Introduction to Gauge Symmetry, Spontaneous Symmetry Breaking and the Higgs Mechanism

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GIAN Course on Electrowak Symmetry Breaking, Flavor Physics and BSM

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The Greatest Equations Ever?

Survery by Physics World (2004)

$$e^{i\pi} + 1 = 0$$

(Euler's Equation)

"The most powerful mathematical statement ever written!"

"What could be more mystical than an imaginary number interacting with real numbers to produce nothing?"

Tied for 1st place:

$$\partial_\mu F^{\mu\nu} = J^\nu$$
$$\partial_\mu {}^*F^{\mu\nu} = 0$$

(Maxwell's Equations)

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$
$$\nabla \times \mathbf{B} = 4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Top Finishers

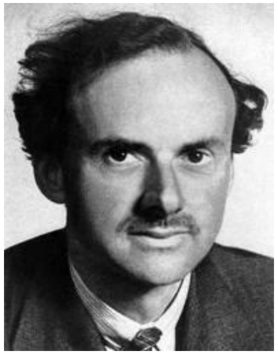
$$\begin{aligned}
\mathbf{F} &= m\mathbf{a} \\
a^2 &= b^2 + c^2 \\
H\Psi &= E\Psi \\
E &= m\,c^2 \\
S &= k\,\ln W \\
1 + 1 &= 2 \\
\delta S &= 0 \\
p &= h/\lambda \\
G_{\mu\nu} &= 8\,\pi\,G\,T_{\mu\nu} \\
C &= 2\pi r \\
i\gamma\cdot\partial\Psi &= m\,\Psi
\end{aligned}$$

Quantum Mechanics & Special Relativity

$$i\hbar\frac{\partial}{\partial t}|\psi,t\rangle = H|\psi,t\rangle$$

Schrodinger equation not Lorentz covariant

Dirac's Equation (1927)



$$H = c\boldsymbol{\alpha}\cdot\mathbf{P} + \boldsymbol{m}\boldsymbol{c}^{2}\,\boldsymbol{\beta}$$

$\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ anticommutning 4×4 matrices

Results in 4 states with energy $\{E, E, -E, -E\}$

$$E^2 = \mathbf{p}^2c^2 + m^2c^4$$

(Nobel Prize 1933, Schrodinger & Dirac)

Antiparticles

Dirac identified negative energy solutions as antiparticle

Predicted the existence of positron

Positron discovered by C. Anderson in 1933

(Nobel Prize 1936)

Every particle has an antiparticle with same mass, but with opposite charge

Clear mathematical understanding of antiparticles given by Stueckelberg & Feynman

Quantum Electrodynamics

Relativistic quantum theory of electrons, positrons and photons

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}_e (i D_\mu \gamma^\mu - m) \psi_e$$

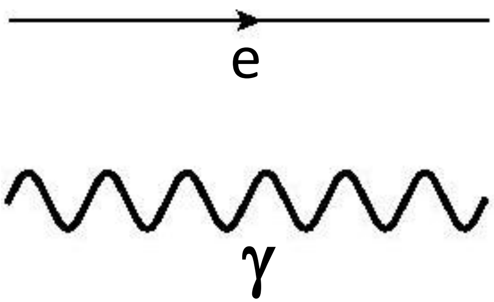
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad D_\mu = \partial_\mu - ie A_\mu$$

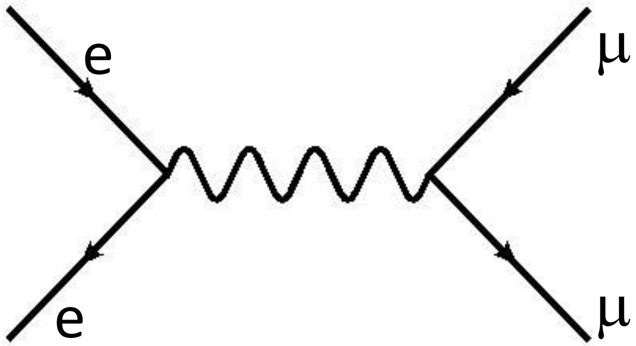
Lagrangian has the symmetry under space–time dependent phase rotations:

$$\psi_e \to e^{i\alpha(x)} \psi_e \qquad A_\mu \to A_\mu + \frac{1}{e} \partial_\mu \alpha$$

This gauge invariance keeps photon massless

Great Success of Quantum Electrodynamics





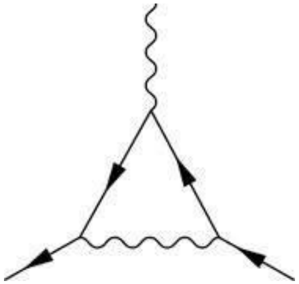
$e^+e^- \rightarrow \mu^+\mu^-$

Calculable theory of physical observables

Feynman, Schwinger, Tomonaga, Dyson – Nobel Prize 1965

Example

Anomalous magnetic moment of the electron:



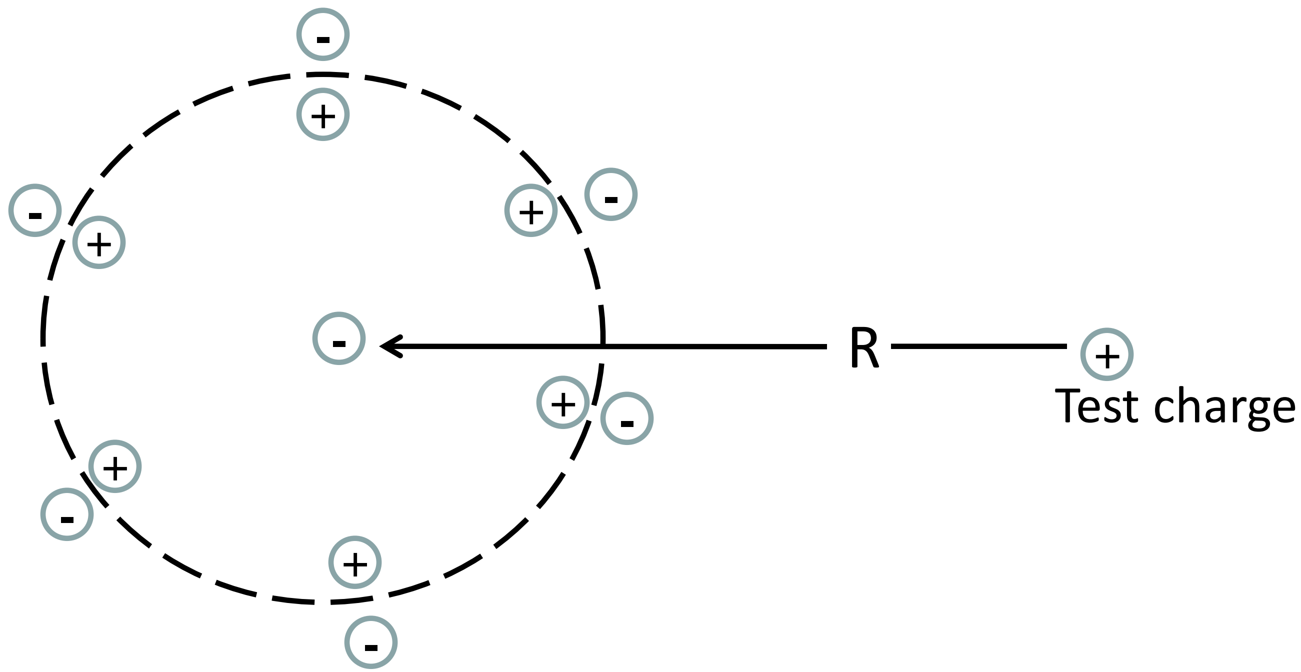
$$a_e^{\mathrm{exp}} = 0.00115965218073 \pm 0.00000000000028$$

$$a_e^{\mathrm{exp}} - a_e^{\mathrm{theory}} = -2.06(7.72) \times 10^{-12}$$

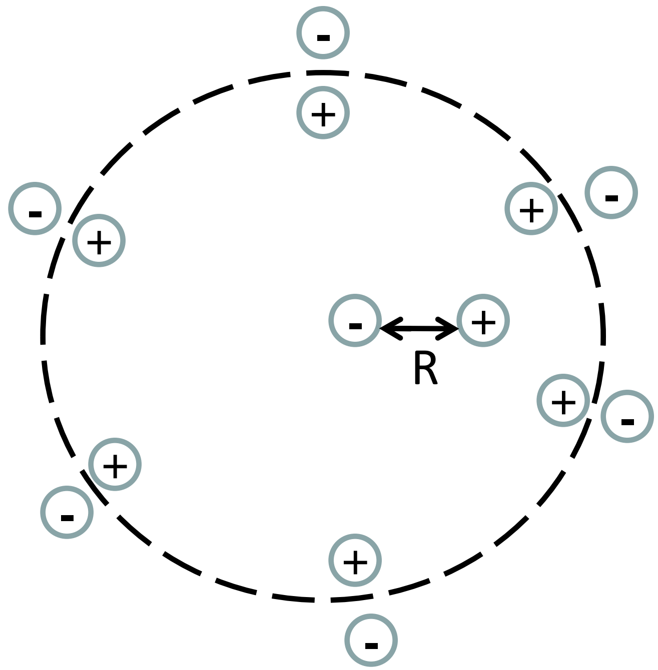
Best measurement of fine structure constant α

$$\alpha^{-1} = 137.035999085(12)(37)(33) \ [0.37\mathrm{ppb}]$$

Charge Screening in QED



With low energy positively charged probe, electron-positron cloud effectively reduces the measured charge



With high energy test charge, screening effect is reduced

Agrees with experiments

$$\alpha^{-1}(\mu = 91 \text{ GeV}) = 127.916 \pm 0.015$$

(LEP experiments at CERN, SLD at SLAC)

Strong Interactions

Unlike electromagnetism, strong force is short range $\sim$ Fermi

Yet, strong force has essentially the same structure as QED

Lagrangian for strong interactions:

$$\mathcal{L}_{\text{QCD}} = \overline{q}(iD_\mu\gamma^\mu - m)q - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

$$D_\mu = \partial_\mu - igT^a G^a_\mu \ , \quad G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f_{abc} G^b_\mu G^c_\nu$$

G^a_μ are the gluon – analogs of photon in strong interactions

$a = 1 - 8$ is the internal symmetry – called color

3 quarks and 8 gluons form $SU(3)$ symmetry

QCD Lagrangian invariant under $SU(3)$ gauge transformations:

$$q(x) \to e^{i\alpha_a(x)T^a} q(x), \qquad G^a_\mu \to G^a_\mu - \frac{1}{g}\partial_\mu \alpha_a - f_{abc}\alpha_b G^c_\mu$$

Generators of $SU(3)$ transformations:

$$[T_a, T_b] = i f_{abc} T^c$$

T_a: Gell–Mann matrices, generalize Pauli matrices to 3D

Matrix structure of QCD interactions ⇒

Gluons carry color – unlike photons, which are charge neutral

This feature makes QCD force short–range and *asymptocially free*

Gell-Mann Matrices

$$T^a = \frac{\lambda^a}{2}$$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

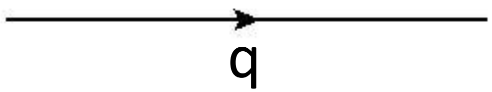
$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

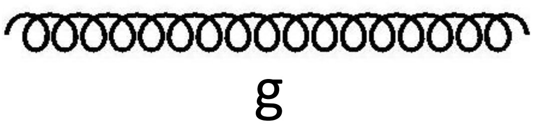
$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

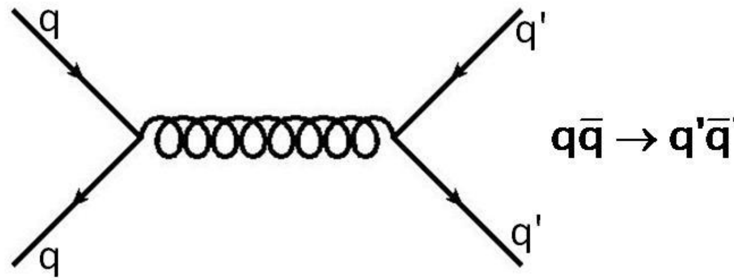
$$[T^a, T^b] = i f^{abc} T^c$$

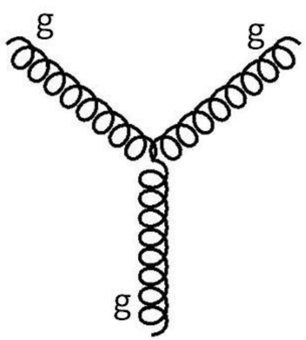
$$f^{123} = 1, \quad f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}$$

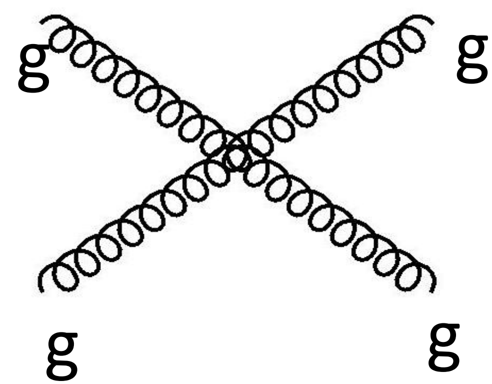
QCD Interactions



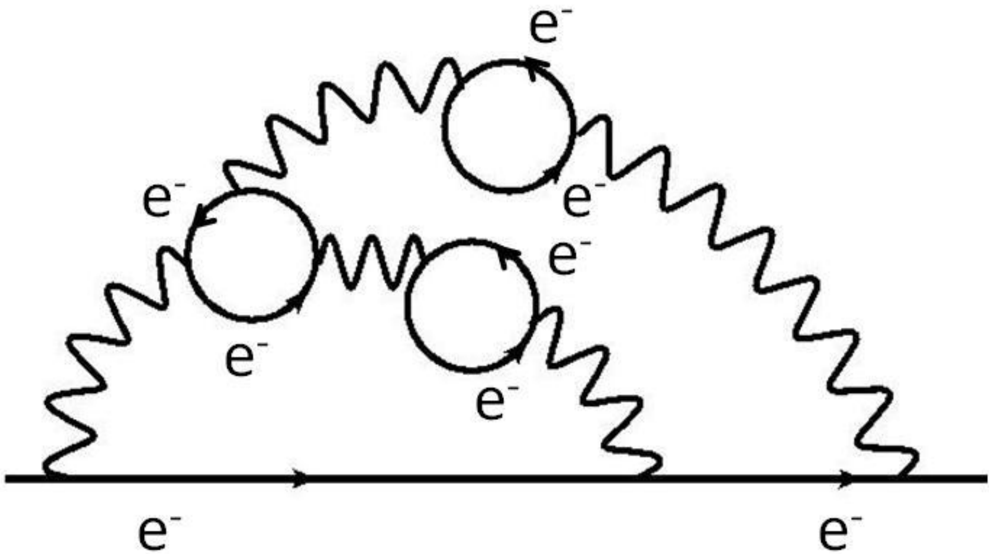






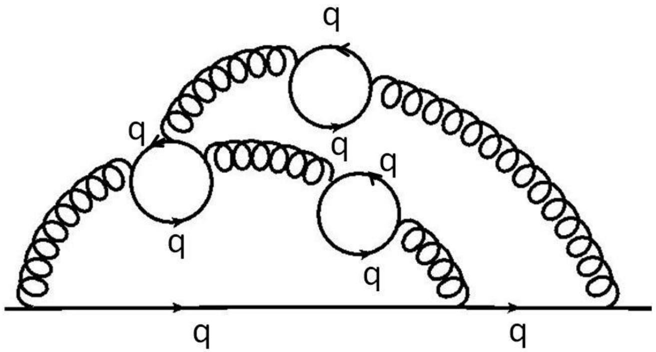


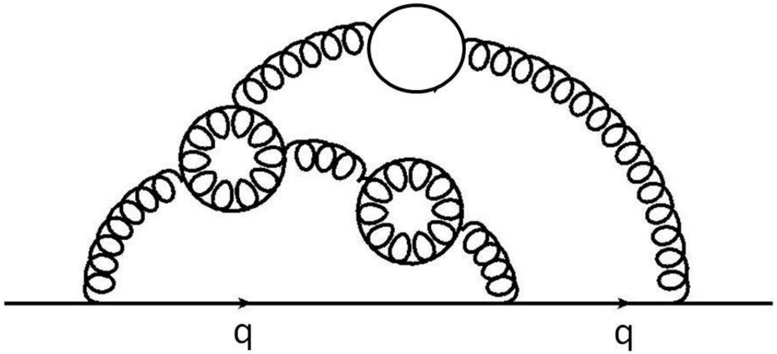
Effective charge of electron in QED



Causes charge screening

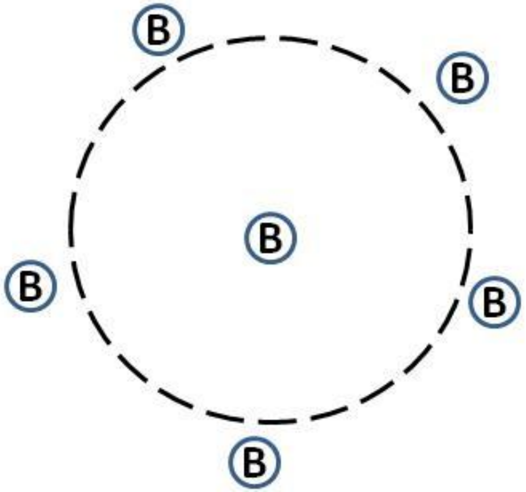
Effective color charge of quark in QCD





Causes color charge anti-screening

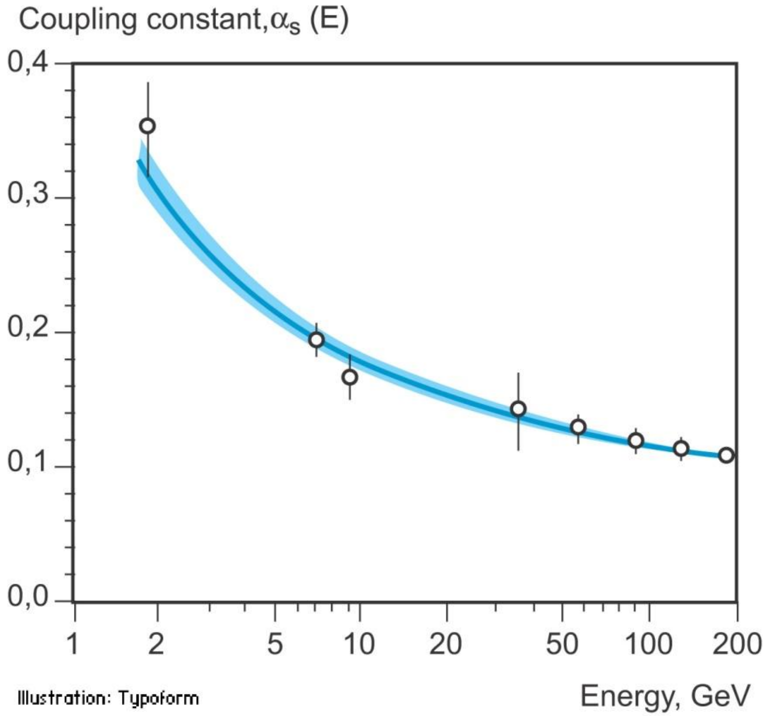
Color charge anti–screening



Quarks cannnot be separated from a hadron

Confinemet of quarks & short range of strong force

Effective color charge in QCD decreases with energy



Asymptotic freedom

Gross, Wilczek, Politzer
Nobel Prize, 2004

Weak Force

Similar structure as QCD, but based on $SU(2)$ internal symmetry

$$\mathcal{L}_{\text{weak}} = \overline{\psi}(iD_\mu\gamma^\mu - m)\psi - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu}_a$$

$$\psi(x) \to e^{i\alpha_a(x)\tau^a}\psi(x), \quad W^a_\mu \to W^a_\mu - \frac{1}{g_w}\partial_\mu\alpha_a - \epsilon_{abc}\alpha_b W^c_\mu, \quad [\tau_a, \tau_b] = i\epsilon_{abc}\tau^c$$

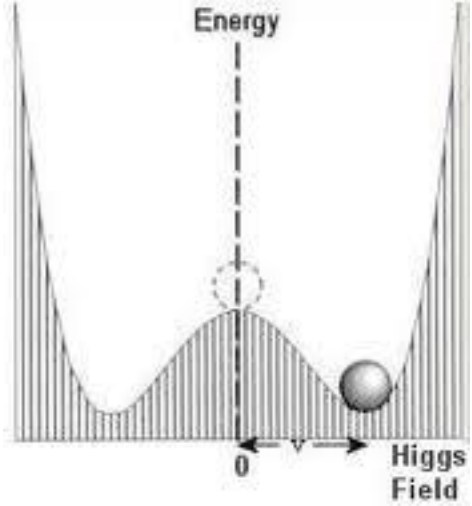
Weak force is short–range due to spontaneous symmetry breaking

Force carriers, $W^\pm$, Z^0 become massive via Higgs mechanism

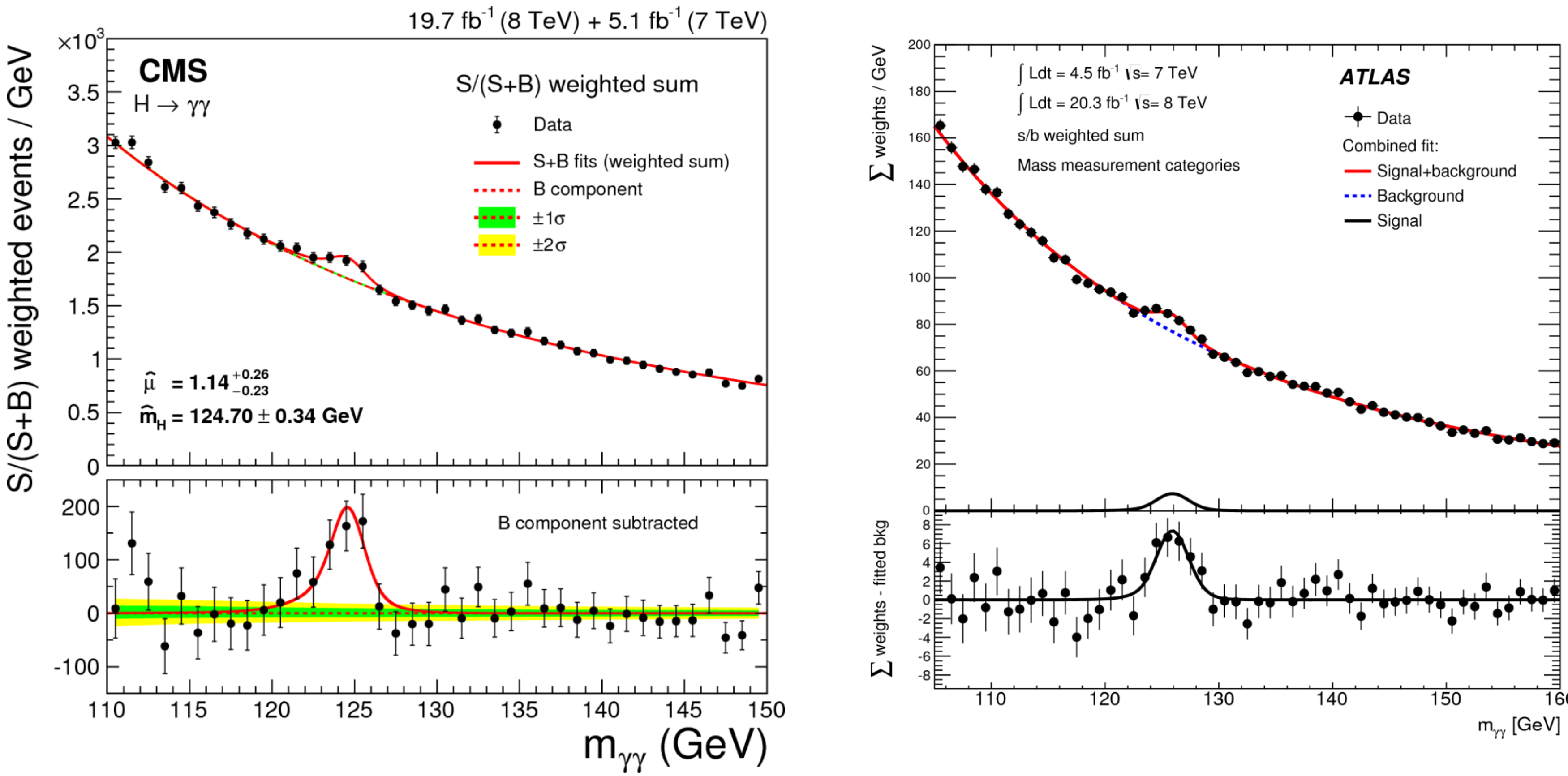
$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

At the minimum of potential $\langle\phi^0\rangle \neq 0$

Predicts a spin zero particle – the Higgs boson



Discovery of the Higgs boson



CMS and ATLAS Experimets at CERN, 2012

Different phases of the same theory?

Electromagnetism, Strong and Weak forces described by very similar mathematics

Electromagnetism: Coulomb phase: $U(1)$

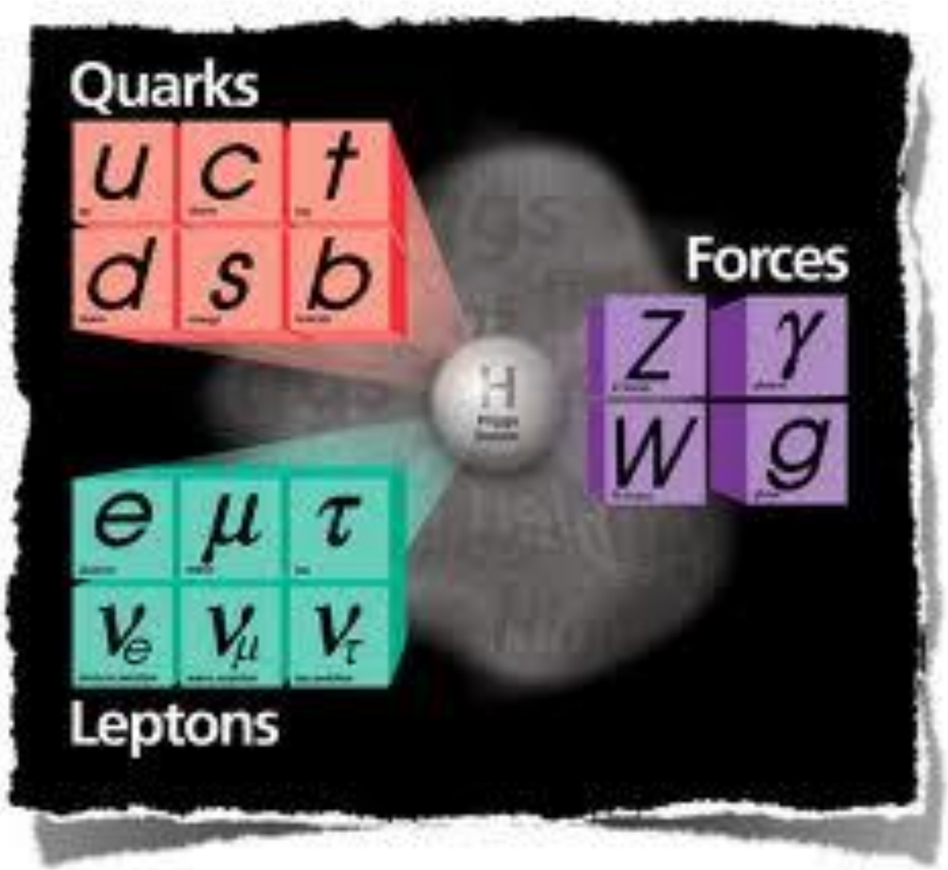
Strong Force: Confining phase: $SU(3)$

Weak Force: Higgs phase: $SU(2)$

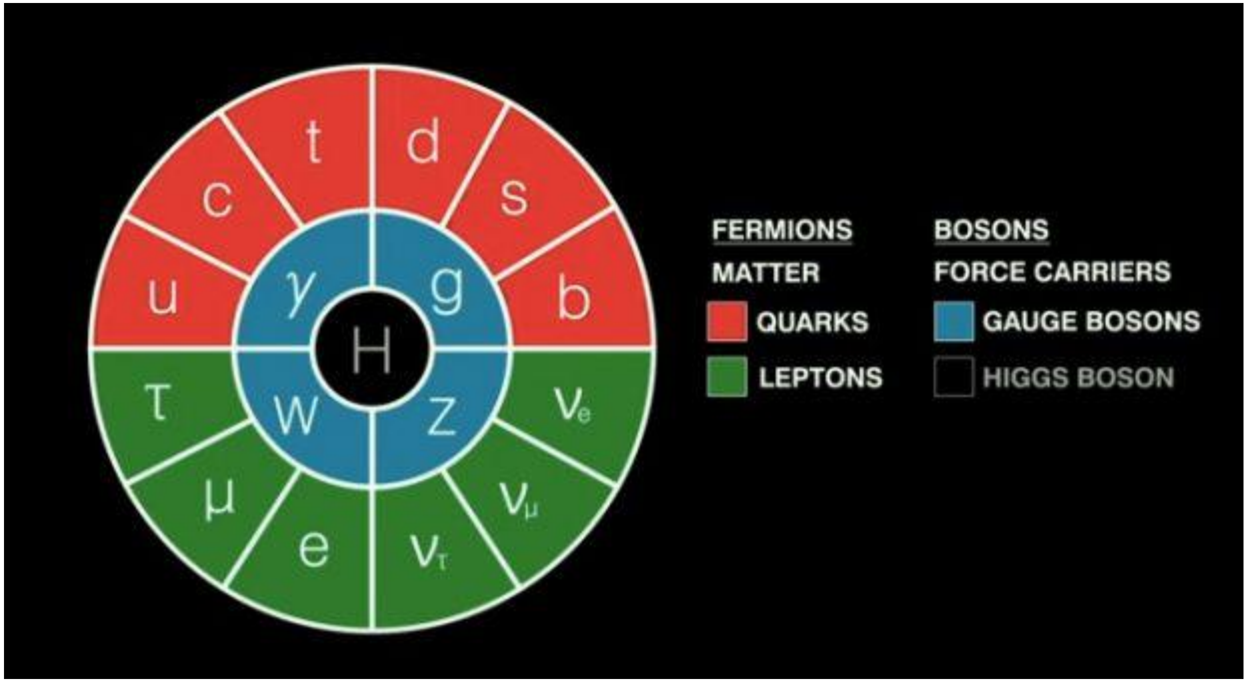
Possible to embed all three symmetry groups into a single symmetry

Grand Unified Symmetry: $SU(5)$ and $SO(10)$, ..

Particles and Force Carriers

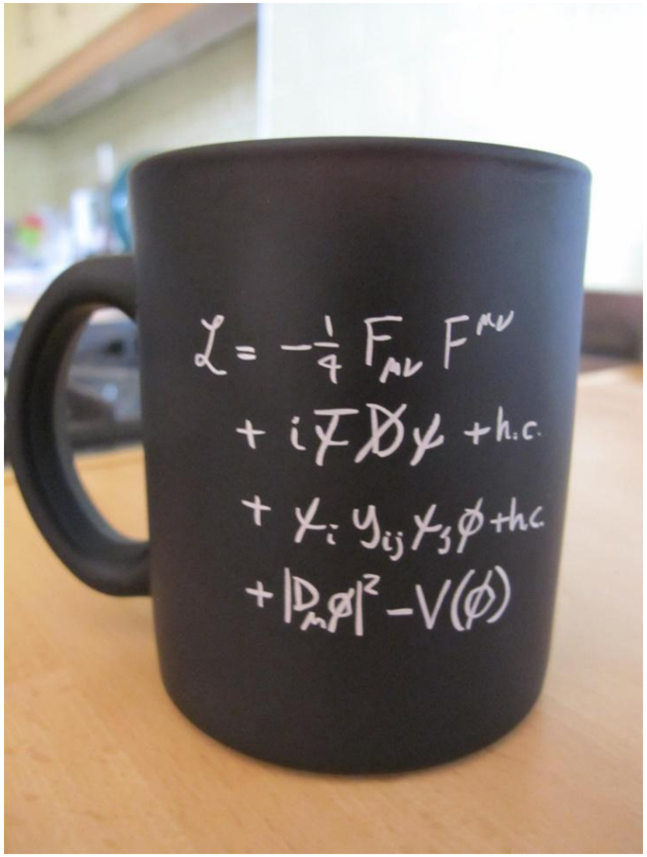


Higgs boson as a key player



The Standard Model Lagrangian

Based on $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry



The Lagrangian in Detail

Kinetic terms for the gauge bosons:

$$\mathcal{L}_{gauge} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu,$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu,$$

Fermion content:

	$Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	$L_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R
Hypercharge	$1/6$	$2/3$	$-1/3$	$-1/2$	-1
Color	triplet	triplet	triplet	singlet	singlet

The Lagrangian in Detail

Interaction of fermions with gauge bosons involves covariant derivative:

$$\mathcal{D}_\mu = \partial_\mu - ig' B_\mu Y - igW^a_\mu T^a - ig_s G^a_\mu t^a,$$

T^a and t^a: $SU(2))_L$ and $SU(3)_c$ generators; $T^a = \sigma^a/2$,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Coresspponding gauge transformations:

$$\mathrm{U}(1)_Y: \qquad \psi \to \exp[i\lambda_Y(x)Y]\psi, \qquad B_\mu \to B_\mu + \frac{1}{g'}\partial_\mu \lambda_Y(x)$$

$$\mathrm{SU}(2)_L: \qquad \psi \to \exp[i\lambda^a_L(x)T^a]\psi, \qquad W^a_\mu \to W^a_\mu + \frac{1}{g}\partial_\mu \lambda^a_L(x) + \epsilon^{abc}W^b_\mu \lambda^c_L(x)$$

$$\mathrm{SU}(3)_c: \qquad \psi \to \exp[i\lambda^a_c(x)t^a]\psi, \qquad G^a_\mu \to G^a_\mu + \frac{1}{g_s}\partial_\mu \lambda^a_c(x) + f^{abc}G^b_\mu \lambda^c_c(x).$$

The Standard Model Higgs Mechanism

A complex scalar field Φ transforming as $(1, 2, 1/2)$ is employed

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix},$$

Higgs field interactions:

$$\mathcal{L}_\Phi = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - V(\Phi) + \mathcal{L}_{\text{Yukawa}}$$

Higgs potential:

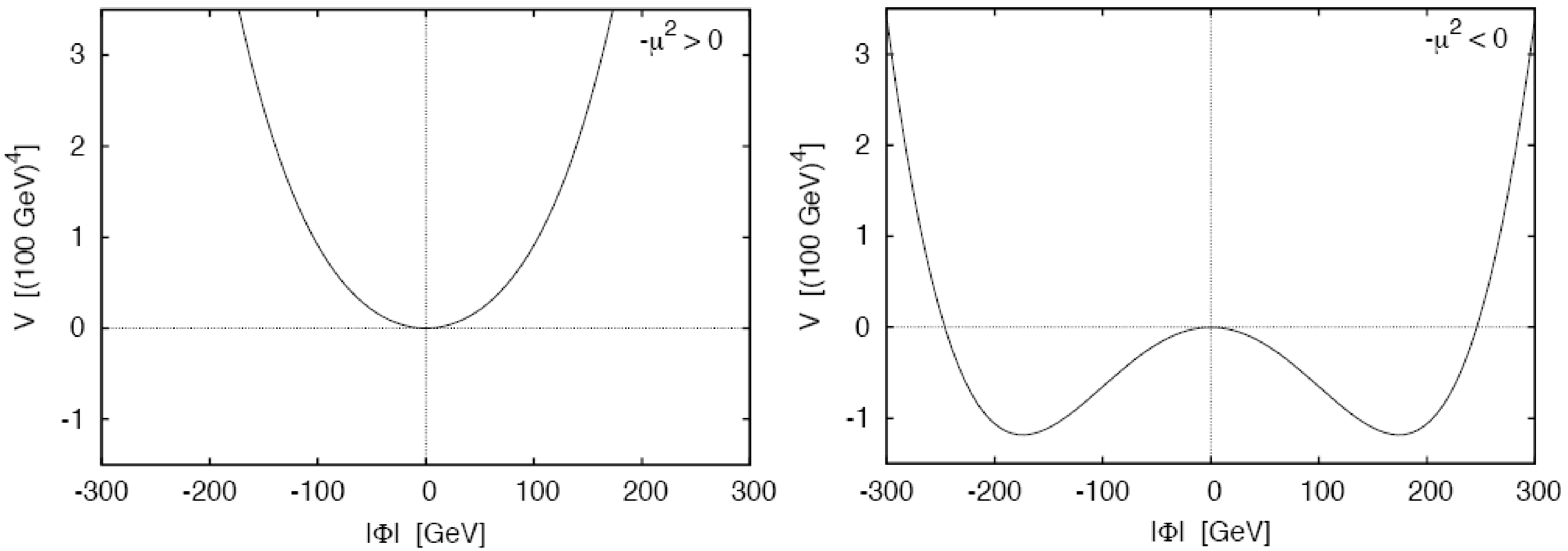
$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2.$$

$\lambda > 0$ needed for a bounded potential

$-\mu^2 > 0$ leads to unbroken $SU(2)_L \times U(1)_Y$ symmetry

$-\mu^2 < 0$ leads to spontaneous breaking of $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$

The Higgs Potential



$$-\mu^2 > 0 \qquad\qquad -\mu^2 < 0$$

$|\Phi| = 0$ is the minimum — At minimum $|\Phi| \neq 0$

$|\Phi| = v/\sqrt{2} = 174$ GeV, $\lambda = 0.129$, $|\mu^2| = (88.4\,\text{GeV})^2$
(for $m_h = 125$ GeV)

Logan TASI Lectures (2013)

Higgs Potential Analysis

At the minimum for $-\mu^2 < 0$ we have

$$\Phi^\dagger \Phi = \frac{1}{2}\left(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right) = \frac{\mu^2}{2\lambda}$$

We can choose the orientation of ϕ_i fields such that only $\langle \phi_3 \rangle \neq 0$

$$\langle \phi_3 \rangle \equiv v = \sqrt{\frac{\mu^2}{\lambda}}, \qquad \langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_4 \rangle = 0.$$

Also define a shifted field $\phi_3 = v + h$ with $\langle h \rangle = 0$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ v + h + i\phi_4 \end{pmatrix}$$

$$V = -\frac{\mu^2}{2}\left(\phi_1^2 + \phi_2^2 + (h+v)^2 + \phi_4^2\right) + \frac{\lambda}{4}\left(\phi_1^2 + \phi_2^2 + (h+v)^2 + \phi_4^2\right)^2$$

Higgs Potential Analysis (cont.)

Collecting qudratic terms, we find

$$m^2_{\phi_1,\phi_2,\phi_4} = 0, \quad m_h^2 = 2\lambda v^2$$

The zero modes can be seen by a gauge transformation:

$$\Phi = \frac{1}{\sqrt{2}} \exp\left(\frac{i\xi^a \sigma^a}{v}\right) \begin{pmatrix} 0 \\ v+h \end{pmatrix}.$$

Here h and ξ^a are fields, σ^a are Pauli matrices

Consider a gauge transformation

$$\mathrm{U}(1)_Y : \qquad \Phi \to \exp\left(i\lambda_Y(x) \cdot \frac{1}{2}\right) \Phi,$$

$$\mathrm{SU}(2)_L : \qquad \Phi \to \exp\left(i\lambda_L^a(x) \frac{\sigma^a}{2}\right) \Phi.$$

Higgs Potential Analysis (cont.)

By choosing $\lambda_L^a(x) = -2\xi^a/v$ at each point in spacetime, Φ becomes

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

This is the unitary gauge. Note that ϕ_1, ϕ_2, ϕ_4 have disappeared. These are unphysical degrees

These degrees are absorbed by the W^+, W^- and Z^0 gauge bosons which have become massive

$SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken down to $U(1)_{em}$

Of the 3+1 generators of $SU(2)_L \times U(1)_Y$, all except one, correp-sonding to unbroken $U(1)_{em}$ have acquired mass

Gauge boson mass generation

Examine the gauge kinetic term for the Higgs field

$$\mathcal{L} \supset (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi)$$

$$\mathcal{D}_\mu = \partial_\mu - i\frac{g'}{2}B_\mu - i\frac{g}{2}W_\mu^a \sigma^a = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{i}{2}g(W_\mu^1 - iW_\mu^2)(v+h) \\ \partial_\mu h + \frac{i}{2}(gW_\mu^3 - g'B_\mu)(v+h) \end{pmatrix}$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) =$$

$$\frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{8}g^2(v+h)^2(W_\mu^1 - iW_\mu^2)(W^{1\mu} + iW^{2\mu}) + \frac{1}{8}(v+h)^2\left(-g'B_\mu + gW_\mu^3\right)^2$$

Define the following gauge boson mass eigenstate fields:

$$W_\mu^+ = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}, \qquad W_\mu^- = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}.$$

$$Z_\mu = \frac{(-g'B_\mu + gW_{3\mu})}{\sqrt{g^2 + g'^2}}, \qquad A_\mu = \frac{(gB_\mu + g'W_{3\mu})}{\sqrt{g^2 + g'^2}}$$

Photon A_μ does not appear; $W^\pm$ and Z acquire masses!

Gauge boson mass generation (cont.)

In terms of mass eigenstates

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{4} g^2 (v+h)^2 W_\mu^+ W^{\mu -} + \frac{1}{8}(g^2 + g'^2)(v+h)^2 Z_\mu Z^\mu$$

We can read off the mass terms:

$$M_W^2 = \frac{g^2 v^2}{4}, \qquad M_Z^2 = \frac{(g^2 + g'^2)\, v^2}{4}$$

Interactions of the Higgs boson with $W^\pm$ and Z are fixed:

$$\mathcal{L}_{Wh} = \frac{g^2 v}{2} h W_\mu W^\mu + \frac{g^2}{4} h^2 W_\mu W^\mu$$

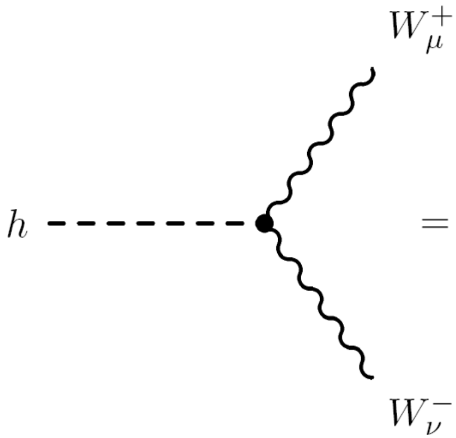
$$\mathcal{L}_{Zh} = \frac{(g^2 + g'^2) v}{2} h Z_\mu Z^\mu + \frac{g^2 + g'^2}{4} h^2 Z_\mu Z^\mu$$

These lead to the following Feynman amplitudes:

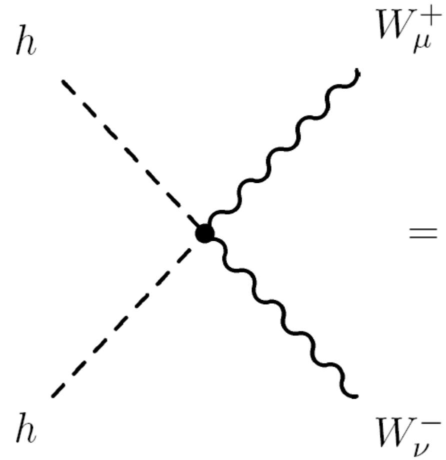
W boson interaction with Higgs

$$hW_\mu^+ W_\nu^- : \qquad i\frac{g^2 v}{2} g_{\mu\nu} = igM_W g_{\mu\nu} = 2i\frac{M_W^2}{v} g_{\mu\nu},$$

$$hhW_\mu^+ W_\nu^- : \qquad i\frac{g^2}{4} \times 2!\, g_{\mu\nu} = 2i\frac{M_W^2}{v^2} g_{\mu\nu}$$



$$= i\frac{g^2 v}{2} g_{\mu\nu} = 2i\frac{M_W^2}{v} g_{\mu\nu}$$

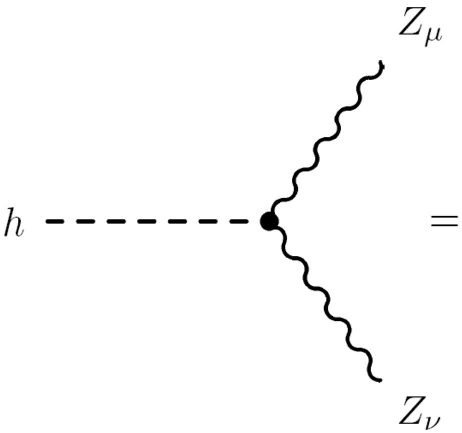


$$= i\frac{g^2}{4} \cdot 2g_{\mu\nu} = 2i\frac{M_W^2}{v^2} g_{\mu\nu}$$

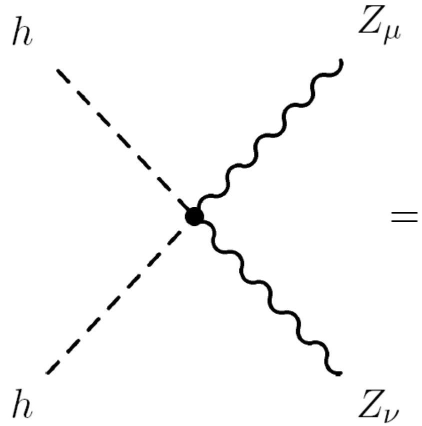
Z boson interaction with Higgs

$$hZ_\mu Z_\nu : \qquad i\frac{(g^2+g'^2)v}{4}\times 2!\, g_{\mu\nu} = i\sqrt{g^2+g'^2}\,M_Z g_{\mu\nu} = 2i\frac{M_Z^2}{v}g_{\mu\nu},$$

$$hhZ_\mu Z_\nu : \qquad i\frac{(g^2+g'^2)}{8}\times 2!\times 2!\, g_{\mu\nu} = 2i\frac{M_Z^2}{v^2}g_{\mu\nu}$$



$$= i\frac{(g^2+g'^2)v}{4}\cdot 2g_{\mu\nu}$$



$$= i\frac{(g^2+g'^2)}{8}\cdot 2\cdot 2g_{\mu\nu} = 2i\frac{M_Z^2}{v^2}g_{\mu\nu}$$

Electroweak Lagrangian

The covariant derivative can be rewritten in terms of W^μ, Z^μ, A^μ

Define $c_W = \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \quad s_W = \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$

$$\begin{aligned} B_\mu &= c_W A_\mu - s_W Z_\mu, \\ W_\mu^3 &= s_W A_\mu + c_W Z_\mu, \\ W^1 T^1 + W^2 T^2 &= \frac{1}{\sqrt{2}}(W^+T^+ + W^-T^-) \end{aligned}$$

$T^\pm = \sigma^\pm$: Raising and lowering operators for $SU(2)$ doublets

$$\mathcal{D}_\mu = \partial_\mu - i g_s G_\mu^a t^a - i\frac{g}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - i Z_\mu\left(g c_W T^3 - g' s_W Y\right) - i A_\mu\left(g s_W T^3 + g' c_W Y\right)$$

Photon couples to electric charge Q

Electroweak Lagrangian (cont.)

Thus we identify:

$$\left(g s_W T^3 + g' c_W Y\right) = \frac{g g'}{\sqrt{g^2 + g'^2}} \left(T^3 + Y\right) \equiv eQ$$

By convention we define

$$e = \frac{g g'}{\sqrt{g^2 + g'^2}} = g s_W = g' c_W, \qquad Q = T^3 + Y$$

Photon coupling is simply $\mathcal{D}_\mu \supset -ieA_\mu Q$

Z coupling:

$$\left(g c_W T^3 - g' s_W Y\right) = \frac{g^2 + g'^2}{\sqrt{g^2 + g'^2}} T^3 - \frac{g'^2}{\sqrt{g^2 + g'^2}} Q = \sqrt{g^2 + g'^2} \left(T^3 - s_W^2 Q\right)$$

Final Electroweak Lagrangian

Collecting all terms, we have the final Lagrangian:

$$\mathcal{D}_\mu = \partial_\mu - i g_s G^a_\mu t^a - i\frac{g}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) - i\frac{e}{s_W c_W} Z_\mu \left(T^3 - s^2_W Q\right) - ieA_\mu Q$$

Interactions of the fermions with gauge bosons follow from:

$$\mathcal{L} \supset \bar{\psi}_L i\mathcal{D}_\mu \gamma^\mu \psi_L + \bar{\psi}_R i\mathcal{D}_\mu \gamma^\mu \psi_R$$

Note: $\mathcal{D}_\mu$ different for left-handed and right-handed fermions

Note: For right-handed fermions, $W^+_\mu + W^-_\mu T^-$ term absent

Note: For leptons the $G^a_\mu t^a$ term is absent

Higgs boson self-couplings

From the Higgs potential

$$\mathcal{L}_V = -V(\Phi) = \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

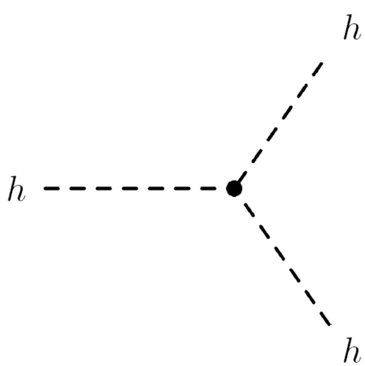
and

$$\Phi^\dagger \Phi = \tfrac{1}{2}(h+v)^2, \; \mu^2 = \lambda v^2,$$

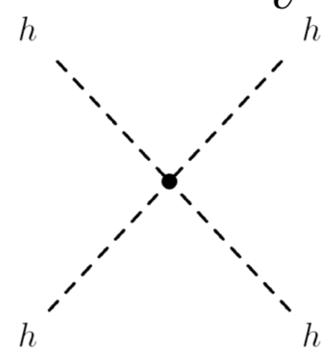
$$\mathcal{L}_V = -\lambda v^2 h^2 - \lambda v h^3 - \frac{\lambda}{4} h^4 + \text{const.}$$

$$hhh: \quad -i\lambda v \times 3! = -6i\lambda v = -3i\frac{m_h^2}{v},$$

$$hhhh: \quad -i\frac{\lambda}{4} \times 4! = -6i\lambda = -3i\frac{m_h^2}{v^2}$$



$$= -i\lambda v \cdot 3! = -6i\lambda v = -3i\frac{m_h^2}{v}$$



$$= -i\frac{\lambda}{4} \cdot 4! = -6i\lambda = -3i\frac{m_h^2}{v^2}$$

Fermion mass generation

The chiral nature of fermions forbids bare masses for fermions

Left-handed fermions are doublets of $SU(2)$ while right-handed ones are singlets

The single Higgs field, which is a doublet of $SU(2)$ can induce masses for all fermions upon spontaneous symmetry breaking

Leptonic Yukawa couplings:

$$\mathcal{L}_{\text{Yukawa}} \supset -\left[y_e \bar{e}_R \Phi^\dagger L_L + y_e^* \bar{L}_L \Phi e_R\right]$$

Substituting $$\Phi = \begin{pmatrix} 0 \\ (v+h)/\sqrt{2} \end{pmatrix}$$

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &\supset -y_e \frac{1}{\sqrt{2}} \left[(v+h)\bar{e}_R e_L + (v+h)\bar{e}_L e_R\right] \\
&= -\frac{y_e}{\sqrt{2}}(v+h)\bar{e}e \\
&= -\left(\frac{y_e v}{\sqrt{2}}\right)\bar{e}e - \frac{y_e}{\sqrt{2}} h\bar{e}e.
\end{aligned}$$

Fermion mass generation (cont.)

Electron mass: $$m_e = \frac{y_e v}{\sqrt{2}}$$

Electron Higgs coupling: $$h\bar{e}e : \quad \frac{-iy_e}{\sqrt{2}} = \frac{-im_e}{v}$$

$$\frac{y_e}{\sqrt{2}} = \frac{m_e}{v} = \frac{0.511\ \text{MeV}}{246\ \text{GeV}} \simeq 2.1 \times 10^{-6}$$

Similarly, for τ lepton:

$$\frac{y_\tau}{\sqrt{2}} = \frac{m_\tau}{v} = \frac{1.78\ \text{GeV}}{246\ \text{GeV}} \simeq 7.2 \times 10^{-3}$$

Higgs boson couples to fermions proportional to their masses

Largest coupling of Higgs is to the top quark

$$\frac{y_t}{\sqrt{2}} = \frac{m_t}{v} = \frac{174\ \text{GeV}}{246\ \text{GeV}} \simeq 0.7$$

Yukawa couplings of quarks

Yukawa coupling to down quarks:

$$\mathcal{L}_{\text{Yukawa}} \supset -\left[y_d \bar{d}_R \Phi^\dagger Q_L + y_d^* \bar{Q}_L \Phi d_R\right]$$

$$\mathcal{L}_{\text{Yukawa}} \supset -\left(\frac{y_d v}{\sqrt{2}}\right) \bar{d}d - \frac{y_d}{\sqrt{2}} h\bar{d}d$$

$$m_d = \frac{Y_d v}{\sqrt{2}}$$

Up quarks can couple to $\tilde{\Phi}$ field:

$$\tilde{\Phi} \equiv i\sigma^2 \Phi^* = i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$

$$\mathcal{L}_{\text{Yukawa}} \supset -\left[y_u \bar{u}_R \tilde{\Phi}^\dagger Q_L + y_u^* \bar{Q}_L \tilde{\Phi} u_R\right]$$

Yukawa couplings of quarks (cont.)

Up quark masses:

$$\mathcal{L}_{\text{Yukawa}} \supset -\left(\frac{y_u v}{\sqrt{2}}\right)\bar{u}u - \frac{y_u}{\sqrt{2}} h\bar{u}u$$

There are three generations of quarks (and leptons)

$$Q_{Lj}, \quad u_{Rj}, \quad d_{Rj}, \qquad j = 1, 2, 3$$

$$\mathcal{L}^q_{\text{Yukawa}} = -\sum_{i=1}^{3}\sum_{j=1}^{3}\left[y^u_{ij}\bar{u}_{Ri}\tilde{\Phi}^\dagger Q_{Lj} + y^d_{ij}\bar{d}_{Ri}\Phi^\dagger Q_{Lj}\right] + \text{h.c.}$$

$$\mathcal{L}^q_{\text{Yukawa}} \supset -(\bar{u}_1, \bar{u}_2, \bar{u}_3)_R\, \mathcal{M}^u \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L - \left(\bar{d}_1, \bar{d}_2, \bar{d}_3\right)_R \mathcal{M}^d \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L + \text{h.c.}$$

$$\mathcal{M}^u_{ij} = \frac{v}{\sqrt{2}} y^u_{ij}, \qquad \mathcal{M}^d_{ij} = \frac{v}{\sqrt{2}} y^d_{ij}$$

These mass matrices should be diagonalized by unitary rotations

Yukawa couplings of quarks (cont.)

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R}, \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R},$$

Choose the unitary matrices $U_{L,R}$ and $D_{L,R}$ so that

$$U_R^{-1} \mathcal{M}^u U_L = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \qquad D_R^{-1} \mathcal{M}^d D_L = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

The Yukawa matrices are simultaneously diagonalized

$$y^u_{ij} = \frac{\sqrt{2}}{v} \mathcal{M}^u_{ij} \text{ and } y^d_{ij} = \frac{\sqrt{2}}{v} \mathcal{M}^d_{ij}$$

The Higgs couplings to fermions are diagonal and real!

What about $W^\pm$, Z and A couplings?

CKM angles and absence of FCNC

A unitary matrix (CKM matrix) becomes physical in charged current

$$J_L^{+\mu} = (\bar{u}_1, \bar{u}_2, \bar{u}_3)_L \, \gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L \, U_L^\dagger \gamma^\mu D_L \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L \, \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

Here $V = U_L^\dagger D_L$ is unitary: $VV^\dagger = V^\dagger V = 1$

In neutral current Z and A couplings:

$$(\bar{u}_1, \bar{u}_2, \bar{u}_3)_L \, \gamma^\mu \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L \, U_L^\dagger \gamma^\mu U_L \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L \, \gamma^\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$$

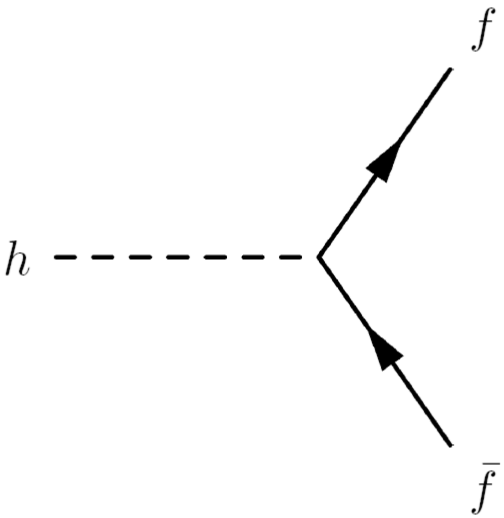
There is no trace of the unitary matrices

Thus, there are no tree-level flavor changing couplings of Z and A

This is the Glashow-Iliopoulose-Miani (GIM) mechanism

Higgs Decays

One dominant mode is the decay $h \to f\bar{f}$



$$i\mathcal{M} = \bar{u}_f \left(\frac{-im_f}{v} \right) v_{\bar{f}}$$

$$\Gamma(h \to f\bar{f}) = \frac{N_c}{8\pi} \frac{m_f^2}{v^2} m_h \left[1 - \frac{4m_f^2}{m_h^2} \right]^{3/2}$$

$N_c = 3$ for quarks and $N_c = 1$ for leptons

$h \to b\bar{b}$ is the dominant mode in this category

Higgs Decays to vector bosons

$h \to W^+W^-$ and $h \to ZZ$ are kinematically forbidden

$h \to W^+W^{-*}$ and $h \to ZZ^*$ are allowed

$h \to \gamma\gamma$ is a discovery channel

If $h \to W^+W^-$ were allowed (i.e., for a heavy Higgs),

$$i\mathcal{M} = 2i\frac{M_W^2}{v} g^{\mu\nu} \epsilon_\mu^*(W^+)\epsilon_\nu(W^-)$$

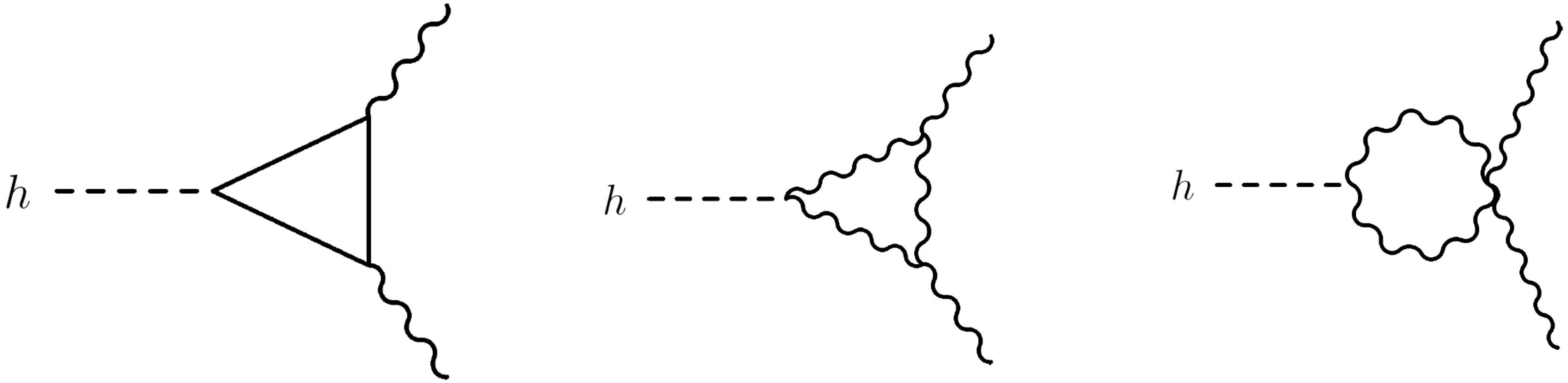
Decay width:

$$\Gamma(h \to W^+W^-) = \frac{1}{16\pi}\left(\frac{M_W^4}{v^2}\right)\frac{m_h^3}{M_W^4}\sqrt{1 - x_W}\left(1 - x_W + \frac{3}{4}x_W^2\right)$$

$$x_W \equiv 4M_W^2/m_h^2$$

Higgs Decays to vector bosons

$h \to \gamma\gamma$ and $h \to gg$ occurs through loops



$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2}{256\pi^3}\frac{m_h^3}{v^2}\left|\sum_i N_{ci} Q_i^2 F_i(\tau_i)\right|^2 ,$$

$$\begin{aligned} F_1(\tau) &= 2 + 3\tau + 3\tau(2-\tau) f(\tau), \\ F_{1/2}(\tau) &= -2\tau\left[1 + (1-\tau) f(\tau)\right] \end{aligned}$$

$$\tau = 4m_i^2/m_h^2$$

$$f(\tau) = \begin{cases} \left[\sin^{-1}(1/\sqrt{\tau})\right]^2 & \text{for } m_h < 2m_i, \\ -\frac{1}{4}\left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right) - i\pi\right]^2 & \text{for } m_h > 2m_i. \end{cases}$$

Higgs Decay Branching Ratios

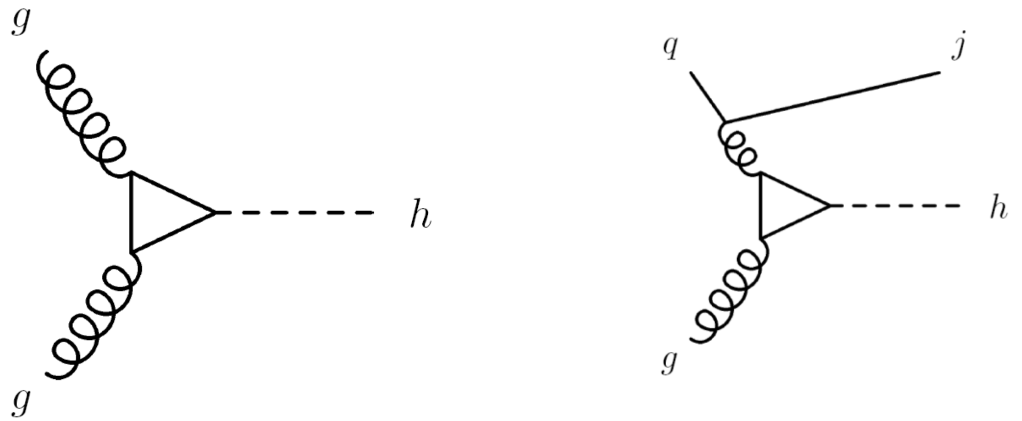
Decay mode	BR
$b\bar{b}$	58%
WW^*	22%
gg	8.6%
$\tau\tau$	6.3%
$c\bar{c}$	2.9%
ZZ^*	2.6%
$\gamma\gamma$	0.23%
$Z\gamma$	0.15%
$\mu\mu$	0.022%
Γ_{tot}	4.1 MeV

$\gamma\gamma$ and ZZ^* are discovery channels

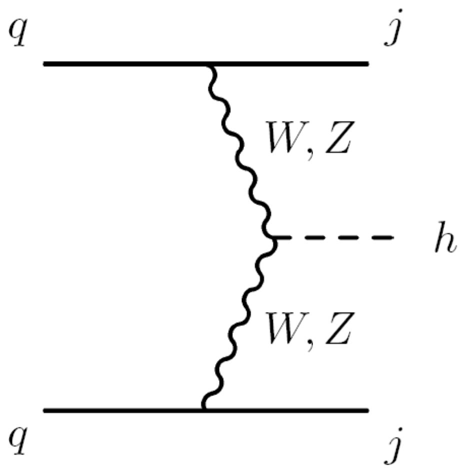
WW^* and $\tau\tau$ have been measured

Hints for $b\bar{b}$ seen

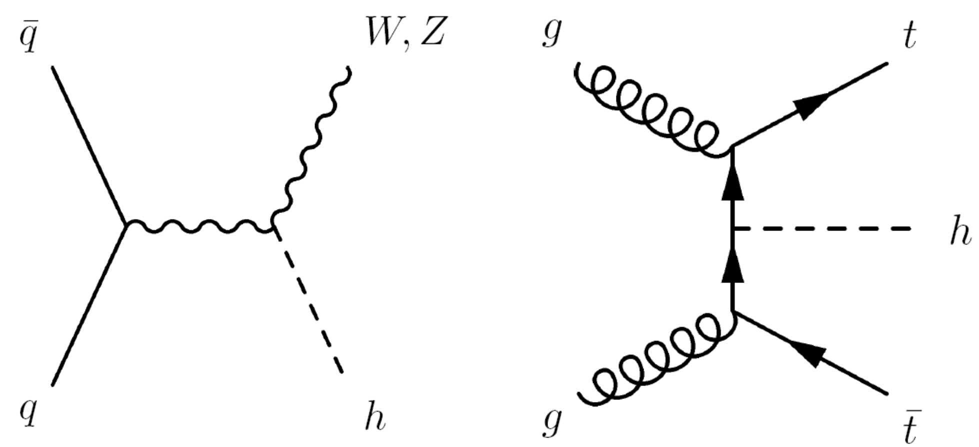
Higgs Production at LHC



Gluon fusion to Next to Leading Order (NLO)



Weak boson fusion



Associated production