified load (input displacement) is applied. The output region is the third region of interest, that is, a fixed point on the structure which traces out the desired path defined by user.

Authors of this paper thought that it is a compulsory task of the compliant mechanisms to trace the prescribed path. Therefore, a formulation based on the precision points representing the prescribed path was proposed using the different bi-objective sets [26], [27], [28]. These studies show the competency and efficiency of the proposed formulation of designing the CM to trace the different sets of prescribed paths, for example, curvilinear or straight line, within the user-defined allowed deviation η at the precision points.

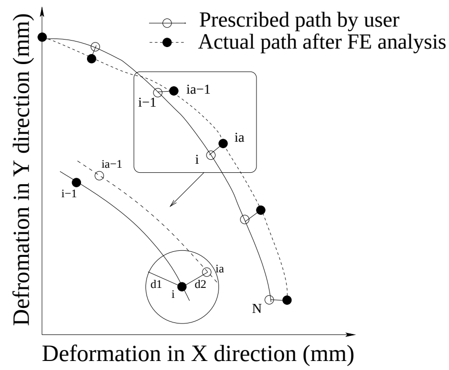


Fig. 2. An actual path traced by a structure after FE analysis and the prescribed path.

For describing the PGCM formulation, the prescribed path and an actual path traced by a structure after FE analysis are drawn in Figure 2 for a hypothetical case. Here, the prescribed path is represented by the precision points. The corresponding points on an actual path traced by a structure is evaluated from geometrical non-linear FE analysis based on equal time steps. For physically representing the formulation, first an euclidean distance (say d_1) between the current (i) and previous ($i-1$) precision points representing the prescribed path is evaluated and multiplied by a factor η called as allowed deviation. Then, another euclidean distance (say d_2) between the current precision point (i) and the corresponding point (i_a) of the actual path is calculated. Based on these calculation, a constraint is imposed at each precision point which ensures that $d_2 \leq d_1$. The mathematical representation of constraints at each N precision points is given in Equation 1. Any structure which satisfies these constraints will guarantee to accomplish the task of tracing the path within the user-defined allowed deviation (η). In the study, an additional constraint limiting the maximum stress developed in the structure is also taken into the consideration for the feasible PGCM designs. For providing some resistance at the output region and for some work meant to be done, a spring of constant stiffness (κ) is also attached.

In the present study, a strategy is developed for generating an initial random population for the NSGA-II algorithm which helps in evolving the topologies of compliant mechanism generating a user-defined path. Motivation behind the present study is to generate topologies of compliant mechanism which perform the same task of generating a prescribed trajectory in one run of an optimization procedure. Both single and two-objective optimization problems are solved using a local search based evolutionary multi-objective optimization procedure. The single objective study minimizes the weight of structure whereas, the two-objective optimization problem simultaneous minimizes the weight and input energy supplied

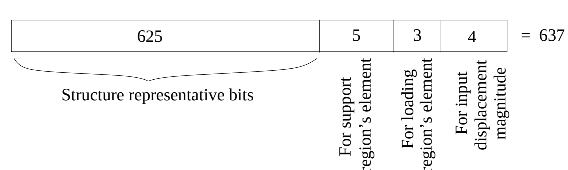


Fig. 3. A binary string representation.

to the structure. Both optimization problems as shown in Equation 1 are subjected to constraints on stress and allowed deviation at the precision points of a prescribed path.

Single-objective optimization
Minimize: Weight of structure

Two-objective optimization
Minimize: Weight of structure,
Minimize: Supplied Input energy to structure, (1)

Both problems are subjected to:

$$1 - \frac{\sqrt{(x_{ia}-x_i)^2+(y_{ia}-y_i)^2}}{\eta\times\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}} \geq 0, \quad i = 1, 2, ..., N$$

$$\sigma_{flexural} - \sigma \geq 0,$$

where $\eta = 15\%$ is the permissible deviation (kept fixed in this paper), and $\sigma_{flexural}$ and σ are flexural yield strength of material and maximum stress developed in the structure, respectively.

III. A LOCAL SEARCH BASED NSGA-II PROCEDURE

Popularly used elitist non-dominated sorting genetic algorithm (known as NSGA-II which is developed by second author of this paper and his students) is used in the paper which has shown to have a good convergence property to the global Pareto-optimal front as well as to maintain the diversity of population on the Pareto-optimal front for two objective problems. A detailed description of NSGA-II can be found from [18]. In short, NSGA-II is population based evolutionary optimization procedure which uses mathematical partial-ordering principle to emphasize non-dominated population members and a crowding distance scheme to emphasize isolated population members in every iteration. An elite-preserving procedure also ensures inclusion of previously found better solutions to further iterations. The overall procedure with N population members has a computational complexity of O(NlogN) for two and three objectives problems and has been popularly used in many studies. NSGA-II is also adopted by a few commercial softwares (such as iSIGHT and modeFRONTIER). A code implementing NSGA-II is available at *http://www.iitk.ac.in/kangal/codes.shtml* website.

As topology optimization of compliant mechanism problem is non-linear and discrete in nature, NSGA-II with local search procedure is used in the present study. A population of 240, crossover probability of 0.95 and mutation probability of ($1/$string length) are assigned and NSGA-II is run for a maximum of 100 generations. For each NSGA-II population member, a binary string length of 637 bits is used in which first 625 bits are used to represent a structure (representing 25×25 grids of material or void) and additional 12 bits are decoded to determine the support and loading region's elements, and the magnitude of input displacement boundary condition. Figure 3 shows a pictorial view of a binary string. Here, the 12 additional bits are further divided into three

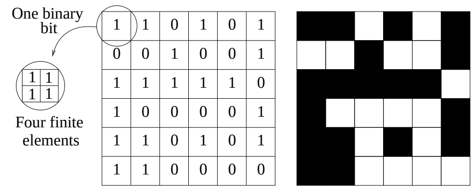


Fig. 4. Representation of structure using binary string and material connectivity.

sets of five, three and four bits. First five bits indicate the support region's element number, whereas the three bits help in determining the loading region's element number. The decoded value of last four bits are used to evaluate the range of input displacement magnitude which varies from 1 mm to 16 mm at step of 1 mm.

A. Representation Scheme

Continuum structure is discretized by 4 node rectangular elements and each element is represented either by 0 or 1, where 1 signifies the presence of material and 0 represents the void. This makes a binary string which is copied to two dimensional array as per the sequence shown in Figure 4. In the present study, one bit of the binary string represents four elements for FE analysis with same gene value as shown in Figure 4.

B. Custom Initialization

In the later studies [29], [30], [26], [27], [28], an initial random population is generated by flipping a coin and decided whether an element is filled with the material. Similarly, the gene value (0-1) is assigned at random which results in impracticalities such as 'checker-board' pattern and disconnected 'floating' material in an initial population. The connectivity among the three regions of interest (refer Figure 1) is also checked before FE analysis of a structure.

In the present work, a strategy of generating an initial random population using a (0-1) binary representation of continuum structures is developed in which the three regions of interest are connected through the intermediate points. A pictorial view is drawn in Figure 5 to show a connectivity between the support and loading region's elements. A random number is generated first to decide the number of intermediate points through which the two regions are connected and it varies from 1 to 5. Depending upon the number of intermediate points (four in this case), coordinates of each intermediate point within the designs domain is randomly generated. Points P1, P2, P3 and P4 in Figure 5 shows the location of intermediate points and, the support and loading region's elements are connected through these points by straight lines. Thereafter, a material is assigned to those elements only where these straight lines pass. A material connectivity of the above mentioned regions is also shown in Figure 5. Similarly, a set of piece-wise linear line segments between the support and output regions and another set between the loading and output regions are explained. Therefore depending on the randomly generated intermediate points, an initial population for the NSGA-II algorithm is generated.

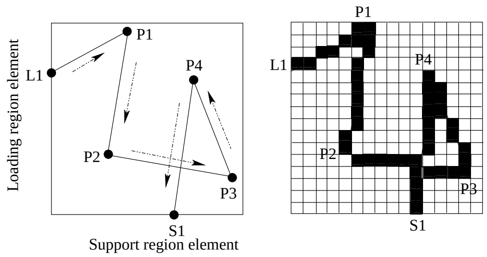


Fig. 5. Connectivity between support and loading regions.

A repairing scheme is also employed in which if two elements generate a point connection, then the given procedure puts one extra material at the nearby element (according to the nature of connectivity) to eliminate the problem of high stress at the point connectivity.

C. Parallel Computing

A distributed computing platform is used to reduce the computational time of designing and synthesis of compliant mechanisms. In this parallelization process, the root processor initializes a random population and performs the NSGA-II operators, like selection, crossover and mutation operators, Pareto-optimal front ranking etc. on the population and replaces it with good individuals. Slave processors calculate the values of objective function and constraints and send them to the root processor. The above process is repeated till the termination criterion of NSGA-II is met. The parallel implementation of NSGA-II is done in the context of FE analysis through $ANSYS$ software which consumes the maximum time of the optimization procedure [26], [27], [28]. A MPI based Linux cluster with 24 processors is used in the present study to solve the computationally extensive evaluation procedure of compliant mechanisms. A detailed specification and configuration of Linux cluster is given at *http://www.iitk.ac.in/kangal/facilities.shtml*.

D. GA Operators

A two-dimensional crossover is used in the present study. This operator has shown a successfully applications in shape optimization [29], [30] and in compliant mechanism design problems by the authors of this paper [26], [27], [28]. In the present recombination operator, two parent solutions are selected and a coin is flipped to decide for row or column-wise crossover. If a row crossover is done, a row is chosen with an equal probability of (P_{xover}/no. of rows) for swapping. The same is done if a column-wise crossover has to be done. During crossover, a random number is generated to identify the number of rows (columns) to be swapped and then, another generated random number helps in getting the first row (column) number of patches. A range of row (column) index is calculated and swapped with other parent. Mutation is done with a low probability on the each bit of a string to change from a void to a filled or from a filled to a void element. Detailed discussion of these crossover and mutation operators are given elsewhere [31], [32].

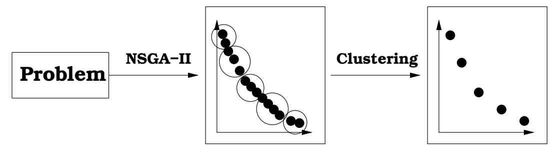


Fig. 6. Clustering Procedure.

E. Clustering Procedure

The number of feasible solutions after the NSGA-II run is equal to or less than the population size. But actually at the end of NSGA-II run, there are not many distinct solutions. So it is not advisable to represent all solutions to the end users. But for the convergence near to the global Pareto-optimal front, a GA needs a fairly large number of population members and generations depending on the problem complexity. To get a meaningful idea of the type of solutions at the end of a NSGA-II run, a clustering procedure is used. The neighboring solutions are grouped together and solutions from each group representing that zone of the Pareto-optimal front are selected as representative solutions [33]. Figure 6 shows the procedure pictorially.

F. Local Search Method

The local search method used here is a combination of evolutionary and classical methods. It is a variant of classical hill climbing process. As a single objective function is needed for the hill climbing, the multi-objective problem is reduced to a single objective problem. This is done by taking a weighted sum of different objectives. The scaled single objective function is minimized in the present study and it is shown in Equation 2:

$$F(x) = \sum_{j=1}^{n} \frac{\overline{w}_j^x (f_{j_{max}}^x - f_j^x)}{f_{j_{max}}^x - f_{j_{min}}^x}, \tag{2}$$

where, f_j^x is j^{th} objective function, $f_{j_{min}}^x$ and $f_{j_{max}}^x$ are minimum and maximum values of j^{th} objective function in the population respectively, n is number of objectives and $\overline{w}_j^x$ is the corresponding weight to the j^{th} objective function which is computed as:

$$\overline{w}_j^x = \frac{(f_{j_{max}}^x - f_j^x) \setminus (f_{j_{max}}^x - f_{j_{min}}^x)}{\sum_{k=0}^{M} (f_{k_{max}}^x - f_k^x) \setminus (f_{k_{max}}^x - f_{k_{min}}^x)}, \tag{3}$$

where M is the number of representative solutions after clustering procedure.

In the Equation 2, the values of the objective functions are normalized to avoid bias towards any objective function. In this approach, the weight vector decides the importance of different objectives, in other words it gives the direction of local search in the objective space.

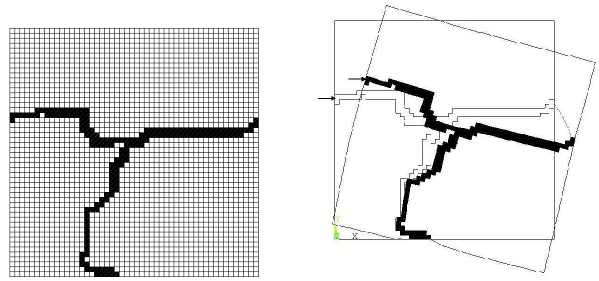
First the weighted sum of the scaled fitness of a selected representative solution after the clustering procedure is executed as given in Equation 2. One bit of representative solution is mutated at a time and the design is extracted from the new string. This new string is now ready for FE analysis and after an *ANSYS* simulation, objective functions and constraint functions are evaluated. If the new design does not satisfy the constraints, then the change in the new string is discarded and old values are restored. Otherwise, the weighted sum of scaled fitness of new string is calculated and compared with the old string. In case of mutating a '0' to '1' , a change is only accepted when the weighted sum of scaled fitness of new string is strictly better than that in the old string, else it is rejected. For the case of mutating '1' to '0', if the weighted sum of scaled fitness of new string is better than or equal to the old strings weighted sum value, then it is accepted else discarded the change. In the case of rejection, the previous bit values are restored.

Before mutating any bit, a binary string is converted into a two-dimensional array and checked for the elements having a material. Then, one by one, all nine neighboring bits including its own bit value are mutated. If a change brings an improvement in scaled fitness, then the change is accepted. This process is repeated till all bits are mutated once. If there is no change in the values of weighted sum of scaled fitness, the local search is terminated. In the same way, all representative solutions are mutated to achieve a local search. As discussed in Section III-A, that one binary bit represents four elements for FE analysis, therefore the local search is performed on these elements. A detailed discussion of the local search is given in the literature [32], [34].

IV. CURVILINEAR PATH GENERATING COMPLIANT MECHANISM TOPOLOGIES

Using an improved initial population strategy, topologies of curvilinear path generating compliant mechanism are evolved using a local search based NSGA-II procedure. The design domain of compliant mechanism is discretized with 50 by 50 rectangular elements in x and y directions respectively. A material with Young's modulus of 3.3 GPa, flexural yield stress of 6.9 MPa, density of 1.114 gm/cm^3 and Poisson ratio of 0.40, is assumed for synthesis of compliant mechanism. Here, a prescribed path is divided into five precision points and the trajectory traced by output point of a structure is evaluated through a geometric nonlinear FE analysis using $ANSYS$ software. During the FE analysis, a small region near the support position is declared as plastic zone and is not considered for stress constraint evaluation. A spring of constant stiffness ($\kappa = 0.4$ KN/m) is attached to the output port for providing some resistance to simulate a real application. Among the non-dominated solutions of NSGA-II, six solutions are selected as representative solutions with the help of a clustering procedure.

As a compliant mechanism is designed to have any support and loading regions with varying input displacement magnitude, GA operations on 12 bits assist in developing the non-dominated topologies. During the whole study, the structures are subjected to fixed output region, direction of input displacement magnitude and a prescribed path. Here, an input displacement is applied in equal steps at the loading region in x direction as shown in Figure 1. It also shows the coordinates of output point $(50, 32)$, and the support and loading regions at the bottom and right-hand side of a design domain of structure respectively.



(a) Undeformed topology (b) Deformed topology

Fig. 7. A minimum weight design.

A. Minimum Weight Design

A single objective study of minimizing the weight of structure subjected to constraints limiting the allowed deviation at each precision point of a used-defined path and limiting the maximum stress developed within the flexural strength of material is solved using the given optimization procedure (described in Section III). It evolves a *minimum weight design* of 0.5247 gms which is supported at 20^{th} element and loaded at 32^{th} element with 7 mm of input displacement. The deformed and undeformed design of minimum weight study is presented in Figure 7. It shows the shapes and connectivity between the members of support, loading and output regions in which the members of loading and output regions get deformed by which the minimum weight design accomplishes the task of tracing a prescribed path, as shown in Figure 7(b).

B. Non-Dominated Topologies

In this section, non-dominated topologies of compliant mechanism generating curvilinear path are evolved by solving a two-objective problem (refer Equation 1) using a local search based evolutionary (NSGA-II) optimization procedure. A two-objective space is drawn in Figure 8 to display the locations of representative NSGA-II solutions and local search solutions. It reveals that out of six representative solutions of NSGA-II, five of them become a part of non-dominated front as listed

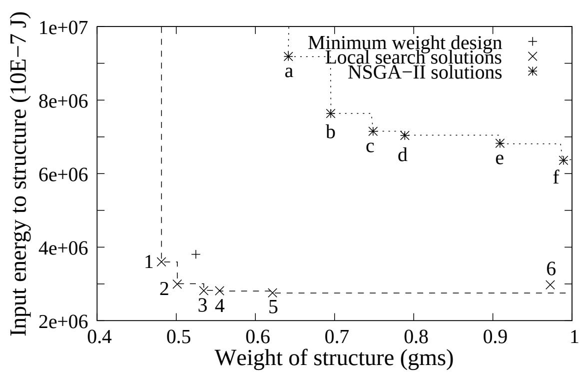


Fig. 8. NSGA-II solutions before and after local search are compared with the single-objective 'minimum-weight' design.

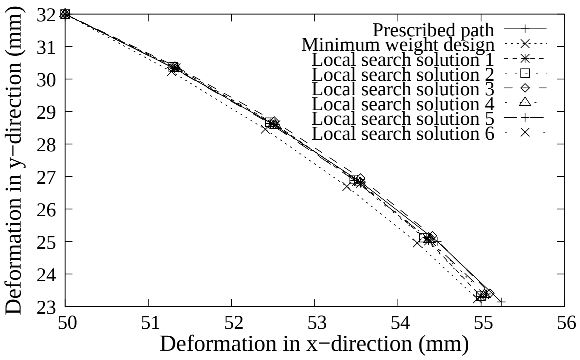


Fig. 9. Prescribed path and path traced by the minimum weight design and local search solutions.

from solution 1 to 5 after the local search. These solutions show a trade-off between the objectives of weight and supplied input energy to the structures. The given figure also shows a position of minimum weight design solution corresponding to its input energy supplied value in two-objective space. It indicates that solutions 1 and 2 are evolved as the lighter weight solutions and also required less supplied energy in comparison with the minimum weight design solution. Hence, these solutions dominate the minimum weight solution of single-objective study. It reveals a fact that the secondary objective of minimizing supplied input energy not only helps in evolving diverse topologies in one run of optimization but also generates non-dominated solutions with the primary objective of minimizing the weight of structures. On the other hand, a single objective study only deals with the optimization of one objective and may result in a premature sub-optimal solution.

A pictorial view of paths generated by the solutions obtained after the local search of single and two-objective studies is shown in Figure 9 along with a prescribed path. For quantitative study of these paths, a Table I is drawn in which the values of maximum allowed d_1 at each precision point and distance d_2 between the precision point and corresponding point on the actual path are given. It can be seen here that the value of d_2 increases as the designs follow the precision points from 1 to 5 for all evolved solutions. This shows that the precision points defining the extreme parts of prescribed path become critical. Both Figure 9 and Table I show that

TABLE I

DEVIATION AT PRECISION POINTS.

Precision points (PP)	1	2	3	4	5
Maximum allowed $\mathbf{d}_1$	0.3196	0.3142	0.3074	0.3084	0.3092
Single-objective study 'Minimum weight solution'					
$\mathbf{d}_2$	0.1032	0.1739	0.2127	0.2470	0.3091
Two-objective study					
Solution 1: $\mathbf{d}_2$	0.0290	0.0301	0.0324	0.1047	0.3058
Solution 2: $\mathbf{d}_2$	0.0531	0.0829	0.1220	0.1979	0.3091
Solution 3: $\mathbf{d}_2$	0.0761	0.1084	0.1262	0.1812	0.3091
Solution 4: $\mathbf{d}_2$	0.0226	0.0407	0.0256	0.1064	0.3091
Solution 5: $\mathbf{d}_2$	0.0278	0.0516	0.0939	0.1766	0.3091
Solution 6: $\mathbf{d}_2$	0.0161	0.0217	0.0588	0.1502	0.3083

TABLE II

TIME TAKEN BY GIVEN OPTIMIZATION PROCEDURE.

Problem	NSGA-II Time (hrs)	Local search Time (hrs)
Single-objective	5.59	12.19
Two-objective	5.47	Solution 1: 8.65 Solution 2: 9.15 Solution 3: 9.61 Solution 4: 10.93 Solution 5: 11.52 Solution 6: 25.07

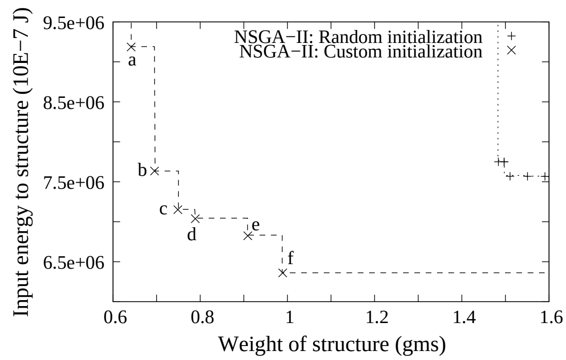


Fig. 10. NSGA-II solutions of both initial population initializations.

the formulation [26] used in the present study successfully accomplishes the task of generating a user-defined path within the allowed deviation of $\eta = 15\%$ at the precision points. The output point of these structures is deformed to 10.486% in x direction and 17.72% in y direction with respect to the size of design domain.

The times taken by the given optimization procedure for solving the single and bi-objective optimization problems are shown in Table II. In the parallel computing, the maximum time is consumed in the function evaluations of an optimization procedure whereas, the communication among the processors takes a smaller time. Therefore, the parallel implementation of NSGA-II helps in reducing the computational time almost in proportion to the number of processors available, that is, 24 in the present study. The local search is performed individually in different processors which take a considerable amount of time of a given procedure to improve the representative NSGA-II solutions.

In the present study, a comparison is also made between the solutions of two-objective studies which are obtained by incorporating a custom improved initial population strategy and by an usual way of creating an initial population of continuum structures in which the material in each element of a structure is assigned at random (authors refer this as a 'Random Initialization'). Therefore, another two-objective study incorporating an usual way of creating an initial population with a local search based NSGA-II procedure is performed for comparison. Figure 10 shows the representative NSGA-II solutions of both two-objective studies and it clearly reveals that the NSGA-II solutions of an improved initial population strategy dominate and explore the larger area of two-objective

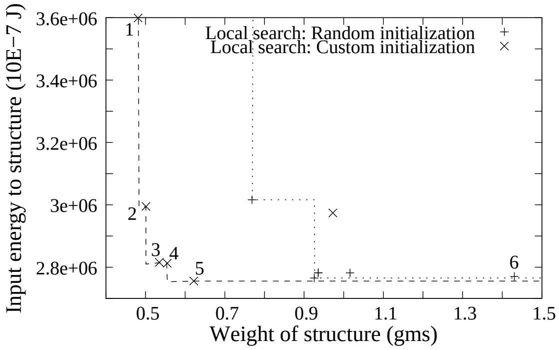
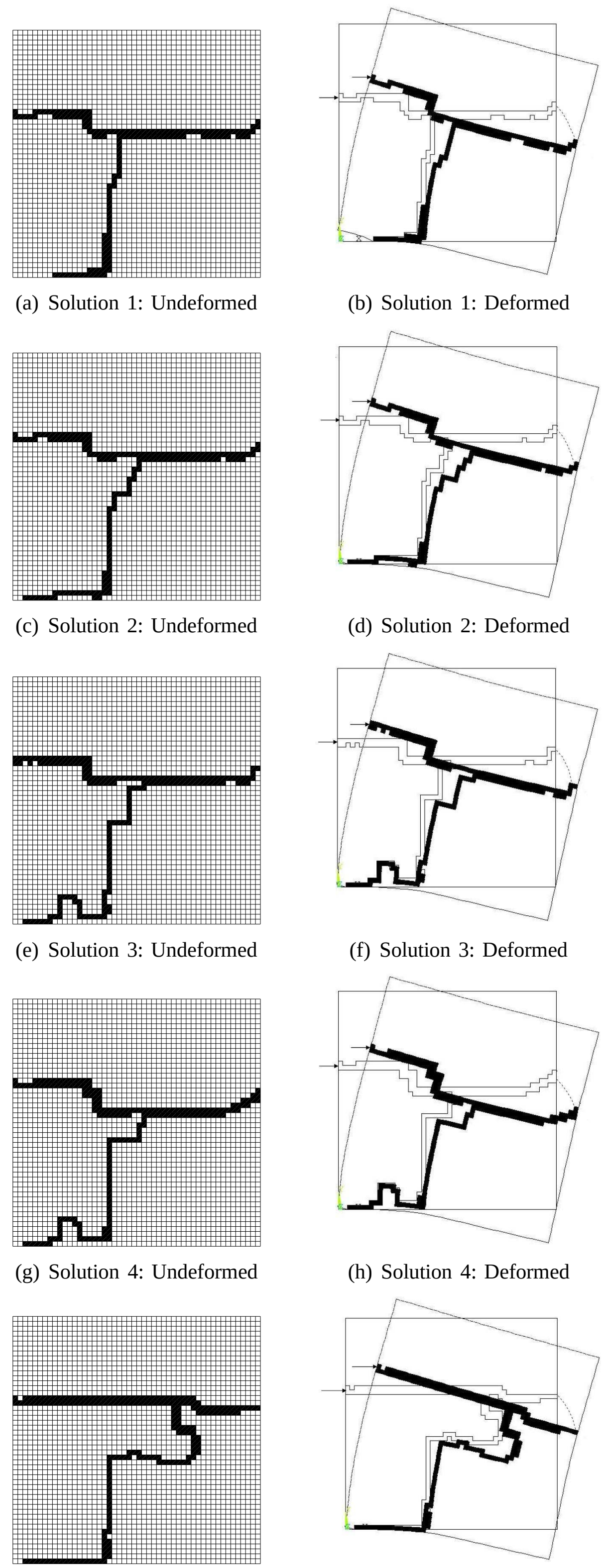


Fig. 11. Local search solutions of both initial population initializations.

space as well, in comparison with the NSGA-II solutions of an usual way of generating an initial population.

The performance of a local search procedure is dependent on the position of representative NSGA-II solutions in the objective space. As a good platform is provided by an improved initial population strategy, the non-dominated local search solutions outperform that of the usual study of generating an initial population as shown in Figure 11. Therefore, solutions 1 to 5 become the part 'Pareto-optimal' front and their respective deformed and undeformed topologies are presented in Figure 12 in which solution 1 is supported at 10^{th} element and remaining non-dominated solutions are supported at 4^{th} element. Solution 6 is a dominated solution and hence, it is not a part of 'Pareto-optimal' front. All the non-dominated topologies show the different shapes and connectivity of the members of three regions of interest. When the deformed topologies of extreme solutions are observed then Figure 12(b) reveals that the members of support and loading regions get deformed such that the output point of the design traces a user-defined path whereas, Figure 12(j) indicates that a 'winding' shape of support region's member near to the junction of the members of three regions helps in deforming the topology to trace a prescribed path by the output point. Therefore, these solutions will allow the designers to understand the topological changes which will be helpful in choosing a particular solution among the trade-off solutions for practice.

The progress of non-dominated feasible solutions with respect to the support region for both two-objective studies is shown in Figure 13. The figure helps us in understand the evolution of compliant mechanisms during the NSGA-II run. Using an improved strategy of generating the initial population, structures with diverse support regions are obtained during NSGA-II run and after the completion of its run, representative solutions supported at 4^{th}, 10^{th} and 32^{nd} elements are found to be present. Similar diversities are also observed when the progress of non-dominated feasible solutions are drawn with respect to the loading region and input displacement magnitude. But finally, all structures are loaded at 32^{nd} element with an input displacement of 7 mm. It is interested to note that NSGA-II procedure finds identical loading location and an identical input displacement for all obtained trade-off solutions. The only way the solutions differ



(a) Solution 1: Undeformed (b) Solution 1: Deformed

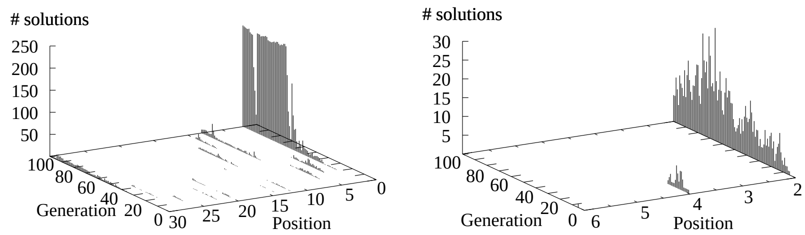
(c) Solution 2: Undeformed (d) Solution 2: Deformed

(e) Solution 3: Undeformed (f) Solution 3: Deformed

(g) Solution 4: Undeformed (h) Solution 4: Deformed

(i) Solution 5: Undeformed (j) Solution 5: Deformed

Fig. 12. Non-dominated topologies of compliant mechanisms generating curvilinear path. Solution 6 is dominated, hence not shown.



(1) Growth of support region locations with the improved strategy.

(2) Growth of support region locations with the usual way.

Fig. 13. Progress of non-dominated feasible solutions during NSGA-II run. Some diversity is maintained in the improved strategy

from each other is by altering their topologies, as shown in Figure 12. The minimum weight structure seems to be a quite rigid and requires a larger amount of supplied energy (refer Figure 12(b)). To reduce the supplied energy, NSGA-II finds that the structures should have a 'winding' shape so that they are more elastic (refer Figure 12(j)). Although this increases the weight of the structure but the saving in supplied energy is also significant and hence, results as a trade-off solution. Similarly, the topologies of Figures 12(d), 12(f) and 12(h) also become a part of compromised solutions. Using an usual way of generating an initial population, all obtained structures are supported at 2^{nd} and loaded at 50^{th} elements, and requires 9 mm of input displacement for accomplishing the task of generating a prescribed path. As Figure 13 shows that all structures are supported with identical location in the early iterations of NSGA-II for the usual study, therefore it results in a set of similar solutions and prematurely converges close to a sub-optimal feasible solutions. The key change needed to get better and diverse solutions is to alter the locations of the support and loading elements as well as loading displacement magnitude.

V. CONCLUSIONS

A successful attempt of incorporating an improved initial population strategy was made for evolving the topologies of path generating complaint mechanism. The two-objective problem of primary and secondary objectives not only evolved the diverse topologies but also provided a flexibility to designers for understanding the topological changes in the designs and choosing a particular solution from a set of non-dominated solutions in one run of a local search based evolutionary (NSGA-II) optimization procedure. The shape and nature of connectivity of the members of three regions helped in generating diverse topologies and resulted in the trade-off between the two-objectives. The comparison between the two strategies of generating an initial population also showed that the obtained non-dominated solutions of improved initial population strategy outperformed that of the usual study. The formulation used for designing the path generating compliant mechanism also helped in evolving the topologies which accomplish the task of generating a user-defined path. Parallel implementation of NSGA-II in the present study also helped in reducing the computational time, whereas the local search significantly improved the NSGA-II solutions in a considerable amount of time of the given optimization procedure.

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