Discrete Mathematics (CS202/MA221) — IIT

Problem set #3 — **Guwahati, India**

September 14, 2011

1. Consider numbers of the form $10^n - 1$. Which of the following statements are true?
   (i) No number in this sequence is divisible by 7.
   (ii) No number in this sequence is divisible by 71.
   (iii) No number in this sequence is divisible by 77.
   (iv) No number is this sequence is divisible by 18.

2. Consider a class of n students. Every day after the lecture chocolates were given to students who answered the questions posed in class. Let c_i and m_i be respectively the number of chocolates and marks that student i received during the course. No student got more that 13 chocolates. It was observed that each student had a distinct (c_i, m_i) pair. What is the maximum value of n. (Assume that maximum marks awarded was 98, minimum was 10 and that all marks were integers).

3. Show that any set of 7 integers will have two integers in them whose sum or difference is divisible by 10. Also construct a set of six integers for which this is not true.

4. Consider particles moving in a closed box of dimension $2m \times 5m \times 10m$. It was observed that no two particles got within a distance of $\sqrt{3}m$. Show that there can be no more than 101 particles in the box.

5. Show using PMI that every natural number can be uniquely represented as a sum of numbers of the form $2^i, i \in \mathbb{N}$. Is this true for numbers of the form $2^i - 1$? Where does the proof go wrong?

6. Count the number of even sized subsets for a set of size n.

7. Let $k = a_1 a_2 \ldots a_n$ be an n digit number with its digits being $a_1, a_2, \ldots a_n$ and $a_1 \neq 0$. By "rotation" of an $n-$digit positive integer $k = a_1 a_2$ where a_i is the ith digit, we mean the cyclic shifts of its digits. Count the number of n digit positive integers such that all the rotations result in n digit numbers.

8. Count the number 6 digit numbers that contains 7 and are divisible by 10.

9. Show that $2^{n-i} \times n \times (n-1) \times (n-2) \ldots (n-i+1) = \sum_{k=i}^{n} k \times (k-1) \times (k-2) \ldots (k-i+1) \times \binom{n}{k}$

10. How many n digit numbers contain at least one even number and one odd number?