Lecture 2: Engineering Curves

Engineering Curves

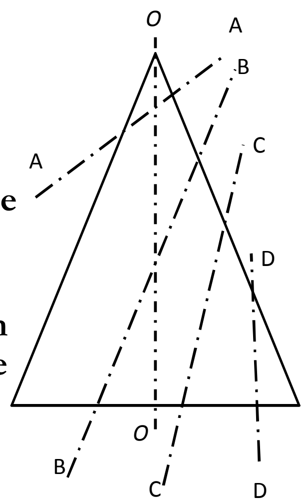
- used in designing certain objects

Conic Sections

- Sections of a right circular cone obtained by cutting the cone in different ways
- Depending on the position of the cutting plane relative to the axis of cone, three conic sections can be obtained
  - ellipse,
  - parabola and
  - hyperbola

Conic Sections

- An *ellipse* is obtained when a section plane *A–A*, inclined to the axis cuts all the generators of the cone.
- A *parabola* is obtained when a section plane *B–B*, parallel to one of the generators cuts the cone. Obviously, the section plane will cut the base of the cone.
- A *hyperbola* is obtained when a section plane *C–C*, inclined to the axis cuts the cone on one side of the axis.
- A *rectangular hyperbola* is obtained when a section plane *D–D*, parallel to the axis cuts the cone.



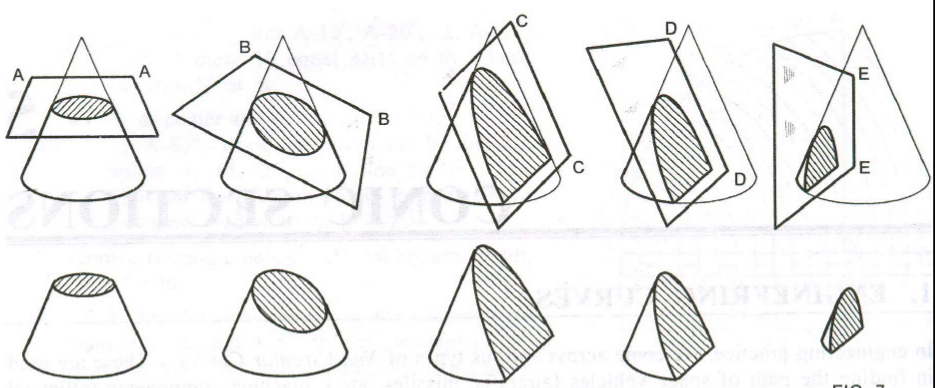


FIG. 3 CIRCLE

FIG. 4 ELLIPSE

FIG. 5 PARABOLA

FIG. 6 HYPERBOLA

FIG. 7 RECTANGULAR HYPERBOLA

Conic is defined as locus of a point moving in a plane such that the ratio of its distance from a fixed point (F) to the fixed straight line is always a constant. This ratio is called as eccentricity.

Ellipse: eccentricity is always <1

Parabola: eccentricity is always=1

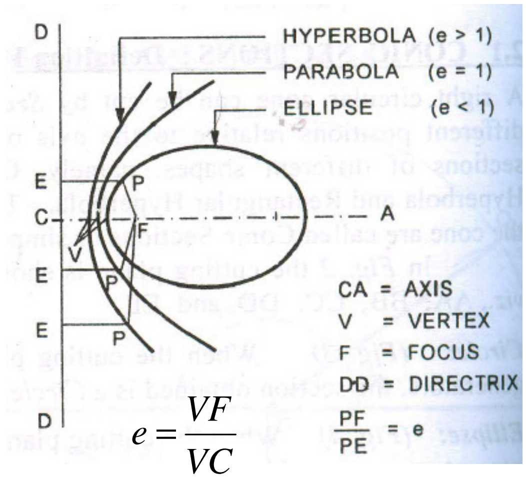
Hyperbola: eccentricity is >1

The fixed point is called the **Focus**

The fixed line is called the **Directrix**

Axis is the line passing though the focus and perpendicular to the directrix

Vertex is a point at which the conic cuts its axis



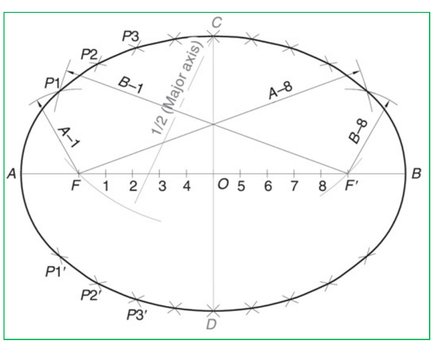
$$e = \frac{VF}{VC}$$

Ellipse

- Eccentricity is less than 1.
- Closed curve.
- The fixed points represent the foci.

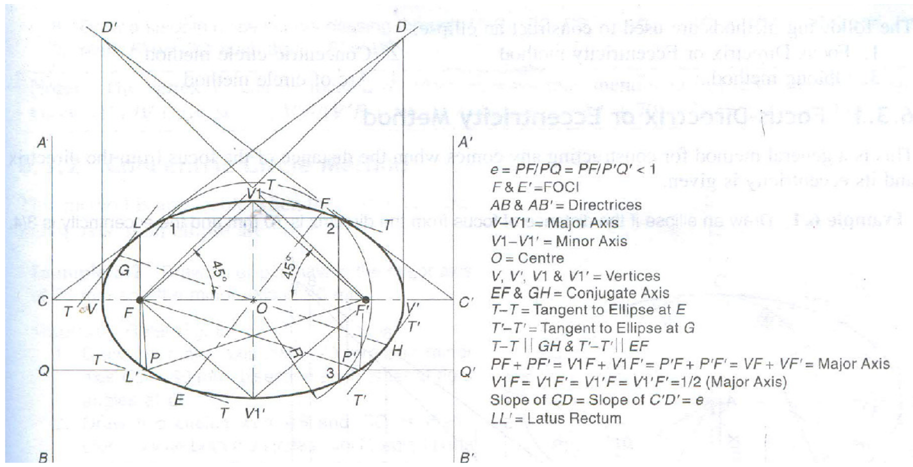
Relationship between Major axis, Minor axis and Foci

- If minor axis is given instead of the distance between the foci, then locate the foci F and F' by cutting the arcs on major axis with C as a center and radius= ½ major axis= OA



- If major axis and minor axis are given, the two fixed points F_1 and F_2 can be located with the following fact
- The sum of the distances of a point on the ellipse from the two foci is equal to the major axis
- The distance of any end of the minor axis from any focus is equal to the half of the major axis

An ellipse has two foci (F and F'), two directrices (AB and $A'B'$), two axes (V–V' and $V1$–$V1'$) and four vertices (V, V', $V1$ and $V1'$). The two axes are called the *major axis* and *minor axis*.



Methods for Generating Ellipse

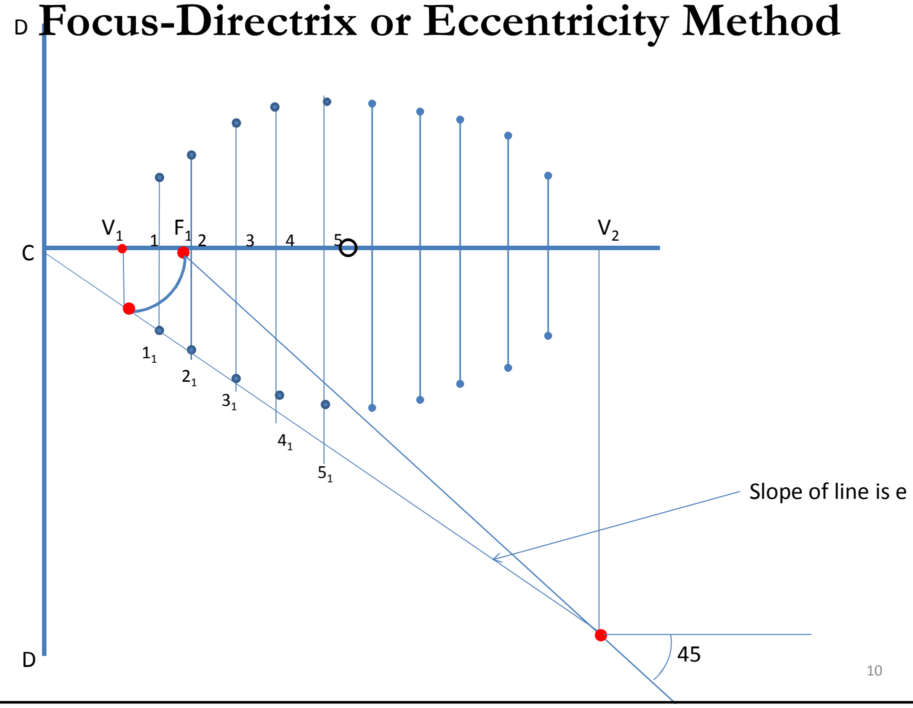
1. **Focus-Directrix Or Eccentricity Method**
   - General method of constructing any conics when the distance of the focus from the directrix and its eccentricity are given.
2. **Concentric Method**
   - This method is applicable when the major axis and minor axis of an ellipse are given.
3. **Oblong Method**
   - This method is applicable when the major axis and minor axis or the conjugate axes with the angle between them is given.

Types of Problems

- **Focus-Directrix Or Eccentricity Method**
  – Draw an ellipse if the distance of the focus from the directrix 50 mm and the eccentricity is 2/3
  – Draw a parabola if the distance of the focus from the directrix is 55 mm
  – Draw a hyperbola of e = 4/3 if the distance of the focus from the directrix = 60 mm
- **Concentric Method**
  – Draw an ellipse having the major axis of 60 mm and the minor axis of 40 mm
- **Oblong Method**
  – Draw an ellipse having conjugate axes of 60 mm and 40 mm long and inclined at 75^0 to each other

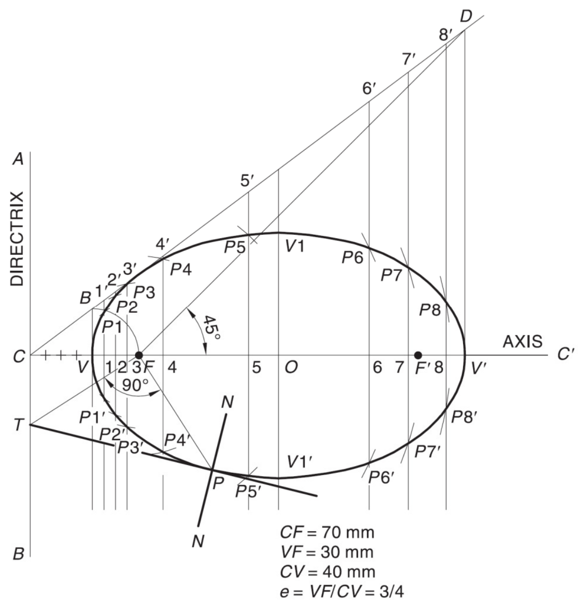
9

Focus-Directrix or Eccentricity Method



10

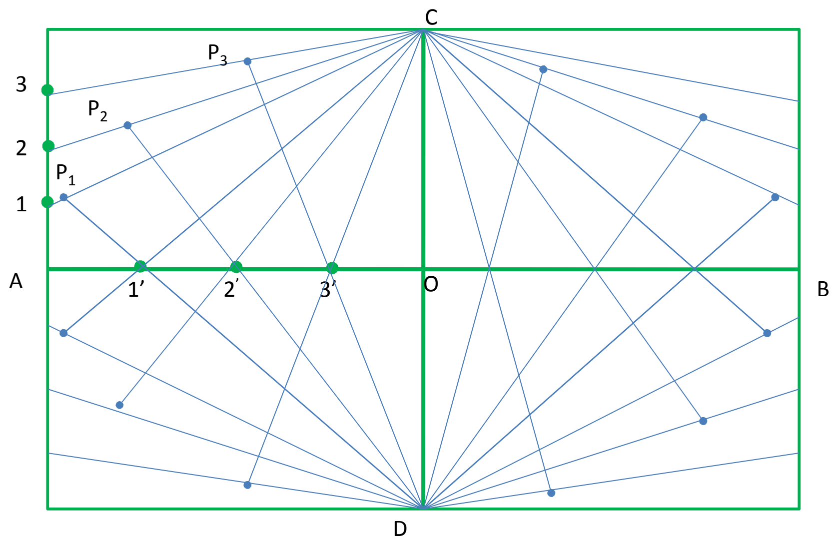
Focus-Directrix or Eccentricity Method



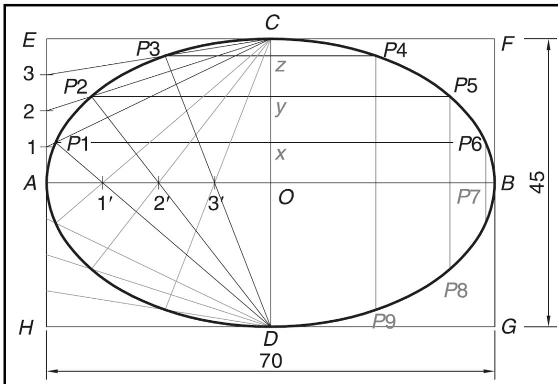
Steps for Focus-Directrix or Eccentricity Method

Q.1: Draw an ellipse if the distance of focus from the directrix is 70 mm and the eccentricity is 3/4.

1. **Draw the directrix and axis as shown.**
2. **Mark F on axis such that CF $_1$= 70 mm.**
3. **Divide CF into 3 + 4 = 7 equal parts and mark V at the fourth division from C. Now, e = FV/ CV = 3/4.**
4. **At V, erect a perpendicular VB = VF. Join CB.**
5. **Through F, draw a line at 45° to meet CB produced at D. Through D, drop a perpendicular DV' on CC'. Mark O at the midpoint of V– V'.**
6. **Mark a few points, 1, 2, 3, … on V– V' and erect perpendiculars though them meeting CD at 1', 2', 3'…. Also erect a perpendicular through O.**
7. **With F as a centre and radius = 1–1', cut two arcs on the perpendicular through 1 to locate P1 and P1¢. Similarly, with F as a centre and radii = 2–2', 3–3', etc., cut arcs on the corresponding perpendiculars to locate P/2 and P/2', P/3 and P/3', etc. Also, cut similar arcs on the perpendicular through O to locate V1 and V1'.**



Oblong Method



Steps for Oblong Method

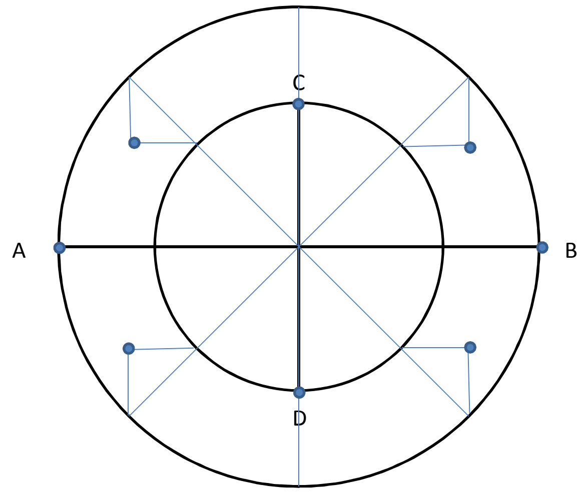
Draw an ellipse with a 70 mm long major axis and a 45 mm long minor axis.

or

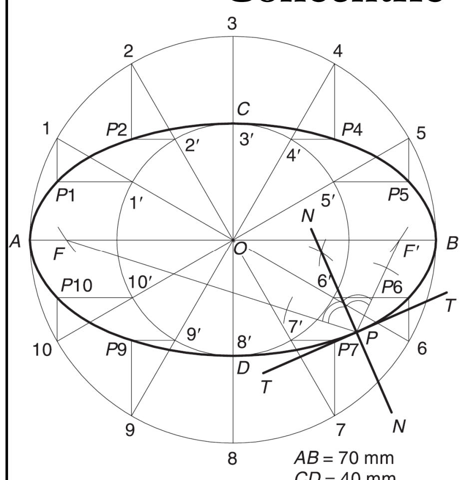
Draw an ellipse circumscribing a rectangle having sides 70 mm and 45 mm.

1. Draw the major axis AB = 70 mm and minor axis CD = 45 mm, bisecting each other at right angles at O.
2. Draw a rectangle EFGH such that EF = AB and FG = CD.
3. Divide AO and AE into same number of equal parts, say 4. Number the divisions as 1, 2, 3 and 1', 2', 3', starting from A.
4. Join C with 1, 2 and 3.
5. Join D with 1' and extend it to meet C–1 at P1. Similarly, join D with 2' and 3' and extend them to meet C–2 and C–3 respectively to locate P/2 and P/3.

Concentric Circle Method



Concentric Circle Method



Draw an ellipse having the major axis of 70 mm and the minor axis of 40 mm.

Draw the major axis AB = 70 mm and minor axis CD = 40 mm, bisecting each other at right angles at O.

Draw two circles with AB and CD as diameters. Divide both the circles into 12 equal parts and number the divisions as A, 1, 2, 3, … 10, B and C, 1', 2', 3' … 10', D.

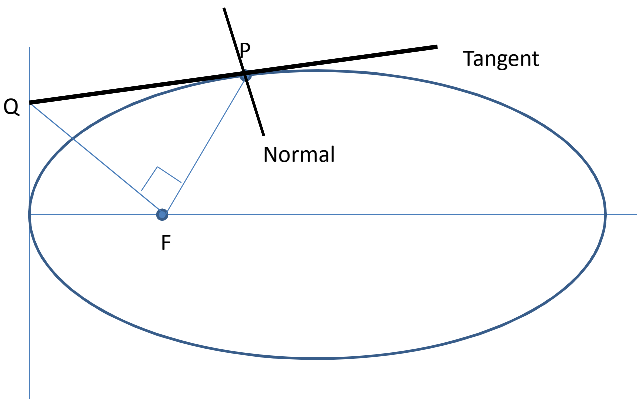
Through 1, draw a line parallel to CD. Through 1', draw a line parallel to AB. Mark P1 at their intersection.

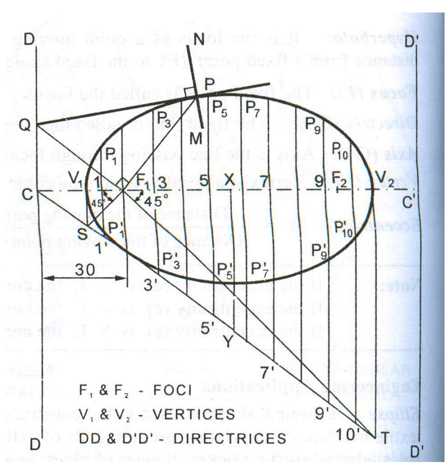
Obtain P/2, P4, P5, etc., in a similar way.

Draw a smooth closed curve through A– P1–P/2– C– P4– P5– B– P6– P7– D– P/9– P10– A.

Tangent and Normal at any point P

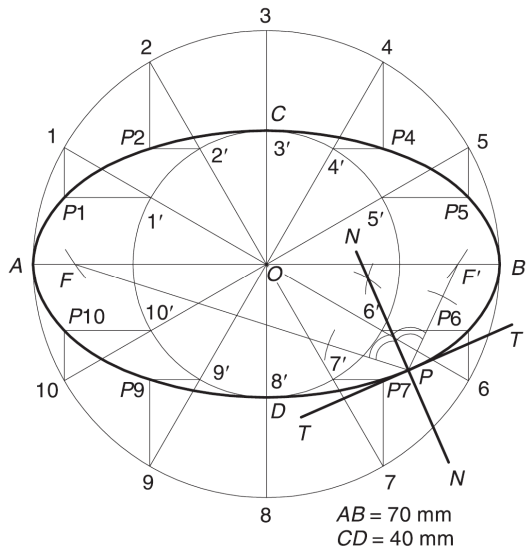
Draw ellipse using Focus-Directrix or Eccentricity Method





1. Mark the given point P and join PF_1 .
2. At F_1 draw a line perpendicular to PF_1 to cut DD at Q.
3. Join QP and extend it. QP is the tangent at P
4. Through P, draw a line NM perpendicular to QP. NM is the normal at P

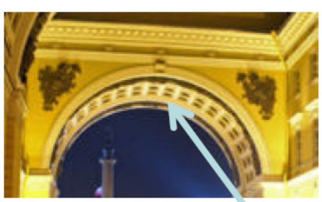
Tangent and Normal at any point P when Focus and Directrix are not known



1. First obtain the foci F and F′ by cutting the arcs on major axis with C as a centre and radius =OA
2. Obtain NN, the bisector of ∠FPF′. N-N is the required normal
3. Draw TT perpendicular to N-N at P. T-T is the required tangent

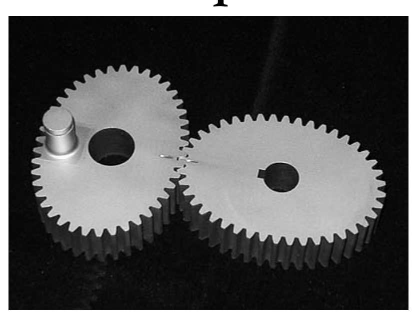
19

Few Applications of Ellipse



Arch

Elliptical shape



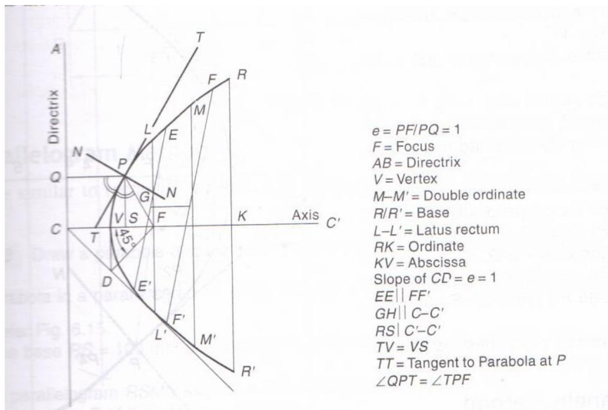
Elliptical gear

Bullet nose

20

Parabola

- **A parabola is a conic whose eccentricity is equal to 1. It is an open-end curve with a focus, a directrix and an axis.**
- **Any chord perpendicular to the axis is called a** *double ordinate.*
- **The double ordinate passing through the focus . i.e LL' represents the** *latus rectum*
- **The shortest distance of the vertex from any ordinate, is known as the** *abscissa.*



21

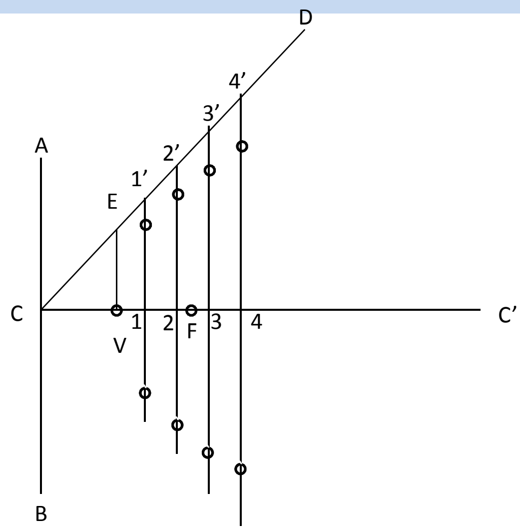
Methods for Generating Parabola

1. **Focus-Directrix Or Eccentricity Method**
   - General method of constructing any conics when the distance of the focus from the directrix
   - For example, draw a parabola if the distance of the focus from the directrix is 55 mm.
2. **Rectangle Method and Parallelogram Method**
   - This method is applicable when the axis (or abscissa) and the base (or double ordinate) of a parabola are given or the conjugte axes with the angle between them is given
   - For example, draw a parabola having an abscissa of 30 mm and the double ordinate are 70 mm, or
   - Draw an parabola having conjugate axes of 60 mm and 40 mm long and inclined at 75^0 to each other.
3. **Tangent Method**
   - This method is applicable when the base and the inclination of tangents at open ends of the parabola with the base are given
   - For example, draw a parabola if the base is 70 mm and the tangents at the base ends make 60° to the base..

22

Focus-Directrix Or Eccentricity Method

- **Distance of the focus from the directrix is known.**



CV = VF

EV = VF

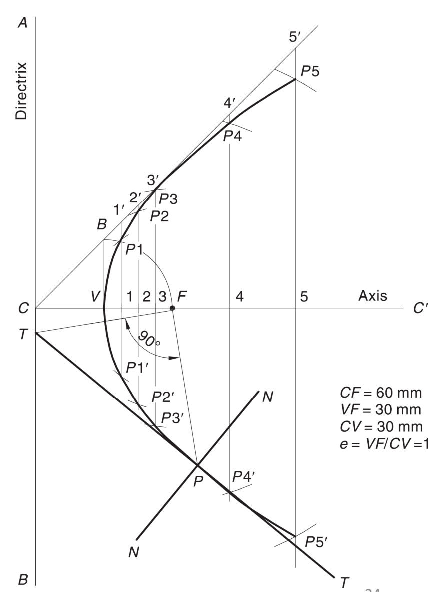
Slope of CD is e = 1

Center = F
Radius = 1-1'

Steps for Focus-Directrix or Eccentricity Method

Draw a parabola if the distance of the focus from the directrix is 60 mm.

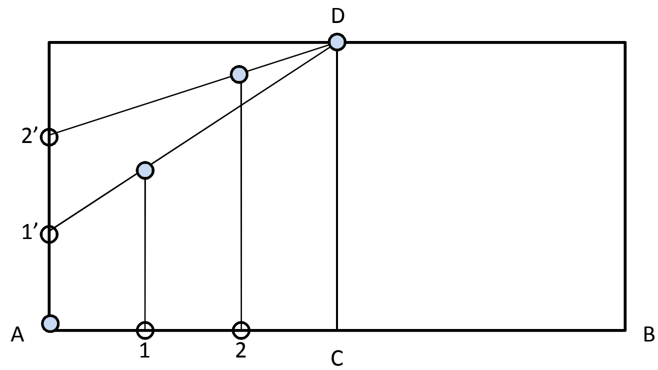
1. **Draw directrix AB and axis CC' as shown.**
2. **Mark F on CC' such that CF = 60 mm.**
3. **Mark V at the midpoint of CF. Therefore, e = VF/ VC = 1.**
4. **At V, erect a perpendicular VB = VF. Join CB.**
5. **Mark a few points, say, 1, 2, 3, … on VC' and erect perpendiculars through them meeting CB produced at 1', 2', 3', …**
6. **With F as a centre and radius = 1–1', cut two arcs on the perpendicular through 1 to locate P1 and P1'. Similarly, with F as a centre and radii = 2–2', 3–3', etc., cut arcs on the corresponding perpendiculars to locate P2 and P2', P3 and P3', etc.**
7. **Draw a smooth curve passing through V, P1, P2, P3 … P3**



Rectangle Method

- Abscissa and ordinate are known.

CD-Abscissa

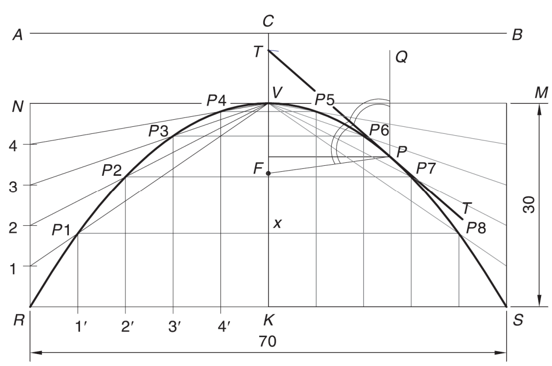


AB-Ordinate

25

Steps for Rectangle Method

Q.1: Draw a parabola having an abscissa of 30 mm and the double ordinate of 70 mm.

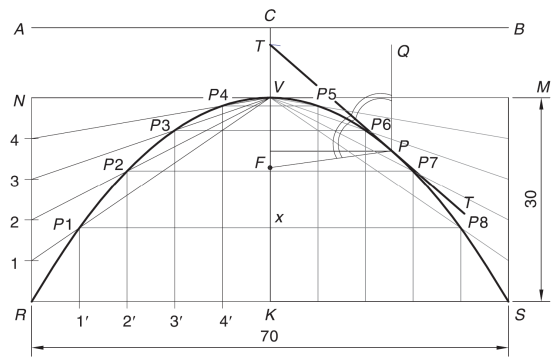


1. **Draw the double ordinate RS = 70 mm. At midpoint K erect a perpendicular KV = 30 mm to represent the abscissa.**
2. **Construct a rectangle RSMN such that SM = KV.**
3. **Divide RN and RK into the same number of equal parts, say 5. Number the divisions as 1, 2, 3, 4 and 1', 2', 3', 4', starting from R.**
4. **Join V–1, V–2, V–3 and V–4.**
5. **Through 1', 2', 3' and 4', draw lines parallel to KV to meet V–1 at P1, V–2 at P2, V–3 at P3 and V–4 at P4, respectively.**
6. **Obtain P5, P6, P7 and P8 in the other half of the rectangle in a similar way. Alternatively, these points can be obtained by drawing lines parallel to RS through P1, P2, P3 and P4. For example, draw P1– P8 such that P1– x = x– P8.**
7. **Join P1, P2, P3 … P8 to obtain the parabola.**

26

Steps for Tangent and Normal at a point p on parabola

1. Join PF. Draw PQ parallel to the axis.
2. Draw the bisector T– T of – FPQ to represent the required tangent.
3. Draw normal N– N perpendicular T– T at P.

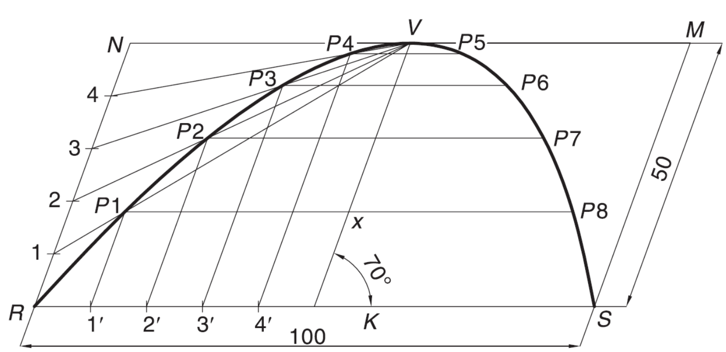


27

Steps for Parallelogram Method

Q.1: Draw a parabola of base 100 mm and axis 50 mm if the axis makes 70° to the base.

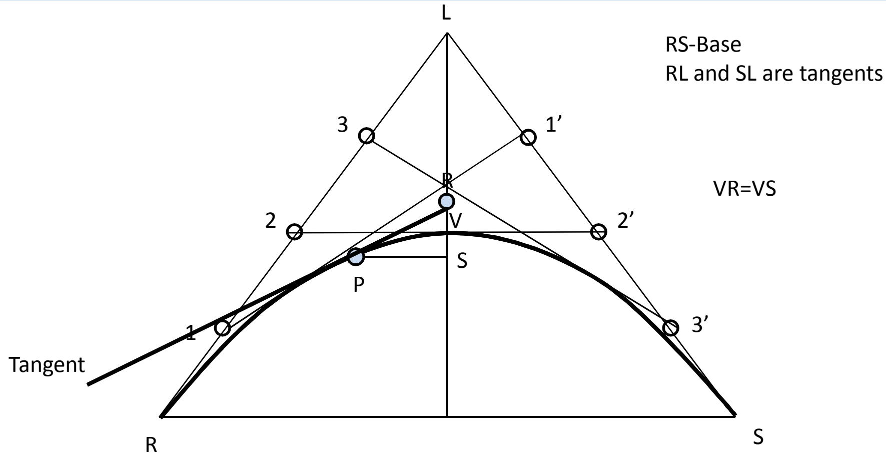
1. Draw the base RS = 100 mm and through its midpoint K, draw the axis KV = 50 mm, inclined at 70° to RS.
2. Draw a parallelogram RSMN such that SM is parallel and equal to KV.
3. Follow steps as in rectangle method



28

Tangent Method

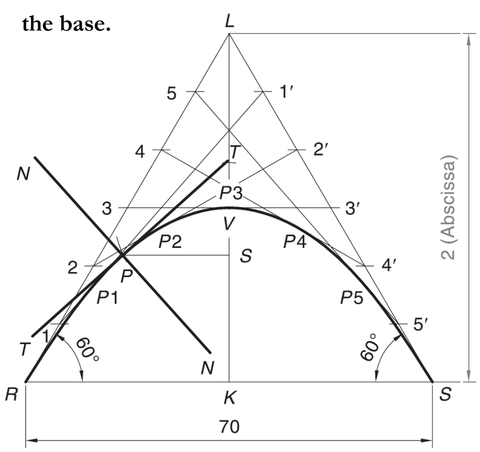
- Base and inclination of tangents are known.



29

Steps for Tangent Method

Q. Draw a parabola if the base is 70 mm and the tangents at the base ends make 60° to the base.



1. **Draw the base RS = 70 mm. Through R and S, draw the lines at 60° to the base, meeting at L.**
2. **Divide RL and SL into the same number of equal parts, say 6. Number the divisions as 1, 2, 3 … and 1', 2', 3', … as shown.**
3. **Join 1–1', 2–2', 3–3', ….**
4. **Draw a smooth curve, starting from R and ending at S and tangent to 1–1', 2–2', 3–3', etc., at P1, P2, P3, etc., respectively**

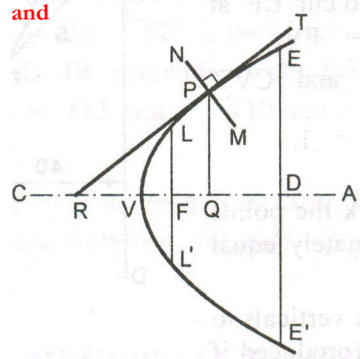
Method to draw tangent at a point on parabola

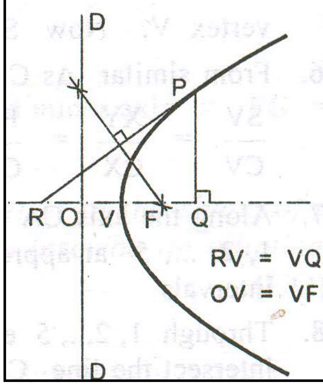
1. First locate the point P on the curve
2. Draw the ordinate PS
3. On LK, mark T such that TV =VS
4. Join TP and extend to obtain tangent TT
5. Draw normal N-N perpendicular to T-T at P

30

Tangent and Normal at any point P when Focus and Directrix are not known

1. Draw the ordinate PQ
2. Find the abscissa VQ
3. Mark R on CA such that RV=VQ
4. Draw the normal NM perpendicular to RP at P





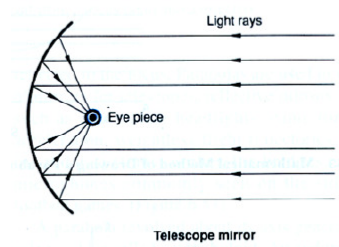
To find the focus and the directrix of a parabola given its axis

1. Mark any point P on the parabola
2. Draw a perpendicular PQ to the given axis
3. Mark a point R on the axis such that RV=VQ
4. Focus: Join RP. Draw a perpendicular bisector of RP cutting the axis at F, F is the focus
5. Directrix: Mark O on the axis such that OV= VF. Through O draw the directrix DD perpendicular to the axis

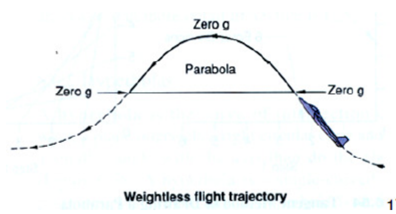
31

Few Applications of Parabola

Golden Gate Bridge in San Francisco



Radio telescopes use a parabolic dish to focus radio signals



32