BSM Physics for Explaining Flavor Puzzles

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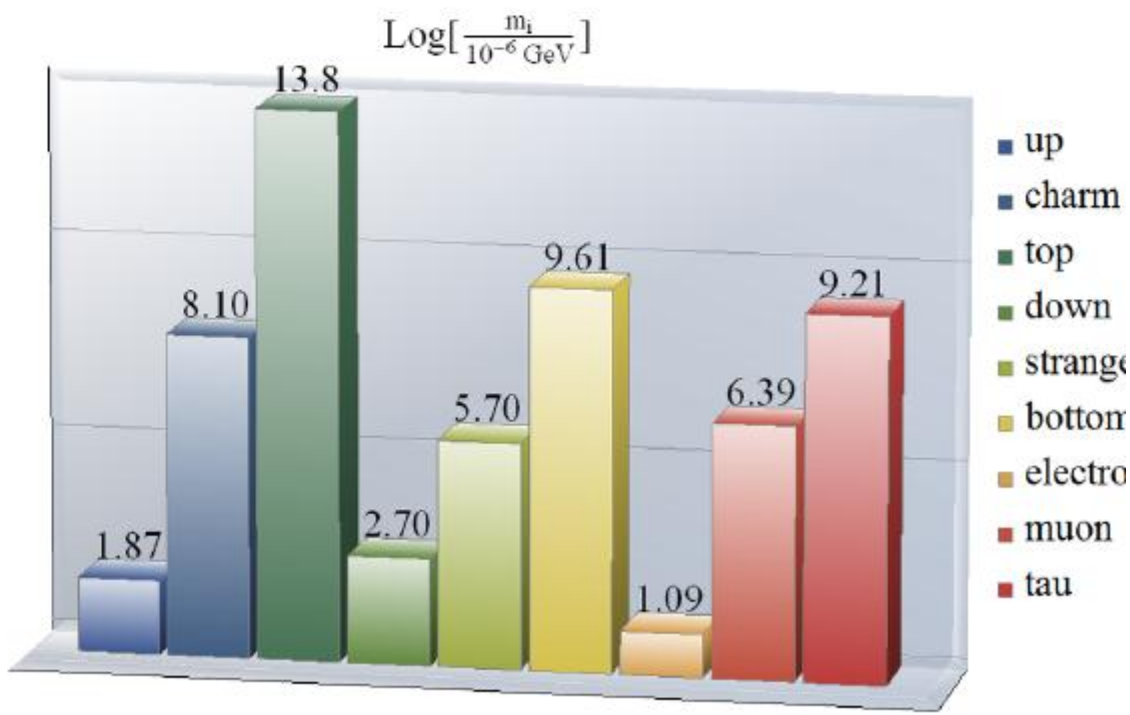
Fermion Mass Puzzle

Charged Fermion Mass Hierarchy

- **up-type quarks**
- $m_u \sim 6.5 \times 10^{-6}$
- $m_c \sim 3.3 \times 10^{-3}$
- $m_t \sim 1$
- **down-type quarks**
- $m_d \sim 1.5 \times 10^{-5}$
- $m_s \sim 3 \times 10^{-4}$
- $m_b \sim 1.5 \times 10^{-2}$
- **charged leptons**
- $m_e \sim 3 \times 10^{-6}$
- $m_\mu \sim 6 \times 10^{-4}$
- $m_\tau \sim 1 \times 10^{-2}$



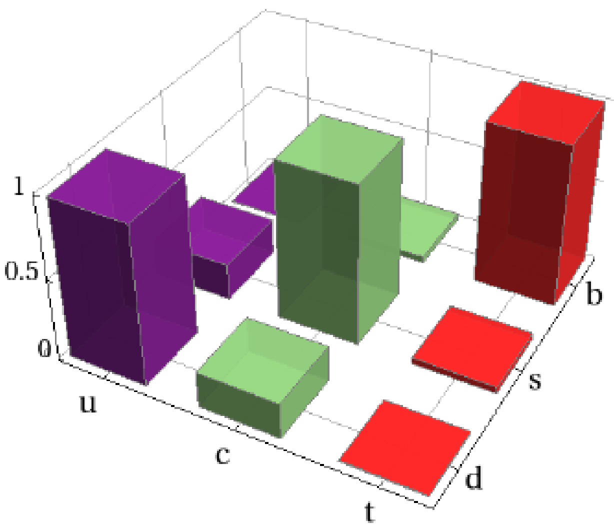
Neutrino masses not strongly hierarchical

3 masses within an order of magnitude consistent!

Quark and Lepton Mixing Parameters

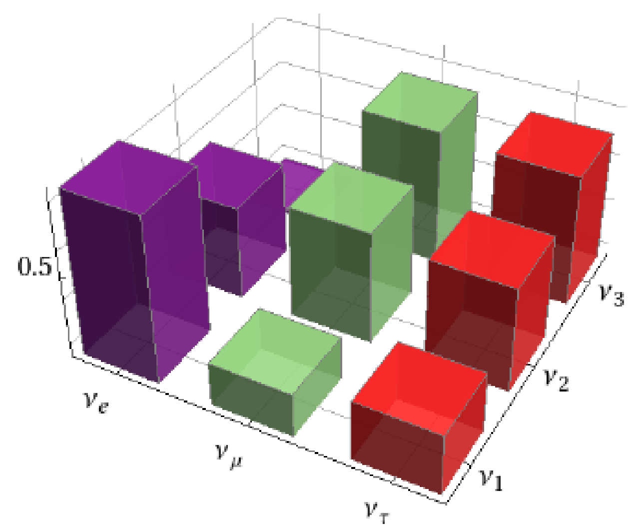
- **Quark Mixings**

$$V_{CKM} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix}$$



- **Leptonic Mixings**

$$U_{PMNS} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix}$$



Cabibbo-Kobayashi-Maskawa (CKM) Matrix

- The unitary matrix V which appears in the charged current interactions enters in a variety of processes. A lot of information has been gained on the matrix elements of V. The general matrix can be written as

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

- V has a single un-removable phase for three families of quarks and leptons. (The phases (α, β) which appeared in the case of Majorana neutrinos can be removed by right–handed quark field redefinition.) The single un-removable phase in V allows for the violation of CP symmetry in the quark sector. Unlike in the leptonic sector, the quark mixing angles turn out to be small.

Wolfenstein form of CKM Matrix

- This enables one to make a perturbative expansion of the mixing matrix a la Wolfenstein . The small parameter is taken to be $\lambda = |V_{us}|$ in terms of which one has

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + \frac{1}{2}A\lambda^4(1 - 2(\rho + i\eta)) & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^5) .$$

- Here the exact correspondence is given by

$$s_{12} \equiv \lambda, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta) .$$

Decays to measure CKM angles

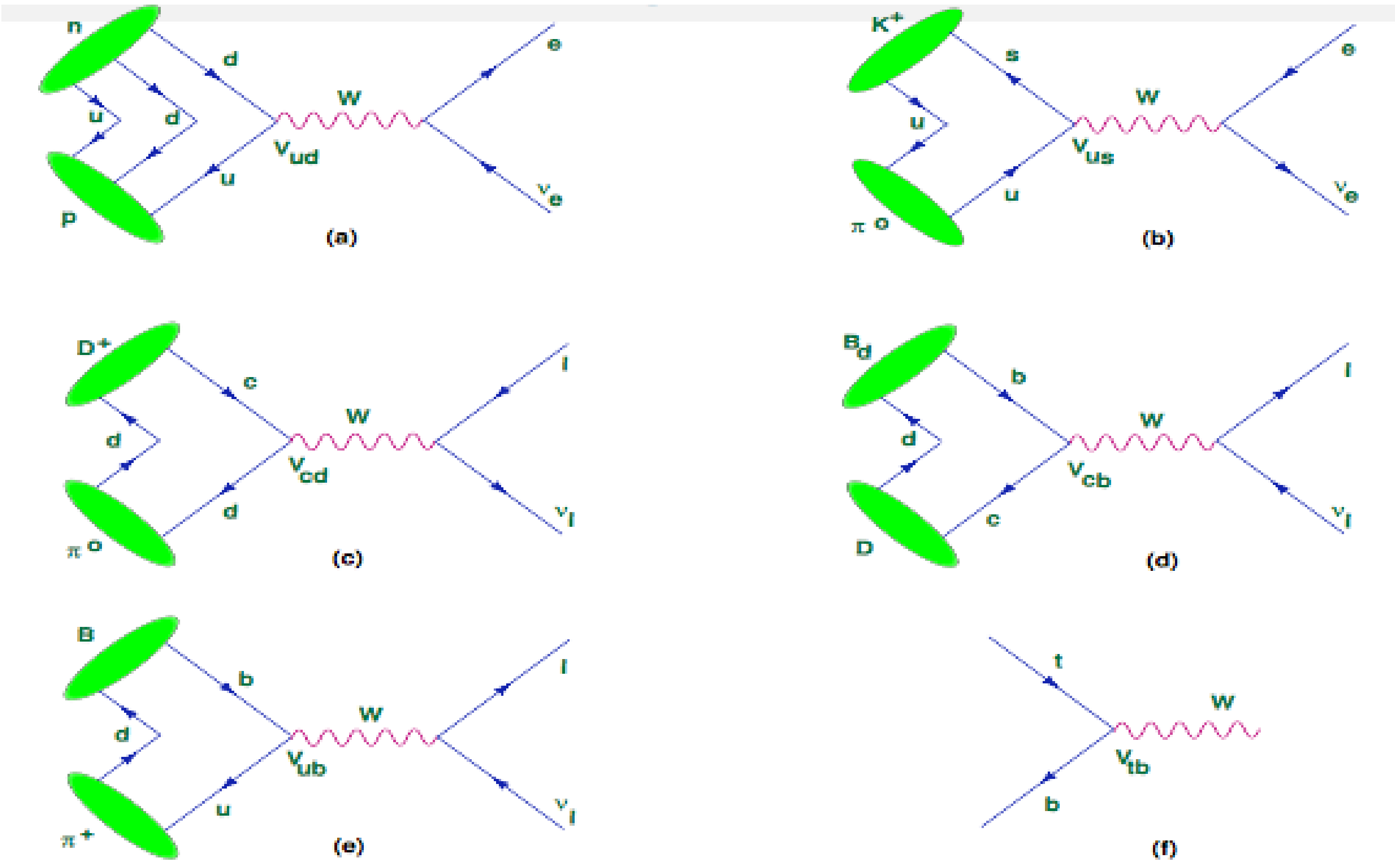


Figure: Processes determining $|V_{ij}|$.

Measuring CKM Angles

Matrix elements of V are determined usually via semileptonic decays of quarks. In Fig., we have displayed the dominant processes enabling determination of these elements.

- (a) is the diagram for nuclear beta decay, from which $|V_{ud}|$ has been extracted rather accurately :

$$|V_{ud}| = 0.97377 \pm 0.00027\,.$$

- (b) shows semileptonic K decay from which the Cabibbo angle $|V_{us}|$ can be extracted. The decays $K_L^0 \to \pi \ell \nu$ and $K^{\pm} \to \pi^0 \ell^{\pm} \nu$ ($\ell = e, \mu$) have been averaged to obtain for the product $|V_{us}| f_+(0) = 0.21668 \pm 0.00045$. Here $f_+(0)$ is the form factor associated with this semileptonic decay evaluated at $q^2 = 0$. Using $f_+(0) = 0.961 \pm 0.008$ (obtained from QCD calculations, which are in agreement with lattice QCD evaluations), one obtains

$$|V_{us}| = 0.2257 \pm 0.0021\,.$$

Measuring CKM angles

- $|V_{cd}|$ is extracted from $D \to K\ell\nu$ and $D \to \pi\ell\nu$ decays with assistance from lattice QCD for the computation of the relevant form factors.
- V_{cs} is determined from semileptonic D decays and from leptonic D_s decay ($D_s^+ \to \mu^+\nu$), combined with lattice calculation of the decay form factor f_{D_s}.
- Both $|V_{cd}|$ and $|V_{cs}|$ have rather large errors currently:

$$\begin{aligned} |V_{cd}| &= 0.230 \pm 0.011\,, \\ |V_{cs}| &= 0.957 \pm 0.010\,. \end{aligned}$$

- $|V_{cb}|$ is determined from both inclusive and exclusive decays of B hadrons into charm, yielding a value

$$|V_{cb}| = (41.6 \pm 0.6) \times 10^{-3}\,.$$

Measuring CKM angles

- $|V_{ub}|$ is determined from charmless B decays and gives

$$|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}\,.$$

- Elements $|V_{td}|$ and $|V_{ts}|$ cannot be currently determined, for a lack of top quark events, but can be inferred from B meson mixings where these elements appear through the box diagram. The result is

$$|V_{td}| = (7.4 \pm 0.8) \times 10^{-3}\,,$$

$$\frac{|V_{td}|}{|V_{ts}|} = 0.208 \pm 0.008\,.$$

CP Violation

- Charge conjugation (C) takes a particle to its antiparticle, Parity (spatial reflection) changes the helicity of the particle. Under CP, e_L^- will transform to e_R^+. Both C and P are broken symmetries in the SM, but the product CP is approximately conserved. Violation of CP has been seen only in weak interactions.

- The CKM mechanism predicts CP violation through a single complex phase that appears in the CKM matrix. Thus in the SM, various CP violating processes in K, B and other systems get correlated. So far such correlations have been consistent with CKM predictions, but more precise determinations in the B and D systems at the LHC may open up new physics possibilities.

CP Violation

- In the $K^0 - \overline{K^0}$ system, CP violation has been observed both in mixing and in direct decays. CP violation in mixing arises in the SM via the W–boson box diagram.

- The CP asymmetry in mixing is parametrized by ϵ, which is a measure of the mixing between the CP even and CP odd states $K^0_{1,2} = (K^0 \pm \overline{K^0})/\sqrt{2}$. It has been measured to be

$$|\epsilon| = (2.229 \pm 0.010) \times 10^{-3} \, .$$

CP Violation

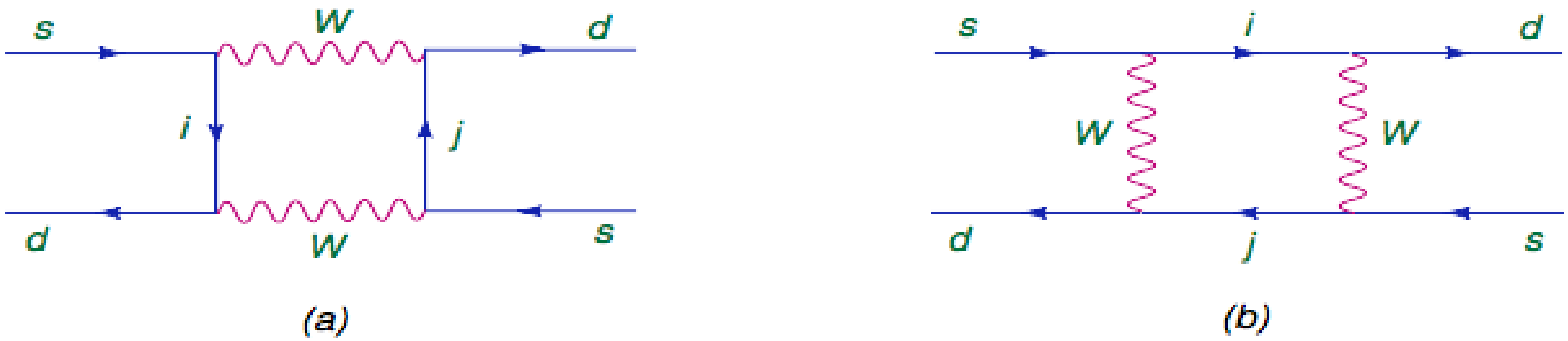


Figure: Box diagram inducing $K^0 - \overline{K^0}$ transition in the SM.

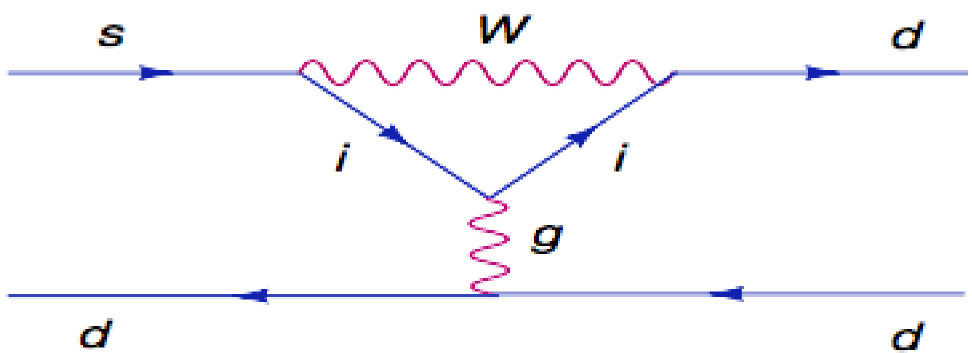


Figure: One loop penguin diagram that generates CP violation in direct $K \to \pi\pi$ decay.

CP Violation

- The measured value in in excellent agreement with expectations from the SM, and enables us to determine the single phase of the CKM matrix.

- The box diagram contribution to ϵ is given by

$$
\begin{aligned}
|\epsilon| &= \frac{G_F^2 f_k^2 m_K m_W^2}{12\sqrt{2}\pi^2 \Delta m_K} \hat{B}_K \left\{ \eta_c S(x_c) \mathrm{Im}[(V_{cs} V_{cd}^*)^2] \right. \\
&+ \left. \eta_t S(x_t) \mathrm{Im}[(V_{ts} V_{td}^*)^2] + 2\eta_{ct} S(x_c, x_t) \mathrm{Im}[V_{cs} V_{cd}^* V_{ts} V_{td}^*] \right\} .
\end{aligned}
$$

Here $S(x)$ and $S(x,y)$ are Inami–Lim functions with $x_{c,t} = m_{c,t}^2 / M_W^2$, and the η factors are QCD correction factors for the running of the effective $\Delta S = 2$ Hamiltonian from M_W to the hadron mass scale.

Unitarity Triangle

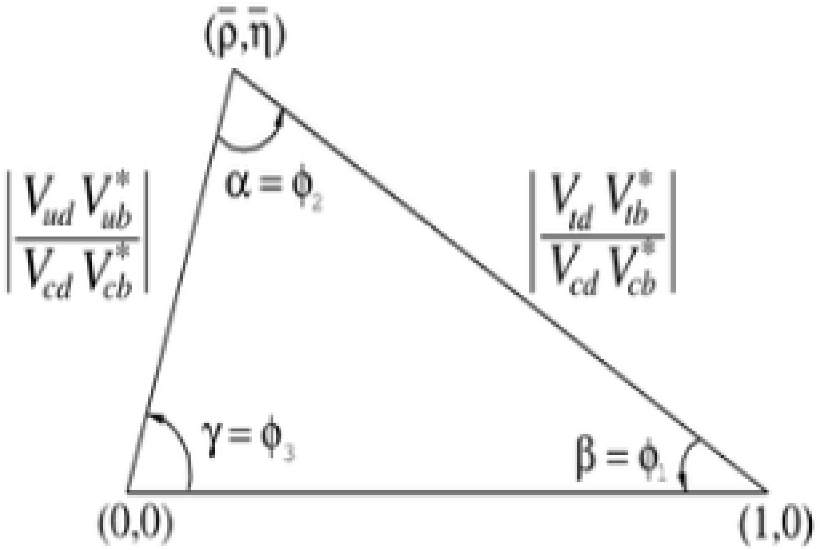


Figure: Unitarity triangle in the CKM model.

Unitarity Triangle

- CP violation in B meson system is now well established. Several CP violating quantities have been measured in B_d meson system, all of which show consistency with the CKM mixing matrix. Unitarity of the CKM matrix implies that $\sum_i V_{ij} V_{ik}^* = \delta_{jk}$ and $\sum_j V_{ij} V_{kj}^* = \delta_{ik}$.
- There are six vanishing combinations, which can be expressed as triangles in the complex plane. The areas of all of these triangles are the same. The most commonly used triangle arises from the relation

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \, .$$

- In the complex plane, the resulting triangle has sides of similar length (of order λ^3). This unitarity triangle relation is shown in Fig.

Unitarity Triangle

- The three interior angles (α, β, γ), also referred to as $(\phi_2,\ \phi_1,\ \phi_3)$, can be written in the CKM model as

$$\begin{aligned}
\alpha &= \arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \simeq \arg\left(-\frac{1-\rho-i\eta}{\rho+i\eta}\right) , \\
\beta &= \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \simeq \arg\left(\frac{1}{1-\rho-i\eta}\right) , \\
\gamma &= \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \simeq \arg\left(\rho+i\eta\right) .
\end{aligned}$$

One experimental test of the CKM mechanism is the measurement of $\alpha+\beta+\gamma = 180^0$.

Angles of the Triangle

- The angle β can be measured with the least theoretical uncertainty from the decay of $B_d \to J/\psi K_S$. It is found to be

$$\sin 2\beta = 0.68 \pm 0.03\,.$$

  This value is in in good agreement with the CKM prediction.

- The angle α is measured from decay modes where $b \to u\bar{u}d$ is dominant. Such decays includ $B \to \pi\pi$, $B \to \rho\rho$ and $B \to \pi\rho$. The value of α extracted is

$$\alpha = (88^{+6}_{-5})^0$$

- The angle γ does not depend on the top quark, and can in principle be measured from tree–level decays of B meson. Strong interaction uncertainties are rather large in decays such as $B^\pm \to D^0 K^\pm$. The current value of the angle γ is

$$\gamma = (77^{+30}_{-32})^0\,.$$

Global it to Flavor Data

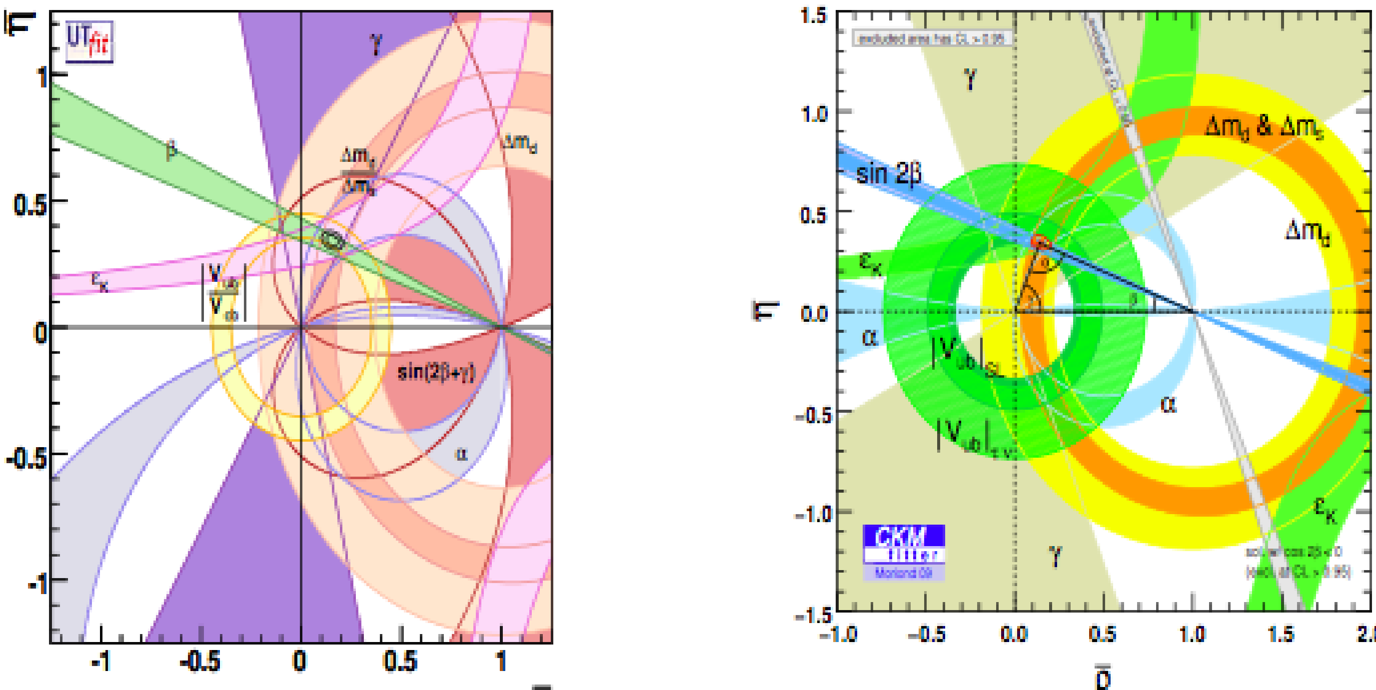


Figure: Global fit to the mixing and CP violation data from the UTFit collaboration (left panel) and the CKMFitter collaboration (right panel).

Best Fit CKM Parameters

- The intersection of the various ellipses gives the best fit value for the Wolfenstein parameters $(\lambda,\ A,\ \rho,\ \eta)$, which are as follows :

$$\lambda = 0.2272 \pm 0.0010,\ \ A = 0.818^{+0.007}_{-0.017},$$
$$\rho = 0.221^{+0.064}_{-0.028},\ \ \eta = 0.340^{+0.017}_{-0.045}.$$

Theories of flavor should provide an understanding of these fundamental parameters.

Relating Quark Mixings with Mass Ratios

- Can the quark mixing angle be computed in terms of the quark mass ratios?

- Clearly such attempts have to go beyond the SM. Here I give a simple two–family example which assumes a flavor $U(1)$ symmetry that distinguishes the two families.

- Consider the mass matrices for $(u,\ c)$ and $(d,\ s)$ quarks given by

$$M_u = \begin{pmatrix} 0 & A_u \\ A_u^* & B_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d \\ A_d^* & B_d \end{pmatrix}.$$

- The crucial features of these matrices are (i) the zeros in the (1,1) entries, and (ii) their hermiticity. Neither of these features can be realized within the SM.

Prediction for Cabibbo Angle

$$M_u = P_u \hat{M}_u P_u^*,$$

- The matrices $\hat{M}_u$ and $\hat{M}_d$, which have all real entries, can be diagonalized readily, yielding for the mixing angles θ_u and θ_d

$$\tan^2\theta_u = \frac{m_u}{m_c},$$
$$\tan^2\theta_d = \frac{m_d}{m_s}.$$

- This yields a prediction for the Cabibbo angle

$$|\sin\theta_C| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\psi}\sqrt{\frac{m_u}{m_c}} \right| .$$

- This formula works rather well, especially since even without the second term, the Cabibbo angle is correctly reproduced. The phase ψ is a parameter, however, its effect is rather restricted. For example, since $\sqrt{m_d/m_s} \simeq 0.22$ and $\sqrt{m_u/m_c} \simeq 0.07$, $|\sin\theta_C|$ must lie between 0.15 and 0.29, independent of the value of ψ.

Realistic Model for Cabibbo Angle

- Since SM interactions do not conserve Parity, it is useful to extend the gauge sector to the left-right symmetric group $G \equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, wherein Parity invariance can be imposed.
- The (1,2) and (2,1) elements of $M_{u,d}$ being complex conjugates of each other will then result. The left–handed and the right–handed quarks transform as $Q_{iL}(3,2,1,1/3) + Q_{iR}(3,1,2,1/3)$ under G.
- Under discrete parity operation $Q_{iL} \leftrightarrow Q_{iR}$. This symmetry can be consistently imposed, as $W_L \leftrightarrow W_R$ in the gauge sector under Parity. The leptons transform as $\psi_{iL}(1,2,1,-1) + \psi_{iR}(1,1,2,-1)$ under the gauge symmetry.
- Note that ψ_R, which is a doublet of $SU(2)_R$, contains the right–handed neutrino, as the partner of e_R. Thus there is a compelling reason for the existence of ν_R, unlike in the SM, where it is optional.

Left-Right Symmetric Model

- The Higgs field that couples to quarks should be $\Phi(1,2,2,0)$, and under Parity $\Phi \to \Phi^\dagger$.
- In matrix form Q_{iL}, Q_{iR}, Φ read as

$$Q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L , \quad Q_{iR} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R , \quad \Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} ,$$

so that the Yukawa Lagrangian for quarks

$$\mathcal{L}_{\text{Yukawa}} = \overline{Q}_L \Phi Y Q_R + \overline{Q}_L \tilde{\Phi} \tilde{Y} Q_R + h.c.$$

is gauge invariant. Here $\tilde{\Phi} \equiv \tau_2 \Phi^* \tau_2$. Imposing Parity, we see that the Yukawa matrices Y and $\tilde{Y}$ must be hermitian, $Y = Y^\dagger$ and $\tilde{Y} = \tilde{Y}^\dagger$.

Left-Right Symmetric Model (cont.)

- The VEVs $\langle \phi_1^0 \rangle$ and $\langle \phi_2^0 \rangle$ can be complex in general, but this will not affect the prediction for the Cabibbo angle, since that only requires $|(M_{u,d})_{12}| = |(M_{u,d})_{21}|$.

- Additional Higgs fields, eg., $\Delta_L(1,3,1,2) + \Delta_R(1,1,3,2)$, would be required for breaking the left–right symmetric gauge group down to the SM and for simultaneously generating large ν_R Majorana masses. However, these fields do not enter into the mass matrices of quarks.

- To enforce zeros in the (1,1) entries of $M_{u,d}$, we can employ the following $U(1)$ flavor symmetry:
  $Q_{1L} : 2,\ Q_{1R} : -2.\ Q_{2L} : 1,\ Q_{2R} : -1.\ \Phi_1 : 2,\ \Phi_2 : 3.$

- Note that two Higgs bidoublet fields are needed. Φ_1 generates the (2,2) entries, while Φ_2 generates the (1,2) and (2,1) entries. There is no (1,1) entry generated, since there is no Higgs field with $U(1)$ charge of $+4$.

Three Family Generalizations

- It can be generalized for the case of three families, a la Fritzsch.
- The up and down quark mass matrices have hermitian nearest neighbor interaction form:

$$M_{u,d} = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}_{u,d} .$$

- Such matrices have factorizable phases, i.e., $M_{u,d} = P_{u,d} \hat{M}_{u,d} P^*_{u,d}$, where $\hat{M}_{u,d}$ are the same, but without any phases, and $P_{u,d}$ are diagonal phase matrices.

Fritzsch Mass Matrix Predictions

- One finds four relations between masses and mixings :

$$
\begin{aligned}
|V_{us}| &\simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\psi}\sqrt{\frac{m_u}{m_c}} \right| , \\
|V_{cb}| &\simeq \left| \sqrt{\frac{m_s}{m_b}} - e^{i\phi}\sqrt{\frac{m_c}{m_t}} \right| , \\
|V_{ub}| &\simeq \left| \frac{m_s}{m_b}\sqrt{\frac{m_d}{m_b}} + e^{i\psi}\sqrt{\frac{m_u}{m_c}} \left(\sqrt{\frac{m_s}{m_b}} - e^{i\phi}\sqrt{\frac{m_c}{m_t}} \right) \right| , \\
|V_{td}| &\simeq \left| \frac{m_c}{m_t}\sqrt{\frac{m_u}{m_t}} + e^{i\psi}\sqrt{\frac{m_d}{m_s}} \left(\sqrt{\frac{m_c}{m_t}} - e^{i\phi}\sqrt{\frac{m_s}{m_b}} \right) \right| .
\end{aligned}
$$

Here the two phases ψ and ϕ are related to the phases in the diagonal matrix P as $\psi = (\alpha - \beta)$ and $\phi = \beta$.

$$
\frac{|V_{ub}|}{|V_{cb}|} \simeq \sqrt{\frac{m_u}{m_c}} , \qquad \frac{|V_{td}|}{|V_{ts}|} \simeq \sqrt{\frac{m_d}{m_s}} .
$$

From V(cb) prediction, top mass predicted to be < 90 GeV!

Froggatt-Nielsen Mechanism for Mass Hierarchy

- The hierarchy in the masses and mixings of quarks and leptons can be understood by assuming a flavor $U(1)$ symmetry under which the fermions are distinguished.

- In this approach developed by Froggatt and Nielsen, there is a "flavon" field S, which is a scalar, usually a SM singlet field, which acquires a VEV and breaks the $U(1)$ symmetry.

- This symmetry breaking is communicated to the fermions at different orders in a small parameter $\epsilon = \langle S \rangle / M_*$. Here M_* is the scale of flavor dynamics, and usually is associated with some heavy fermions which are integrated out.

- The nice feature of this approach is that the mass and mixing hierarchies will be explained as powers of the expansion parameter ϵ without assuming widely different Yukawa couplings.

- The effective theory below M_* is rather simple, while the full theory will have many heavy fermions, called Froggatt–Nielsen fields.

A Two-Family FN Model

- Let me illustrate this idea with a two family example which is realistic when applied to the second and third families of quarks.
- Consider M_u and M_d for the $(c,\ t)$ and $(s,\ b)$ sectors given by

$$M_u = \begin{pmatrix} \epsilon^4 & \epsilon^2 \\ \epsilon^2 & 1 \end{pmatrix} v_u \,, \quad M_d = \begin{pmatrix} \epsilon^3 & \epsilon^3 \\ \epsilon & \epsilon \end{pmatrix} v_d \,.$$

  Here $\epsilon \sim 0.2$ is a flavor symmetry breaking parameter.

- Every term has an order one coefficient which is not displayed.
- We obtain the following relations for quark masses and $|V_{cb}|$:

$$\frac{m_c}{m_t} \sim \epsilon^4 \,, \ \frac{m_s}{m_b} \sim \epsilon^2 \,, \ |V_{cb}| \sim \epsilon^2 \,.$$

A Two-Family FN Model

- First, let us look at the effective Yukawa couplings, which can be obtained from the Lagrangian:

$$\begin{aligned} \mathcal{L}_{FN}^{\text{eff}} &= \left[Q_3 u_3^c H_u + Q_2 u_3^c H_u S^2 + Q_3 u_2^c H_u S^2 + Q_2 u_2^c H_u S^4\right] \\ &+ \left[Q_3 d_3^c H_d S + Q_3 d_2^c H_d S + Q_2 d_2^c H_d S^3 + Q_2 d_3^c H_d S^3\right] + h.c. \end{aligned}$$

- Here I assumed supersymmetry, so that there are two Higgs doublets $H_{u,d}$.
- It is not necessary to assume SUSY, one can simply identify H_u as H of SM, and replace H_d by $\tilde{H}$.
- All couplings are taken to be of order one.
- The symmetry is a $U(1)$ with the following charge assignment.

$$\{Q_3, u_3^c\} : 0; \;\; \{Q_2, u_2^c\} : 2; \; \{d_2^c, d_3^c\} : 1; \; \{H_u, H_d\} : 0; \; S : -1 \, .$$

Spaghetti Diagrams in FN Model

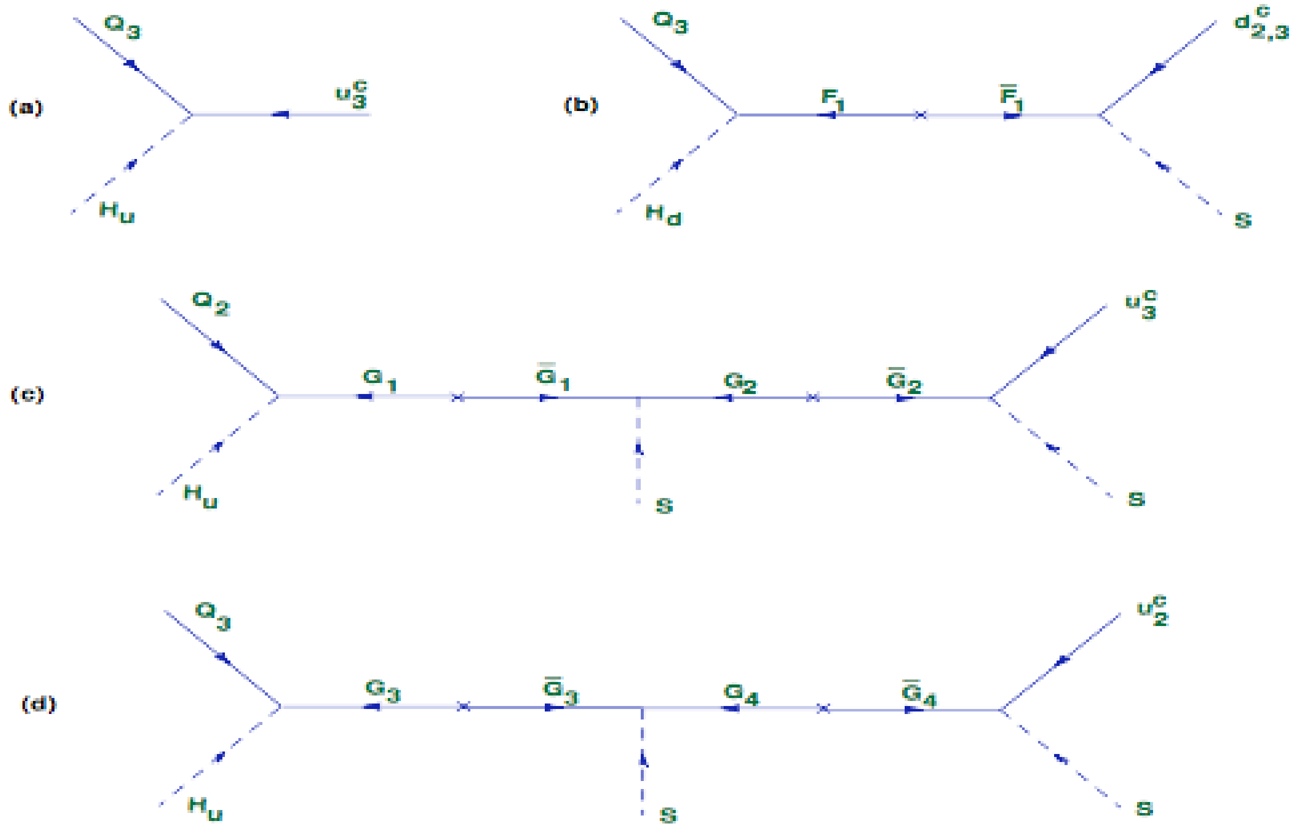


Figure: Froggatt–Nielsen fields generating effective Yukawa couplings.

Spaghetti Diagrams in FN Model

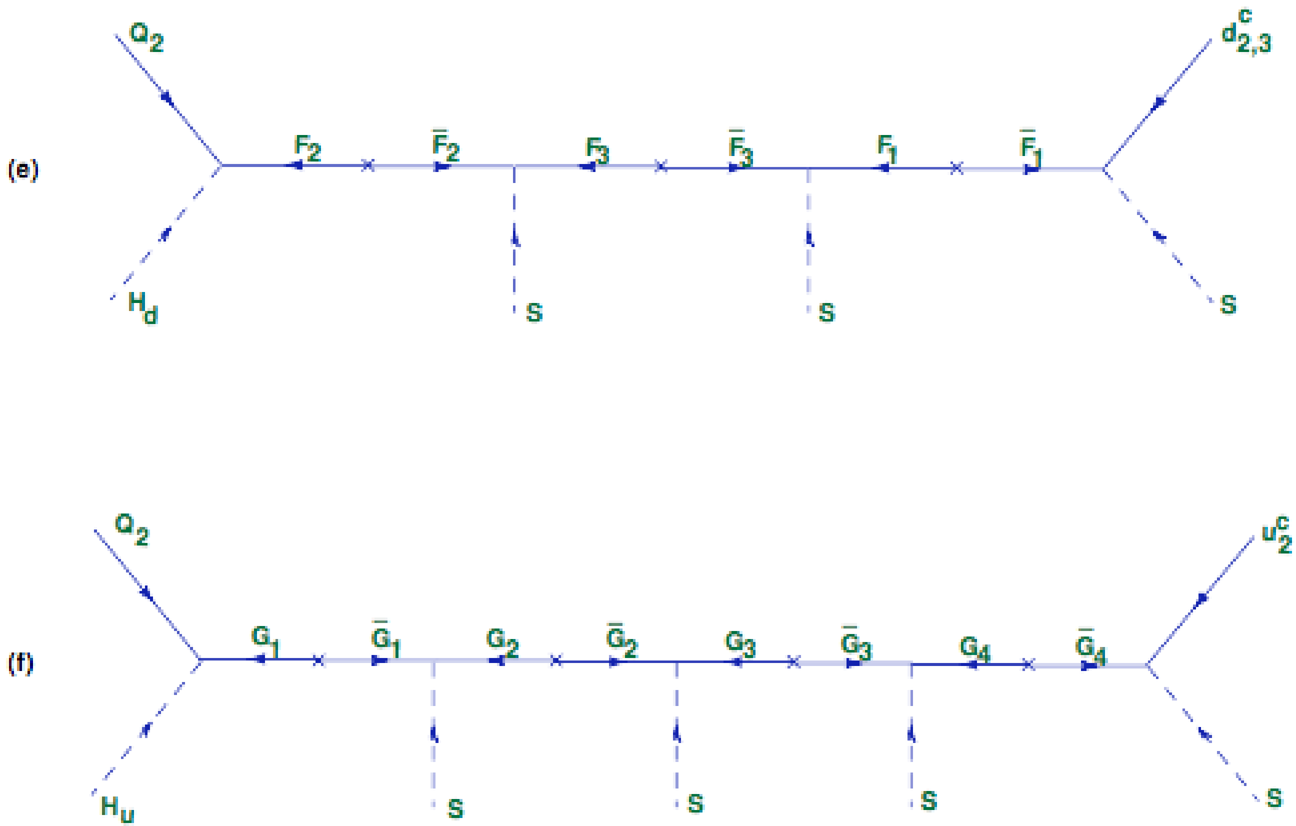


Figure: Froggatt–Nielsen fields generating effective Yukawa couplings.

A Three-Family FN Model

- An explicit and complete anomalous $U(1)$ model that fits well all quark and lepton masses and mixings is constructed below. Consider the quark and lepton mass matrices of the following form :

$$M_u \sim \langle H_u \rangle \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad M_d \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix},$$

$$M_e \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon \\ \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad M_{\nu_D} \sim \langle H_u \rangle \epsilon^s \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$

$$M_{\nu^c} \sim M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \quad \Rightarrow \quad M_\nu^{light} \sim \frac{\langle H_u \rangle^2}{M_R} \epsilon^{2s} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}.$$

This structure explains small CKM mixing and large lepton mixing

U(1) Charges in Three-Family FN Model

Field	$U(1)_A$ Charge	Charge notation
Q_1, Q_2, Q_3	$4, 2, 0$	q_i^Q
L_1, L_2, L_3	$1+s, s, s$	q_i^L
u_1^c, u_2^c, u_3^c	$4, 2, 0$	q_i^u
d_1^c, d_2^c, d_3^c	$1+p, p, p$	q_i^d
e_1^c, e_2^c, e_3^c	$4+p-s, 2+p-s, p-s$	q_i^e
$\nu_1^c, \nu_2^c, \nu_3^c$	$1, 0, 0$	q_i^ν
H_u, H_d, S	$0, 0, -1$	$(h, \bar{h}, q_s)$

Table: The flavor $U(1)_A$ charge assignment for the MSSM fields and the flavon field S.