1. Show that if G is an abelian group then for all $a, b \in G$ and for all $n \in \mathbb{N}$, $(ab)^n = a^n b^n$.
2. Show that if G is a group such that for all $a, b \in G$ $(ab)^2 = a^2 b^2$, then G is abelian.
3. Let $S = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\}$. Is S is a group under usual multiplication? What is the smallest group containing S?
4. Let $n\mathbb{Z}, n \in \mathbb{N}$ denote the set of all multiples(both positive and negative) of n.
    1. Show that $n\mathbb{Z}$ a group under addition.
    2. Consider the groups $3\mathbb{Z}$ and $5\mathbb{Z}$. Let $h : 3\mathbb{Z} \mapsto 5\mathbb{Z}$ be a such that $h(15) = 45$ Can h be a homomorphism?
5. Let G be a finite group with order p where p is a prime number. Show that G is cyclic.
6. Let G be a finite abelian group and let $a, b \in G$. $ord(a) = 3$ and $ord(b) = 5$. What is $ord(ab)$?
7. Show that for all $a \in G$ where G is a group, $ord(a) = ord(a^{-1})$.
8. Which of the following are groups?
    1. $G = \{a + b\sqrt(2)\}, a, b \in \mathbb{Z}$ under addition.
    2. $G = \{a + b\sqrt(2)\} \setminus \{0\}, a, b \in \mathbb{Z}$ under multiplication.
    3. $G = \{a + b\sqrt(2)\}, a, b \in \mathbb{Q}$ under addition.
    4. $G = \{a + b\sqrt(2)\} \setminus \{0\}, a, b \in \mathbb{Q}$ under multiplication.
9. Let G be a group of order 10. Let $a, b \in G$ such that order of a and b are 2 and 5 respectively. Describe all the elements of G.
10. Show that there cannot be an isomorphism from the additive group $\mathbb{Z}$ to the additive group $\mathbb{Q}$.
11. Show that matrices $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ form a group under matrix multiplication. Is this a proper subgroup of 2×2 invertible matrices?