Introduction to Flavor Physics

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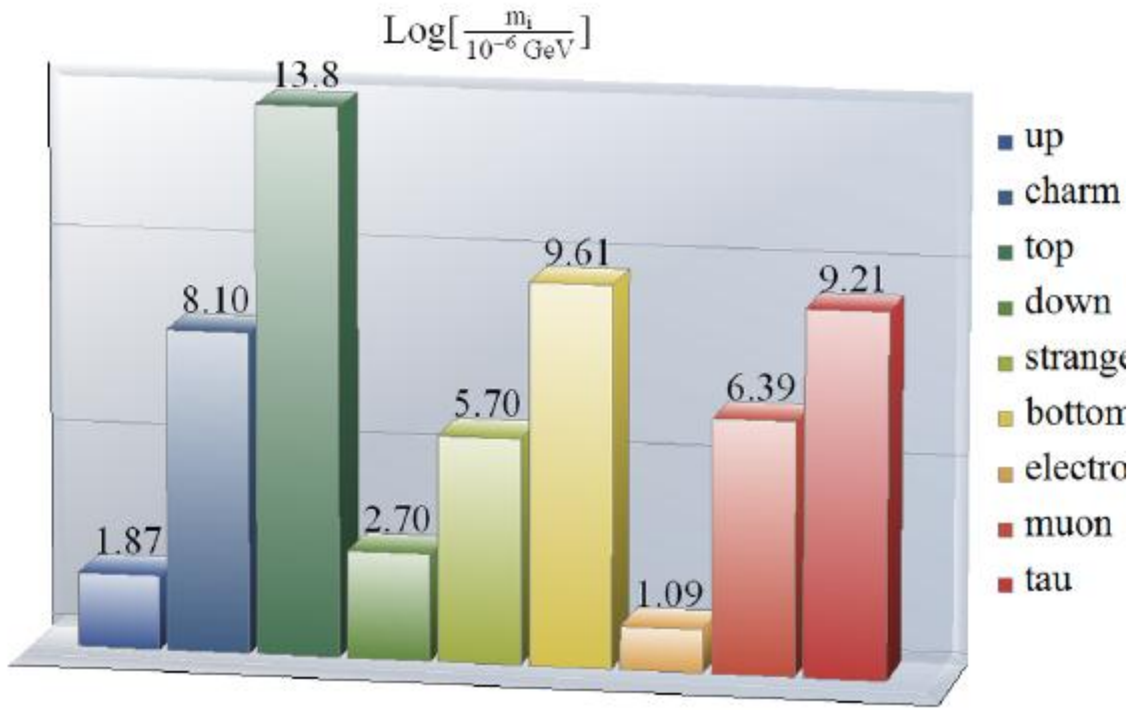
Fermion Mass Puzzle

Charged Fermion Mass Hierarchy

- **up-type quarks**
- $m_u \sim 6.5 \times 10^{-6}$
- $m_c \sim 3.3 \times 10^{-3}$
- $m_t \sim 1$
- **down-type quarks**
- $m_d \sim 1.5 \times 10^{-5}$
- $m_s \sim 3 \times 10^{-4}$
- $m_b \sim 1.5 \times 10^{-2}$
- **charged leptons**
- $m_e \sim 3 \times 10^{-6}$
- $m_\mu \sim 6 \times 10^{-4}$
- $m_\tau \sim 1 \times 10^{-2}$



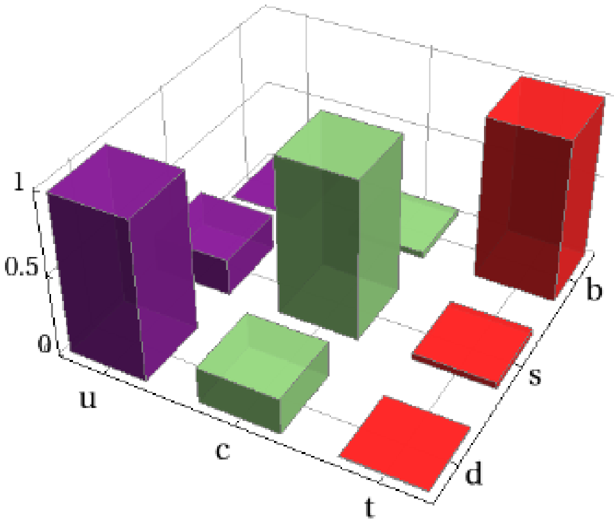
Neutrino masses not strongly hierarchical

3 masses within an order of magnitude consistent!

Quark and Lepton Mixing Parameters

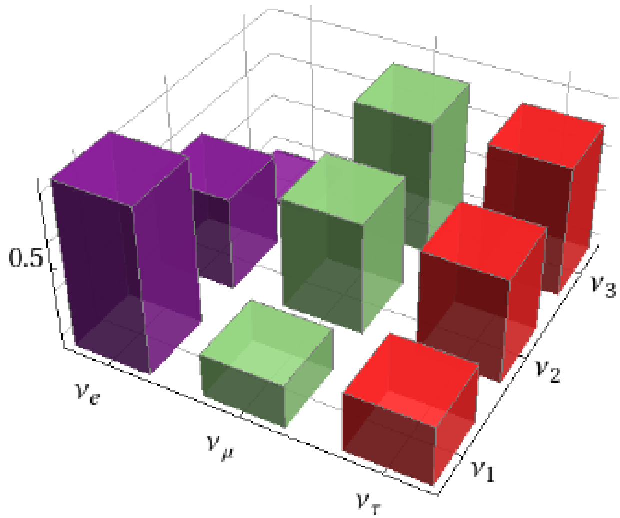
- **Quark Mixings**

$$V_{CKM} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix}$$



- **Leptonic Mixings**

$$U_{PMNS} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix}$$



The Flavor Puzzle

- Why are there three families of quarks and leptons?
- Are the flavor parameters all arbitrary, or are they inter-connected?
- Why do the charged fermion masses exhibit a strong hierarchical structure spanning some six orders of magnitude?
- Why are the mixing angles in the quark sector hierarchical?
- Are the mixing parameters related to the mass ratios?
- Why is $\overline{\theta} < 10^{-9}$?
- What causes the neutrino mixing angles to be much larger than the corresponding quark mixing angles?
- What is the origin of CP violation?

A lack of fundamental understanding of these issues is called “Flavor Puzzle”

Counting Flavor Parameters

- The counting of parameters of the SM:
  - Five flavor universal parameter – three gauge couplings (g_1, g_2, g_3), one Higgs quartic coupling λ, and one Higgs mass-squared μ^2
  - Fourteen parameters associated with the flavor sector – Six quark masses, three charged lepton masses, four quark mixing angles (including one weak CP violating phase), the strong CP violating parameter $\overline{\theta}$
  - If we include small neutrino masses and mixing angles into the SM, an additional nine parameters will have to be introduced (three neutrino masses, three neutrino mixing angles and three CP violating phases, in the case of Majorana neutrinos).
- 23 out of 28 parameters of the SM are from flavor sector!

Possible Solutions to the Flavor Puzzle

Various solutions to the flavor puzzle have been suggested, leading to BSM

If flavor dynamics occurs near TeV, it is accessible to LHC

Flavor dynamics may occur near the Planck scale, which is difficult to test

Remnants of flavor dynamics maybe carried to TeV scale by surviving symmetry such as SUSY

$(g-2)$ of the muon, $\mu \to e\gamma$ and $b \to s\mu^+\mu^-$ decays are possible tests of new flavor physics

Flavor Structure of Standad Model

- Because of the chiral structure of weak interactions, bare fermion masses are not allowed in the Standard Model.

- Fermion masses arise via Yukawa interactions given by the Lagrangian

$$\mathcal{L}_{\rm Yukawa} = Q^T Y_u u^c H - Q^T Y_d d^c \tilde{H} - L^T Y_\ell e^c \tilde{H} + h.c.$$

- Here all fermion fields are left-handed

- A charge conjugation matrix C is understood to be sandwiched between all of the fermion bi-linears.

- $SU(2)_L$ contraction between the fermion doublet and Higgs doublet involves the matrix $i\tau_2$.

$$Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix} \; ; \; L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \; ; \; H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \; ; \; \tilde{H} = \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix} ,$$

Flavor Structure of Standad Model (cont.)

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= (Y_u)_{ij}[u_i u_j^c H^0 - d_i u_j^c H^+] + (Y_d)_{ij}[u_i d_j^c H^- + d_i d_j^c H^{0*}] \\ &+ (Y_\ell)_{ij}[\nu_i e_j^c H^- + e_i e_j^c H^{0*}] + h.c.\end{aligned}$$

$$H^0 = (\frac{h}{\sqrt{2}} + v) \qquad (v = 174 \text{ GeV})$$

$$M_u = Y_u v\,, \qquad M_d = Y_d v\,, \qquad M_\ell = Y_\ell v\,.$$

Higgs coupling to fermions: $(Y_u)_{ij}/\sqrt{2}(uu^c h)$

Mass matrix diagonalization also diagonalizes Yukawa coupling matrix $\Rightarrow$ No Higgs FCNC

Flavor Structure of Standad Model (cont.)

- Unitary rotations on the quark fields in family space:

$$\begin{aligned} u &= V_u\, u^0, \quad u^c = V_{u^c}\, u^{c0}, \\ d &= V_d\, d^0, \quad d^c = V_{d^c}\, d^{c0}, \end{aligned}$$

- Choose unitary matrices such that

$$\begin{aligned} V_u^T (Y_u v) V_{u^c} &= \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}, \\ V_d^T (Y_d v) V_{d^c} &= \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}. \end{aligned}$$

- Unitary rotations leave kinetic terms of fermions canonical
- Bi-unitary rotations can diagonalize non-Hermitian matrices

Flavor Structure of Standad Model (cont.)

- The couplings of the Z^0 boson and the photon to quarks will remain flavor diagonal
- $(\overline{u}\gamma_\mu I u)Z^\mu \Rightarrow (\overline{u^0}\gamma_\mu(V_u^\dagger I V_u)u^0)Z^\mu = (\overline{u^0}\gamma_\mu I u^0)Z^\mu$
- The charged current quark interaction, $\mathcal{L}_{cc} = g/\sqrt{2}(\overline{u}\gamma_\mu d)W^{+\mu} + h.c.$, becomes

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}[\overline{u^0}\gamma_\mu V d^0]\, W^{\mu +} + h.c.$$

  where

$$V = V_u^\dagger V_d$$

  is the quark mixing matrix, or the Cabibbo–Kobayashi–Maskawa (CKM) matrix.

- In the SM, all the flavor violation is contained in V. Being product of unitary matrices, V is itself unitary. This feature has thus far withstood experimental scrutiny

Flavor Structure of Leptons

- We can repeat this process in the leptonic sector. Define

$$\nu = V_\nu\ \nu^0\,, \quad e = V_e\ e^0\,, \quad e^c = V_{e^c}\ e^{c0}\,.$$

Choose Y_e and Y_{e^c} such that

$$Y_e^T (Y_\ell v) Y_{e^c} = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}\,.$$

- Note that there is no right–handed neutrino in the SM and hence there is no neutrino mass. One can choose $V_\nu = V_e$, so that the charged current weak interactions islavor diagonal.

- However, it is now well established that neutrinos have small masses

Flavor Structure of Leptons (cont.)

- Additional terms must be added in order to accommodate them. The simplest possibility is to add a non–renormalizable term

$$\mathcal{L}_{\nu-\mathrm{mass}} = \frac{(L^T Y_\nu L) H H}{2 M_*} + h.c.$$

- This can be realized by integrating out some heavy fields with mass of order M_*.
- Well known example is seesaw, where M_* is mass of ν_R
- Light neutrino mass matrix given by $M_\nu = Y_\nu \frac{v^2}{M_*}$
- Now we choose V_ν so that

$$V_\nu^T Y_\nu \frac{v^2}{M_*} V_\nu = \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} ,$$

with $m_{1,2,3}$ being the tiny masses of the three light neutrinos.

Flavor Structure of Leptons (cont.)

- The leptonic charge current interaction now becomes

$$\mathcal{L}_{cc}^{\ell} = \frac{g}{\sqrt{2}}[\overline{e^0}\gamma_\mu U \nu^0]\; W^{-\mu} + h.c.$$

  where $U = V_e^\dagger V_\nu$ is the leptonic mixing matrix, or the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix.

- Neutrino oscillations observed in experiments are attributed to the off–diagonal entries of the matrix U. As V, U is also unitary.
- We assumed here that the neutrino mass generation mechanism violated total lepton number by two units.
- While this is not established, the seesaw mechanism predicts this
- If neutrinos are Dirac particles, there is no similar explanation for the smallness of their masses

Charged Lepton Masses

- Charged Leptons are propagating states, and their masses are simply the poles in the propagators. Experimental information on charged lepton masses is rather accurate:

$$
\begin{aligned}
m_e &= 0.510998902 \pm 0.000000021 \text{ MeV}\,, \\
m_\mu &= 105.658357 \pm 0.000005 \text{ MeV}\,, \\
m_\tau &= 1777.03^{+0.30}_{-0.26} \text{ MeV}\,.
\end{aligned}
$$

- The direct kinematic limits on the three neutrino masses are:

$$
m_{\nu_e} \leq 3 \text{ eV}\,,\ m_{\nu_\mu} \leq 0.19 \text{ MeV}\,,\ m_{\nu_\tau} \leq 18.2 \text{ MeV}\,.
$$

- Neutrino oscillation date determine the mass splittings:

$$
\Delta m^2_{21} = m^2_2 - m^2_1 \simeq 7.5 \times 10^{-5} \text{eV}^2, \Delta m^2_{23} = m^2_2 - m^2_3 \simeq \pm 2.5 \times 10^{-3} \text{ eV}^2
$$

Leptonic Mixing Matrix

- The PMNS matrix U, being unitary, has N^2 independent components for N families of leptons. Out of these, $N(N-1)/2$ are Euler angles, while the remaining $N(N+1)/2$ are phases. Many of these phases can be absorbed into the fermionic fields and removed.

- If one writes $U = Q\hat{U}P$, where P and Q are diagonal phase matrices, then by redefining the phases of e fields as $e \to Qe$, the N phases in Q can be removed. P has only $N-1$ non–removable phases (an overall phase is irrelevant). For $N=3$, $P = diag.(e^{i\alpha},\ e^{i\beta},\ 1)$. α, β are called the Majorana phases.

Leptonic Mixing Matrix (cont.)

- If the neutrino masses are of the Dirac type, these phases can also be removed by redefining the ν^c fields. $\hat{U}$ will then have $N(N+1)/2 - (2N-1) = \frac{1}{2}(N-1)(N-2)$ phases. For $N = 3$, there is a single "Dirac" phase in U. This single phase will be relevant for neutrino oscillation phenomenology.
- The two Majorana phases (α, β) do not affect neutrino oscillations, but will be relevant for neutrino-less double beta decay.

Leptonic Mixing Matrix (cont.)

- In general, the PMNS matrix for three families of leptons can be written as

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} .$$

- To enforce the unitarity relations it is convenient to adopt specific parametrizations. The "standard parametrization" that is now widely used has $U_{PMNS} = U.P$ where

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} .$$

Here $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$.

Leptonic Mixing Matrix (cont.)

- Our current understanding of these mixing angles arising from neutrino oscillations can be summarized as follows ($2\,\sigma$ error bars quoted) :

$$
\begin{aligned}
\sin^2\theta_{12} &= 0.27 - 0.35\,, \\
\sin^2\theta_{23} &= 0.39 - 0.63\,, \\
\sin^2\theta_{13} &\leq 0.040\,.
\end{aligned}
$$

Here θ_{12} limit arises from solar neutrino data (when combined with KamLand reactor neutrino data), θ_{23} from atmospheric neutrinos (when combined with MINOS accelerator neutrino data), and θ_{13} from reactor neutrino data.

Leptonic Mixing Matrix (cont.)

- It is intriguing that the current understanding of leptonic mixing can be parametrized by the unitary matrix

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P\,.$$

  This mixing is known as tri-bimaximal mixing. This nomenclature is based on the numerology
  $\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0.$

- As we will see, such a geometric structure is far from being similar to the quark mixing matrix. Note that currently θ_{13} is allowed to be zero, in which case the Dirac phase δ becomes irrelevant.

Extracting Quark Masses

Quark masses are extracted differently, since they are not propagating states

QCD Lagrangian exhibits chiral symmetry breaking via condensate of quarks:

$$\langle \overline{u} u \rangle = \langle \overline{d} d \rangle = \langle \overline{s} s \rangle = \Lambda^3_{QCD}$$

The global symmetry $SU(3)_L \times SU(3)_R$ breaks down to $SU(3)_V$

This leads to 8 (pseudo)-Goldstone Bosons:

$$(\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \overline{K^0}, \eta)$$

Hadron masses come from quark condensate and the Higgs VEV

Extracting Quark Masses (cont.)

$$\mathcal{L} = \sum_{k=1}^{N_F=3} \bar{q}_k (i\not{D} - m_k) q_k - \frac{1}{4} G_{\mu\nu} G^{\mu nu}$$

- This Lagrangian has a chiral symmetry in the limit where the quark masses vanish. The three left–handed quarks can be rotated into one another, and the three right–handed quarks can be rotated independently.

- The symmetry is $SU(3)_L \times SU(3)_R \times U(1)_V$, with the axial $U(1)_A$ (of the classical symmetry $U(3)_L \times U(3)_R$) explicitly broken by anomalies. The $U(1)_V$ is baryon number, which remains unbroken even after QCD dynamics. QCD dynamics breaks the $SU(3)_L \times SU(3)_R$ symmetry down to the diagonal subgroup $SU(3)_V$.

Extracting Quark Masses (cont.)

- The explicit breaking of chiral symmetry occurs via the mass term

$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} .$$

  M can be thought of as a spurion field which breaks the chiral symmetry spontaneously.

- Under $SU(3)_L \times SU(3)_R$ symmetry $q_L \to U_L\ q_L,\ \ q_R \to U_R\ q_R$, while $M \to U_L\ M\ U_R^\dagger$. That is, M transforms as a $(3, 3^*)$ of this group. Under the unbroken diagonal $SU(3)_V$ subgroup, both q_L and q_R transform as triplets, while M splits into a $\mathbf{1} + \mathbf{8}$.

Extracting Quark Masses (cont.)

- Thus M can be written as $M = M_1 + M_8$, where M_1 is a singlet of $SU(3)_V$, while M_8 is an octet:

$$
\begin{aligned}
M_1 &= \frac{(m_u + m_d + m_s)}{3} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \\
M_8 &= \frac{(m_u - m_d)}{2} \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} + \frac{(m_u - m_d - 2m_s)}{6} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}
\end{aligned}
$$

Extracting Quark Masses (cont.)

- The octet (under $SU(3)_V$) of mesons can be written down as a (normalized) matrix

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\eta^0 \end{pmatrix} .$$

- The lowest order invariants involving Φ bilinear and M are

$$A \ \mathrm{Tr}(\Phi^2)M_1 + B \ \mathrm{Tr}(\Phi^2 M_8) .$$

Here A and B are arbitrary coefficients.

Extracting Quark Masses (cont.)

- Now, in the limit of $m_u = 0, m_d = 0, m_s \neq 0$, the $SU(2)_L \times SU(2)_R$ chiral symmetry remains unbroken, and so the pion fields should be massless. Working out the mass terms, and demanding that the pion mass vanishes in this limit, one finds a relation $A = 2B$.
- Electromagnetic interactions will split the masses of the neutral and charged members. Then we have

$$\begin{aligned} m_{\pi^0}^2 &= B(m_u + m_d) \\ m_{\pi^\pm}^2 &= B(m_u + m_d) + \Delta_{\mathrm{em}} \\ m_{K^0}^2 &= m_{\overline{K^0}}^2 = B(m_d + m_s) \\ m_{K^\pm}^2 &= B(m_u + m_s) + \Delta_{\mathrm{em}} \\ m_\eta^2 &= \frac{1}{3} B(m_u + m_d + 4m_s) \,. \end{aligned}$$

Extracting Quark Masses (cont.)

- Eliminating B and Δ_{em}, we obtain two relations for quark mass ratios:

$$\frac{m_u}{m_d} = \frac{2m_{\pi^0}^2 - m_{\pi^+}^2 + m_{K^+}^2 - m_{K^0}^2}{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2} = 0.56$$

$$\frac{m_s}{m_d} = \frac{m_{K^0}^2 + m_{K^+}^2 - m_{\pi^+}^2}{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2} = 20.1$$

- This is the lowest order chiral perturbation theory result for the mass ratios. Second order chiral perturbation theory makes important corrections to these ratios. Note that the absolute masses cannot be determined in this way. Alternative techniques, such as QCD sum rules and lattice calculations which provide the most precise numbers have to be applied for this.

Extracting Quark Masses (cont.)

- For heavy quarks (c and b), one can invoke another type of symmetry, the heavy quark effective theory (HQET). When the mass of the quark is heavier than the typical momentum of the partons $\Lambda \sim m_p/3 = 330$ MeV, one can make another type of expansion.

- In analogy with atomic physics, where different isotopes exhibit similar chemical behavior, the behavior of charm hadrons and bottom hadrons will be similar. In fact, there will be an $SU(2)$ symmetry relating the two, to lowest order in HQET expansion.

Extracting Quark Masses (cont.)

- One consequence is that the mass splitting between the vector and scalar mesons in the b and c sector should be related. This leads to a relations $M_{B^*} - M_B = \Lambda^2/m_b$ and $M_{D^*} - M_D = \Lambda^2/m_c$, leading to the prediction
$$\frac{M_{B^*} - M_B}{M_{D^*} - M_D} = \frac{m_c}{m_b},$$
which is in good agreement with experiments.
- The most reliable determination of light quark masses come from lattice QCD. The QCD Lagrangian has only very few parameters, the strong coupling constant, and the three light quark masses. All the hadron masses and decay constants should in principle be calculable in terms of these parameters. Since QCD coupling is strong at low energies, perturbation theory is not reliable.

Quark Masses from Lattice QCD

- The MILC collaboration, which adopted a partially quenched approximation, finds for the light quark masses

$$
\begin{aligned}
m_u(2\ \mathrm{GeV}) &= 1.7 \pm 0.3\ \mathrm{MeV}\,, \\
m_d(2\ \mathrm{GeV}) &= 3.9 \pm 0.46\ \mathrm{MeV}\,, \\
m_s(2\ \mathrm{GeV}) &= 76 \pm 7.6\ \mathrm{MeV}\,.
\end{aligned}
$$

- The ratios of light quark masses are thought to be more reliable, as many of the uncertainties cancel in the ratios. It is customary to define an average mass of up and down quarks $\hat{m} = (m_u + m_d)/2$. The results of MILC collaboration corresponds to the following mass ratios:

$$
\begin{aligned}
\frac{m_u}{m_d} &= 0.43 \pm 0.08\,, \\
\frac{m_s}{\hat{m}} &= 27.4 \pm 4.2\,.
\end{aligned}
$$

Quark Masses from Lattice QCD

- The JLQCD collaboration, which includes three flavors of dynamical quarks finds

$$\begin{aligned} \hat{m}(2\ \mathrm{GeV}) &= 3.55^{+0.65}_{-0.28}\ \mathrm{MeV}\,, \\ m_s(2\ \mathrm{GeV}) &= 90.1^{+17.2}_{-6.1}\ \mathrm{MeV}\,, \\ \frac{m_u}{m_d} &= 0.577 \pm 0.025\,. \end{aligned}$$

- The RBC & UKQCD collaboration, which includes $2+1$ dynamical domain wall quarks finds

$$\begin{aligned} \hat{m}(2\ \mathrm{GeV}) &= 3.72 \pm 0.41\ \mathrm{MeV}\,, \\ m_s(2\ \mathrm{GeV}) &= 107.3 \pm 11.7\ \mathrm{MeV}\,, \\ \hat{m} : m_s &= 1 : 28.8 \pm 1.65\,. \end{aligned}$$

Quark Masses from Lattice QCD

- And finally, the HPQCD collaboration finds

$$
\begin{aligned}
m_u(2\ \mathrm{GeV}) &= 1.9 \pm 0.24\ \mathrm{MeV}\,, \\
m_d(2\ \mathrm{GeV}) &= 4.4 \pm 0.34\ \mathrm{MeV}\,, \\
m_s(2\ \mathrm{GeV}) &= 87 \pm 5.7\ \mathrm{MeV} \\
\hat{m}(2\ \mathrm{GeV}) &= 3.2 \pm 0.89\ \mathrm{MeV}\,, \\
\frac{m_u}{m_d} &= 0.43 \pm 0.08\,.
\end{aligned}
$$

- One sees that the lattice calculations are settling down, and have become quite reliable. It should be mentioned that the same lattice QCD calculations also provide several of the hadronic form factors which enter into the determination of the CKM mixing angles.

Quark Masses from Lattice QCD

- We summarize the masses of these quarks thus obtained, along with the ranges for the light quark masses.

$$
\begin{aligned}
m_u(2\ \mathrm{GeV}) &= 1.5 \text{ to } 3.3\ \mathrm{MeV}\,, \\
m_d(2\ \mathrm{GeV}) &= 3.5 \text{ to } 6.0\ \mathrm{MeV}\,, \\
m_s(2\ \mathrm{GeV}) &= 105^{+25}_{-35}\ \mathrm{MeV}\,, \\
\frac{m_u}{m_d} &= 0.35 \text{ to } 0.60\,, \\
\frac{m_s}{m_d} &= 17 \text{ to } 22\,, \\
\frac{m_s}{(m_u+m_d)/2} &= 25 \text{ to } 30\,, \\
m_c(m_c) &= 1.27^{+0.07}_{-0.11}\ \mathrm{GeV}\,, \\
m_b(m_b) &= 4.20^{+0.17}_{-0.07}\ \mathrm{GeV}\,.
\end{aligned}
$$

Running mass and Pole mass

The physical mass of heavy quark is the "pole mass"

Sometimes we use "running mass" to compare experiments at different energies

$$M_q = m_q(M_q)\left[1 + \frac{4}{3}\frac{\alpha_s(M_q)}{\pi} + \kappa_q^{(2)}\left(\frac{\alpha_s(M_q)}{\pi}\right)^2 + \kappa_q^{(3)}\left(\frac{\alpha_s(M_q)}{\pi}\right)^3\right]$$

Difference is due to QCD and QED corrections

Running mass and Pole mass

- The two–loop and the three–loop QCD correction factors are $\{\kappa_c^{(2)}, \kappa_b^{(2)}, \kappa_t^{(2)}\} = \{11, 21, 10.17, 9.13\}$ and $\{\kappa_c^{(3)}, \kappa_b^{(3)}, \kappa_t^{(3)}\} = \{123.8, 101.5, 80.4\}$.

- There can be significant differences between M_q and $m_q(M_q)$. For example, using $\alpha_s(M_Z) = 0.1176$ and $M_t = 172.5$ GeV, one obtains, with QCD evolution of α_s from M_Z to M_t, $\alpha_s(M_t) = 0.108$, and then $m_t(M_t) = 162.8$ GeV. For c and b quarks the differences are even bigger.

- The running masses of leptons can be defined analogously, but now the QCD corrections are replaced by QED corrections. Consequently the differences between the pole mass M_ℓ and running mass $m_\ell(M_\ell)$ are less significant. The two masses are related via

$$m_\ell(\mu) = M_\ell \left[1 - \frac{\alpha}{\pi}\left\{1 + \frac{3}{2}\ln\frac{\mu}{m_\ell(\mu)}\right\}\right] .$$

Running Quark and Lepton Masses

- While extrapolating the Yukawa coupling above the weak scale one has to specify the theory valid in that regime. Often it will be assumed to be the minimal supersymmetric standard model (MSSM).
- In the fermion Yukawa sector there are significant differences between the MSSM and the SM. The main difference is that supersymmetry requires two Higgs doublets, H_u with $(Y/2) = +1/2$ and H_d with $(Y/2) = -1/2$.
- The extra doublet is needed for anomaly cancelation and also for generating all fermion masses. Recall that in the SM Yukawa interaction we used H for generating the up–type quark masses and its conjugate $\tilde{H}$ for the down–type quark and charged lepton masses. Supersymmetric Yukawa couplings must be derived from a superpotential W, which is required to be holomorphic. This means that if H appears in W, then H^* cannot appear.

SUSY Yukawa Lagrangian

- The MSSM Yukawa interactions arise from the following superpotential.

$$\mathcal{W}^{\mathrm{MSSM}}_{\mathrm{Yukawa}} = Q^T Y_u u^c H_u - Q^T Y_d d^c H_d - L^T Y_\ell e^c H_d \,.$$

If we denote the VEVs of H_u and H_d as v_u and v_d, then the mass matrices for the three charged fermion sectors are

$$M_u = Y_u v_u \,, \qquad M_d = Y_d v_d \,, \qquad M_\ell = Y_\ell v_d \,.$$

$$\tan\beta = \frac{v_u}{v_d}$$

Running quark and lepton masses

$m_i \diagdown \mu$	$m_c(m_c)$	2 GeV	$m_b(m_b)$	$m_t(m_t)$	1 TeV	$\Lambda_{\rm GUT}^{\tan\beta=10}$	$\Lambda_{\rm GUT}^{\tan\beta=50}$
m_u(MeV)	2.57	**2.2**	1.86	1.22	1.10	0.49	0.48
m_d(MeV)	5.85	**5.0**	4.22	2.76	2.50	0.70	0.51
m_s(MeV)	111	**95**	80	52	47	13	10
m_c(GeV)	**1.25**	1.07	0.901	0.590	0.532	0.236	0.237
m_b(GeV)	5.99	5.05	**4.20**	2.75	2.43	0.79	0.61
m_t(GeV)	*384.8*	*318.4*	*259.8*	**162.9**	150.7	92.2	94.7
m_e(MeV)	0.4955	$\sim$	0.4931	0.4853	0.4959	0.2838	0.206
m_μ(MeV)	104.474	$\sim$	103.995	102.467	104.688	59.903	43.502
m_τ(MeV)	1774.90	$\sim$	1767.08	1742.15	1779.74	1021.95	773.44

Table: The running masses of quarks and leptons as a function of momentum μ. The last two columns correspond to the running masses at $\Lambda_{\rm GUT} = 2 \times 10^{16}$ GeV assuming low energy MSSM spectrum with $\tan\beta = 10$ and 50.

CKM Matrix

- The unitary matrix V which appears in the charged current interactions enters in a variety of processes. A lot of information has been gained on the matrix elements of V. The general matrix can be written as

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

- V has a single un-removable phase for three families of quarks and leptons. (The phases (α, β) which appeared in the case of Majorana neutrinos can be removed by right–handed quark field redefinition.) The single un-removable phase in V allows for the violation of CP symmetry in the quark sector. Unlike in the leptonic sector, the quark mixing angles turn out to be small.

CKM Matrix

- This enables one to make a perturbative expansion of the mixing matrix a la Wolfenstein . The small parameter is taken to be $\lambda = |V_{us}|$ in terms of which one has

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2+\frac{1}{2}A\lambda^4(1-2(\rho+i\eta)) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^5).$$

- Here the exact correspondence is given by

$$s_{12} \equiv \lambda, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta).$$

Decays to measure CKM angles

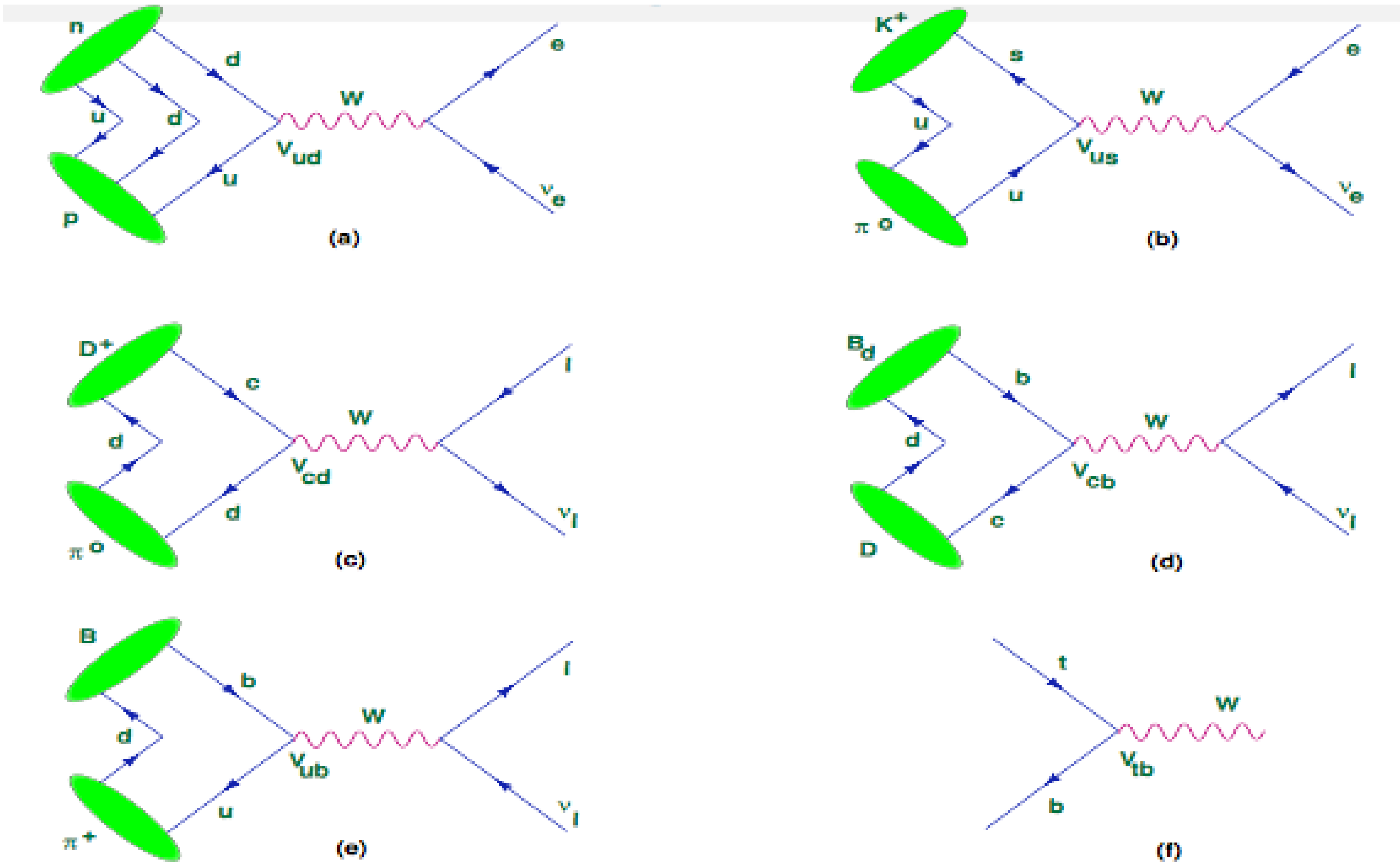


Figure: Processes determining $|V_{ij}|$.

Measuring CKM Angles

Matrix elements of V are determined usually via semileptonic decays of quarks. In Fig., we have displayed the dominant processes enabling determination of these elements.

- (a) is the diagram for nuclear beta decay, from which $|V_{ud}|$ has been extracted rather accurately :

$$|V_{ud}| = 0.97377 \pm 0.00027\,.$$

- (b) shows semileptonic K decay from which the Cabibbo angle $|V_{us}|$ can be extracted. The decays $K_L^0 \to \pi \ell \nu$ and $K^\pm \to \pi^0 \ell^\pm \nu$ ($\ell = e, \mu$) have been averaged to obtain for the product $|V_{us}| f_+(0) = 0.21668 \pm 0.00045$. Here $f_+(0)$ is the form factor associated with this semileptonic decay evaluated at $q^2 = 0$. Using $f_+(0) = 0.961 \pm 0.008$ (obtained from QCD calculations, which are in agreement with lattice QCD evaluations), one obtains

$$|V_{us}| = 0.2257 \pm 0.0021\,.$$

Measuring CKM angles

- $|V_{cd}|$ is extracted from $D \to K\ell\nu$ and $D \to \pi\ell\nu$ decays with assistance from lattice QCD for the computation of the relevant form factors.
- V_{cs} is determined from semileptonic D decays and from leptonic D_s decay ($D_s^+ \to \mu^+\nu$), combined with lattice calculation of the decay form factor f_{D_s}.
- Both $|V_{cd}|$ and $|V_{cs}|$ have rather large errors currently:

$$\begin{aligned} |V_{cd}| &= 0.230 \pm 0.011\,, \\ |V_{cs}| &= 0.957 \pm 0.010\,. \end{aligned}$$

- $|V_{cb}|$ is determined from both inclusive and exclusive decays of B hadrons into charm, yielding a value

$$|V_{cb}| = (41.6 \pm 0.6) \times 10^{-3}\,.$$

Measuring CKM angles

- $|V_{ub}|$ is determined from charmless B decays and gives

$$|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}\,.$$

- Elements $|V_{td}|$ and $|V_{ts}|$ cannot be currently determined, for a lack of top quark events, but can be inferred from B meson mixings where these elements appear through the box diagram. The result is

$$|V_{td}| = (7.4 \pm 0.8) \times 10^{-3}\,,$$

$$\frac{|V_{td}|}{|V_{ts}|} = 0.208 \pm 0.008\,.$$

CP Violation

- Charge conjugation (C) takes a particle to its antiparticle, Parity (spatial reflection) changes the helicity of the particle. Under CP, e_L^- will transform to e_R^+. Both C and P are broken symmetries in the SM, but the product CP is approximately conserved. Violation of CP has been seen only in weak interactions.

- The CKM mechanism predicts CP violation through a single complex phase that appears in the CKM matrix. Thus in the SM, various CP violating processes in K, B and other systems get correlated. So far such correlations have been consistent with CKM predictions, but more precise determinations in the B and D systems at the LHC may open up new physics possibilities.

CP Violation

- In the $K^0 - \overline{K^0}$ system, CP violation has been observed both in mixing and in direct decays. CP violation in mixing arises in the SM via the W–boson box diagram.

- The CP asymmetry in mixing is parametrized by ϵ, which is a measure of the mixing between the CP even and CP odd states $K^0_{1,2} = (K^0 \pm \overline{K^0})/\sqrt{2}$. It has been measured to be

$$|\epsilon| = (2.229 \pm 0.010) \times 10^{-3}\,.$$

CP Violation

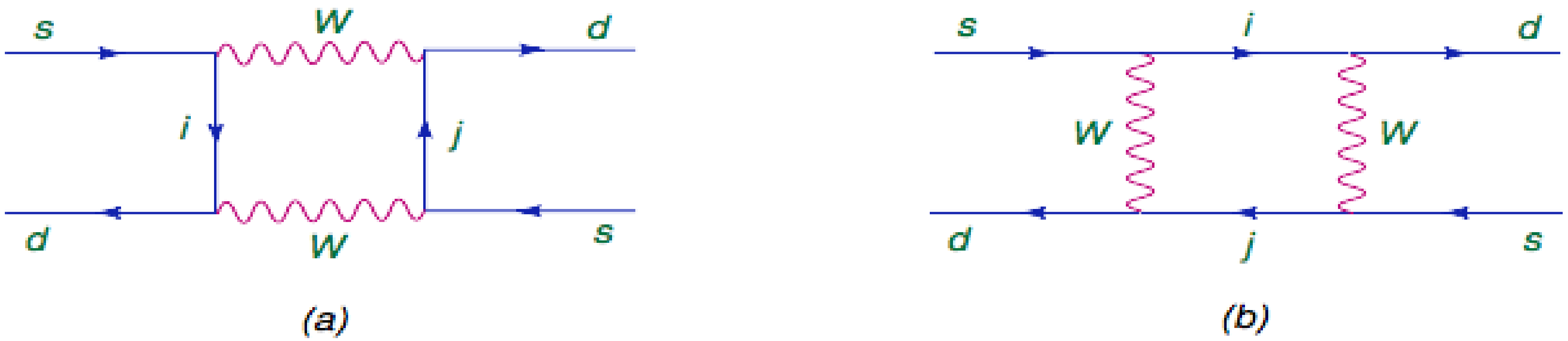


Figure: Box diagram inducing $K^0 - \overline{K^0}$ transition in the SM.

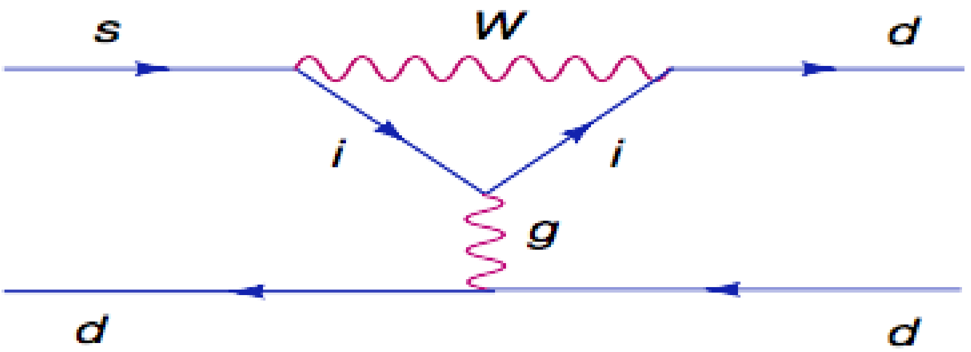


Figure: One loop penguin diagram that generates CP violation in direct $K \to \pi\pi$ decay.

CP Violation

- The measured value in in excellent agreement with expectations from the SM, and enables us to determine the single phase of the CKM matrix.
- The box diagram contribution to ϵ is given by

$$
\begin{aligned}
|\epsilon| &= \frac{G_F^2 f_k^2 m_K m_W^2}{12\sqrt{2}\pi^2 \Delta m_K} \hat{B}_K \{ \eta_c S(x_c) \mathrm{Im}[(V_{cs} V_{cd}^*)^2] \\
&+ \eta_t S(x_t) \mathrm{Im}[(V_{ts} V_{td}^*)^2] + 2\eta_{ct} S(x_c, x_t) \mathrm{Im}[V_{cs} V_{cd}^* V_{ts} V_{td}^*] \} \, .
\end{aligned}
$$

Here $S(x)$ and $S(x, y)$ are Inami–Lim functions with $x_{c,t} = m_{c,t}^2 / M_W^2$, and the η factors are QCD correction factors for the running of the effective $\Delta S = 2$ Hamiltonian from M_W to the hadron mass scale.

Unitarity Triangle

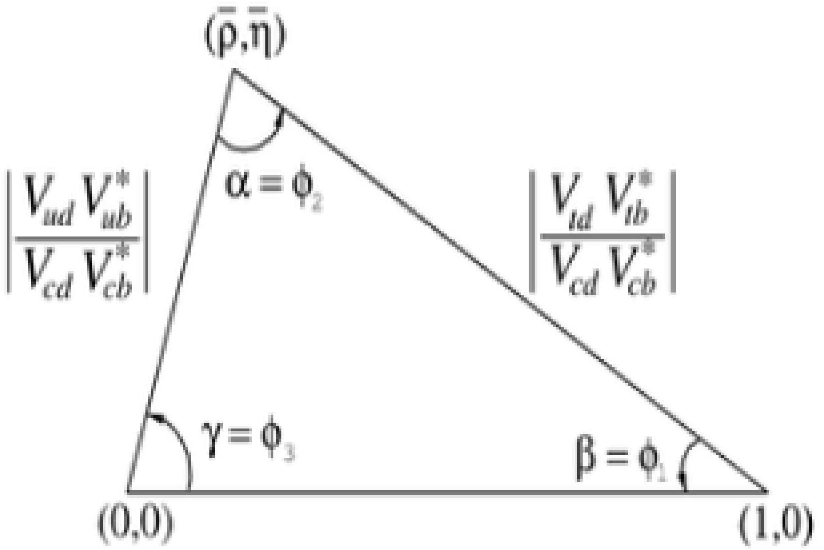


Figure: Unitarity triangle in the CKM model.

Unitarity Triangle

- CP violation in B meson system is now well established. Several CP violating quantities have been measured in B_d meson system, all of which show consistency with the CKM mixing matrix. Unitarity of the CKM matrix implies that $\sum_i V_{ij} V_{ik}^* = \delta_{jk}$ and $\sum_j V_{ij} V_{kj}^* = \delta_{ik}$.
- There are six vanishing combinations, which can be expressed as triangles in the complex plane. The areas of all of these triangles are the same. The most commonly used triangle arises from the relation

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \,.$$

- In the complex plane, the resulting triangle has sides of similar length (of order λ^3). This unitarity triangle relation is shown in Fig.

Unitarity Triangle

- The three interior angles (α, β, γ), also referred to as (ϕ_2, ϕ_1, ϕ_3), can be written in the CKM model as

$$\begin{aligned}
\alpha &= \arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \simeq \arg\left(-\frac{1-\rho-i\eta}{\rho+i\eta}\right) , \\
\beta &= \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \simeq \arg\left(\frac{1}{1-\rho-i\eta}\right) , \\
\gamma &= \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \simeq \arg(\rho+i\eta) .
\end{aligned}$$

One experimental test of the CKM mechanism is the measurement of $\alpha + \beta + \gamma = 180^0$.

Unitarity Triangle

- The angle β can be measured with the least theoretical uncertainty from the decay of $B_d \to J/\psi K_S$. It is found to be

$$\sin 2\beta = 0.68 \pm 0.03\,.$$

  This value is in in good agreement with the CKM prediction.

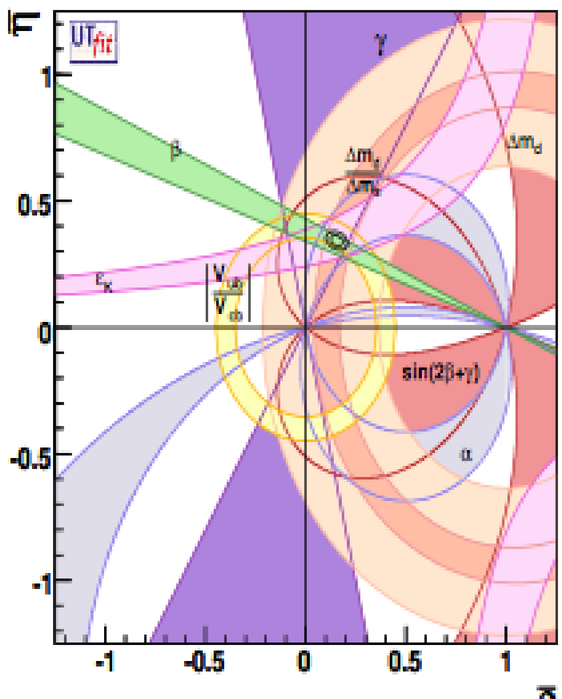
- The angle α is measured from decay modes where $b \to u\bar{u}d$ is dominant. Such decays includ $B \to \pi\pi$, $B \to \rho\rho$ and $B \to \pi\rho$. The value of α extracted is

$$\alpha = (88^{+6}_{-5})^0$$

- The angle γ does not depend on the top quark, and can in principle be measured from tree–level decays of B meson. Strong interaction uncertainties are rather large in decays such as $B^\pm \to D^0 K^\pm$. The current value of the angle γ is

$$\gamma = (77^{+30}_{-32})^0\,.$$

Unitarity Triangle



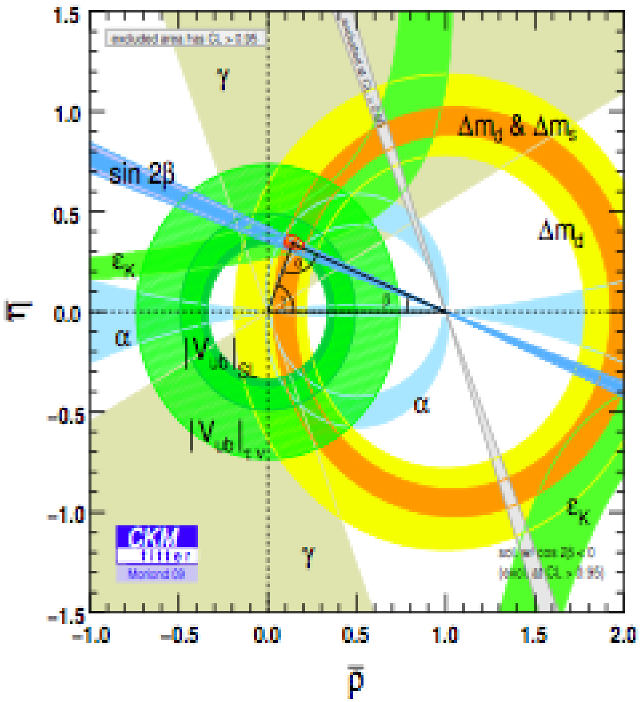


Figure: Global fit to the mixing and CP violation data from the UTFit collaboration (left panel) and the CKMFitter collaboration (right panel).

Unitarity Triangle

- The intersection of the various ellipses gives the best fit value for the Wolfenstein parameters $(\lambda,\ A,\ \rho,\ \eta)$, which are as follows :

$$\lambda = 0.2272 \pm 0.0010,\ \ A = 0.818^{+0.007}_{-0.017},$$
$$\rho = 0.221^{+0.064}_{-0.028},\ \ \eta = 0.340^{+0.017}_{-0.045}\ .$$

Theories of flavor should provide an understanding of these fundamental parameters.