1. Define **girth** to be the length of smallest cycle in a graph. Let G be a graph with girth 5 and let the degree of every vertex be $\geq d$. Show that G has at least $d^2 + 1$ vertices.

2. Let G be a graph on 10 vertices. Show that if G had more than 20 edges then it must have a triangle.

3. True or False

   1. The number of people who have married an odd number of people is even.
   2. Let G be a graph on 7 vertices such that the sum of the vertex degrees is at least 21. There are is a vertex in G with 4 neighbors.

4. Prove or give counter example.

   1. Every graph with an Eulerian cycle has a Hamiltonian cycle.
   2. Every graph with a Hamiltonian cycle has an Eulerian cycle.
   3. Some graph with an Eulerian cycle has a Hamiltonian cycle.
   4. Some graph with a Hamiltonian cycle has an Eulerian cycle

5. What is the maximum number of edges a graph G on n vertices can have if G is not connected?

6. Let G be a graph on n vertices with the following properties:

   - G is not connected.
   - No two edges added will connect the graph.

   What is the maximum number of edges in the graph?

7. In every directed graph, a **king** (or **queen** if you so wish) is a vertex from which all other vertices can be reached by a path of length at most 2. Does every tournament have a king?