The Strong CP Problem and its Resolutions

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The Strong CP Problem

- There is no indication of CP violation in strong interactions. Yet, the QCD Lagrangian admits a term

$$\mathcal{L}_{QCD} = \frac{\theta\, g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

which is P and T violating, and thus, owing to CPT invariance CP violating as well. Here, $\tilde{G}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}$ is the dual field strength for the gluon.

- The Lagrangian is a total divergence, since $G^a_{\mu\nu}\tilde{G}^{a\mu\nu} = \partial_\mu K^\mu = \partial_\mu[\epsilon^{\mu\nu\rho\sigma} A^a_\nu (F^a_{\rho\sigma} - \frac{2}{3}\epsilon^{abc} A^b_\rho A^c_\sigma)]$.

- However, in QCD, the surface term gives rise to non-zero contributions, owing to finite energy "instanton" configurations, causing P and T violation.

The phase of the quark mass matrix gives additional contribution to physical observables

The Strong CP Problem (cont.)

- Specifically, $\mathrm{Det}(M_u) \to \mathrm{Det}(M_u)$ under such a special bi–unitary transformation. If the phases of the quark masses are denoted as $\theta_{u,c,t}$ and $\theta_{d,s,b}$, the combination

$$\theta_{\mathrm{QFD}} = \theta_u + \theta_c + \theta_t + \theta_d + \theta_s + \theta_b = \mathrm{Arg}[\mathrm{Det}(M_q)]$$

  cannot be removed by anomaly–free rotations.

- A chiral rotation on the quark fields is necessary in order to remove this phase. This however will generate an anomaly term in the Lagrangian, of the same form.

- The physical parameter is then

$$\bar{\theta} = \theta + \mathrm{Arg}[\mathrm{Det} M_q] \,.$$

The Strong CP Problem (cont.)

- With $\bar{\theta}$ physical, there will be CP violation in strong interactions. However, there are stringent constraints on the value of $\bar{\theta}$ from experimental limits on the electric dipole moment (EDM) of the neutron: $\bar{\theta} < 10^{-10}$.
- This arises since in the presence of $\bar{\theta}$ neutron EDM can be shown to have a non-zero value given by

$$d_n \simeq \left[10^{-16} \times \bar{\theta}\right] \ \mathrm{e-cm}\,.$$

- From the experimental limit on neutron EDM, $d_n < 10^{-26}$ e-cm, one obtains the limit $\bar{\theta} < 10^{-10}$. Why is it that a fundamental dimensionless parameter of the Lagrangian, which should naturally be of order one, so small is the strong CP problem.
- If CP were a good symmetry of the entire Lagrangian, small $\bar{\theta}$ would have been quite natural. However, weak interactions do break *CP* invariance, which makes the strong CP problem acute.

Solutions to the Strong CP Problem

- There are various proposed solutions to the problem. At some point in time it was thought that the up quark mass may be zero.
- If true, that would solve the strong CP problem, since θ_u is then un-physical, and therefore $\bar{\theta}$ can be removed from the theory.
- But now we know, especially from lattice gauge theory results, that $m_u = 0$ is not an acceptable solution.

Peccei-Quinn Symmetry and Axion Solution

- The most widely studied solution of the strong CP problem is the Peccei–Quinn (PQ) mechanism, which yields a light pseudo–Goldstone boson, the axion.
- Here the parameter $\overline{\theta}$ is promoted to a dynamical filed. This field acquires a non–perturbative potential induced by the QCD anomaly.
- Minimization of the potential yields the desired solution $\overline{\theta} = 0$, solving the strong CP problem.

Peccei-Quinn Symmetry and Axion Solution

- In the presence of the $\bar{\theta}$ term in the Lagrangian, non–perturbative QCD effects will induce a vacuum energy given by

  $$E_{\rm vac} = \mu^4 \cos\bar{\theta}\,,$$

  where $\mu \sim \Lambda_{\rm QCD} \sim 100$ MeV.

- This observation is crucially used in the PQ mechanism. What if $\bar{\theta}$ is a dynamical field?

- Then this non–perturbative potential will have to be minimized to locate the ground state (unlike the case where $\bar{\theta}$ is a constant in the Lagrangian). Minimization of this potential will yield $\bar{\theta} = 0$, as desired.

Axion Solution

- The essence of the PQ mechanism can be explained with a simple toy model.
- Consider QCD with one quark flavor (q) and no weak interactions. Suppose there is a global $U(1)$ symmetry under which $q \to e^{-i\alpha\gamma_5/2} q$. Such a symmetry has a QCD anomaly, and can only be imposed at the classical level. A bare mass for q is then forbidden.
- Introduce now a complex color singlet scalar field ϕ which transforms under this $U(1)$ as $\phi \to e^{i\alpha}\phi$.
- The following Yukawa interaction is then allowed.

$$\mathcal{L}_{\rm Yuk} = Y \overline{q}_L \phi q_R + Y^* \overline{q}_R \phi^* q_L \,.$$

- The potential for ϕ also respects the $U(1)$ symmetry, and is given by

$$V(\phi) = -m_\phi^2 |\phi|^2 + \lambda |\phi|^4$$

Axion Solution (cont.)

- With a negative sign for m_ϕ^2, the ϕ field will acquire a non-zero VEV, spontaneously breaking the $U(1)$.
- In this broken symmetric phase, we can parametrize ϕ as

$$\phi = \left[f_a + \tilde{\phi}(x^\mu)\right] e^{ia(x)/f_a}\,.$$

  Here f_a is a real constant, while $\tilde{\phi}(x^\mu)$ and $a(x^\mu)$ are dynamical (real) fields.

- The quark q now acquires a mass, given by $M_q = Y f_a e^{ia(x)/f_a}$. Making the quark mass real by a field redefinition will induce a $\overline{\theta}$ given by

$$\overline{\theta}_{\rm eff} = \theta + {\rm Arg}[{\rm Det}\ Y] + \frac{1}{f_a} a(x^\mu)\,.$$

Axion Solution (cont.)

- The crucial point is that now $\bar{\theta}$ is a dynamical field, because of the presence of the a field, the axion.
- Without non–perturbative QCD effects, a will be massless, since it is the Goldstone boson associated with the spontaneous breaking of the global $U(1)$.
- The vacuum energy

$$E_{\rm vac} = -\mu^4 \cos \bar{\theta}_{\rm eff} \, .$$

Axion Couplings

- Minimizing this potential with respect the dynamical a field would yield $\bar{\theta}_{\text{eff}} = 0$.
- The field–dependent redefinition on q, $q(x^\mu) \to q(x^\mu) e^{-i(a(x^\mu)/f_a)(\gamma_5/2)}$ would remove the axion field from quark interactions except via derivatives, originating from the kinetic terms.
- The axion also will have couplings to the gluon field strength. These couplings are given by

$$\mathcal{L}_a = -\left(\frac{\partial_\mu a}{f_a}\right) \bar{q}\gamma_\mu\gamma_5 q + \frac{g^2}{32\pi^2}\left(\frac{a}{f_a}\right) G\tilde{G}\,.$$

Axion Couplings

- It is the second term that actually induces the potential for the axion. Because of this potential, axion will have a mass of order $m_a \sim \Lambda_{\rm QCD}^2 / f_a$.
- The essentials of realistic axion model are already present in this toy model. We need to turn on weak interactions, and we need to add three families of quarks.
- The straightforward implementation would involve the SM extended to have two Higgs doublets, one coupling to the up–type quarks, and the other coupling to the down–type quarks.

DFSZ Axion Model

- Acceptable axion models of the "invisible" type involving high scale PQ symmetry breaking are fully consistent.
- In the DFSZ model, in addition to the two Higgs doublets, a complex singlet Higgs scalar S is also introduced.
- The axion decay constant f_a is now the VEV of S, which can be much above the weak scale.
- The axion is primarily in S, with very weak couplings to the SM fermions. There are non–trivial constraints from astrophysics and cosmology on such a weakly interacting light particle.
- For example, axion can be produced inside supernovae. Once produced, they will escape freely, draining the supernova of its energy.
- Consistency with supernova observations requires that $f_a > 10^9$ GeV. Cosmological abundance of the axion requires that $f_a < 10^{12}$ GeV.

Axion is a well motivated dark matter candidate

Parity Symmetry to solve Strong CP Problem

- There is another class of solution to the strong CP problem. One can assume Parity to set $\theta = 0$.
- If the fermion mass matrices have real determinant, then $\bar{\theta}$ can be zero at the tree level.
- Loop induced $\bar{\theta}$ needs to be small, but this is not difficult to realize.
- Let me illustrate this idea with the left–right symmetric model which has Parity invariance. The Yukawa couplings are hermitian in this setup.
- To make the mass matrices also hermitian, we must ensure that the VEVs of scalars are real. This is easily done in the SUSY version, which is what I will describe.
- In SUSY models, one should also take into account the contributions from the gluino to $\bar{\theta}$.

Parity Symmetry to solve Strong CP Problem

- The model is the SUSY version of left–right symmetric model based on the gauge symmetry $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.
- Two bi-doublet scalars $\Phi_i(1,2,2,0)$ $(i = 1,2)$ are used to generate quark and lepton masses as well as CKM mixings.
- The relevant superpotential is given as

$$W = Y_u Q Q^c \Phi_u + Y_d Q Q^c \Phi_d \, .$$

Babu,Dutta, Mohapatra (2002)

Parity Symmetry to solve Strong CP Problem

- The Yukawa coupling matrices Y_u and Y_d will be hermitian, owing to Parity invariance.
- Parity also implies that the QCD Lagrangian parameter $\theta = 0$ and that the gluino mass is real. The soft SUSY breaking A–terms, will also be hermitian. We shall consider the case where these A terms are proportional to the respective Yukawa matrices.
- Furthermore, we assume universal masses for the squarks, as in minimal supergravity, or in gauge mediated SUSY breaking models.
- The quark mass matrices $M_{u,d}$ are hermitian at tree level since the VEVs of the bi-doublet scalars turn out to be real. Therefore $\bar{\theta} = 0$ at tree level.