

Lecture-IV

Contact forces & Newton's laws of
motion

Contact forces:

Force arises from interaction between two bodies.

By contact forces we mean the forces which are transmitted between bodies by short-range atomic or molecular interactions.

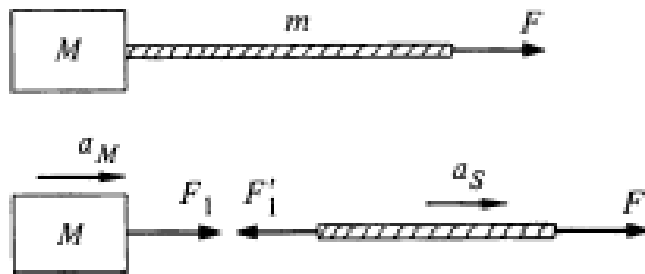
Examples: push, pull, tension of a string, the normal force, the force of friction, etc.

The origin of these forces can be explained in terms of the fundamental properties of matter. However, our approach will emphasize the empirical properties of these forces and the techniques for dealing with them in physical problems, with only brief mention of their microscopic origins.

Tension in a string:

A string consists of long chains of atoms characteristic of the particular material found in the string. When a string is pulled, we say it is under tension. **The long chains of molecules are stretched, and inter-atomic forces between atoms in the molecules prevent the molecules from breaking apart.** A detailed microscopic description would be difficult and unnecessary for our purpose. Instead, a macroscopic model would be developed for the behavior of strings under tension.

Consider a block of mass M in free space pulled by a string of mass m . A force F is applied to the string, as shown. What is the force that the string “transmits” to the block?



$$F_1 = Ma_M$$

$$F - F_1' = ma_S.$$

String is inextensible: $a_S = a_M$.
By Newton's third law: $F_1 = F_1'$

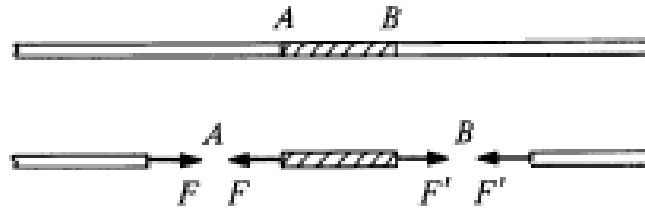
$$a = \frac{F}{M + m},$$

$$F_1 = F_1' = \frac{M}{M + m} F.$$

The force on the block is less than F ; the string does not transmit the full applied force.

If the mass of the string is negligible compared with the block, $F_1 = F$ to good approximation.

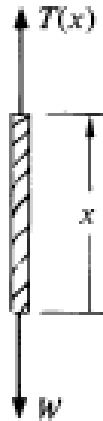
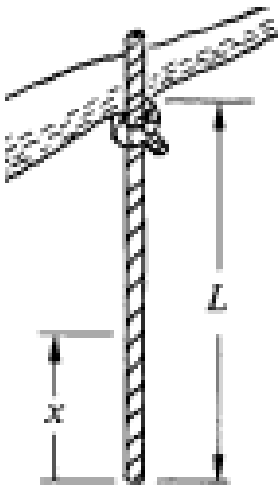
A string is composed of short sections interacting by contact forces. Each section pulls the sections to either side of it, and by Newton's third law, it is pulled by the adjacent sections. The magnitude of the force acting between adjacent sections is called tension. There is no direction associated with tension. In the sketch, the tension at A is F and the tension at B is F' .



Although a string may be under considerable tension, if the tension is uniform, the net string force on each small section is zero and the section remains at rest unless external forces act on it. If there are external forces on the section, or if the string is accelerating, the tension generally varies along the string.

Dangling rope:

A uniform rope of mass M and length L hangs from the limb of a tree. Find the tension a distance x from the bottom.



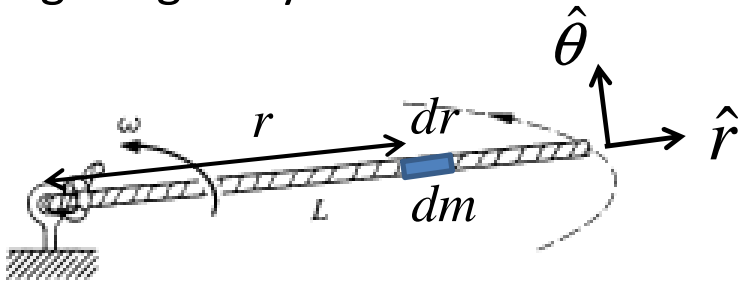
$$W = Mg(x/L).$$

$$T(x) = \frac{Mg}{L} x.$$

At the bottom of the rope the tension is zero, while at the top the tension equals the total weight of the rope Mg .

Whirling rope:

A uniform rope of mass M and length L is pivoted at one end and whirls with uniform angular velocity ω . What is the tension in the rope at distance r from the pivot? Neglect gravity.



$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta};$$

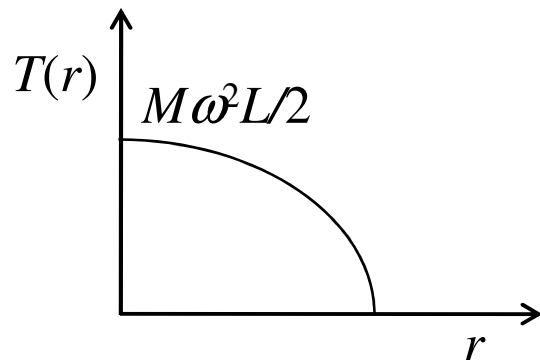
$$\ddot{r} = \dot{r} = 0, \quad \ddot{\theta} = 0 \quad \vec{a} = -r\omega^2\hat{r}$$

$$dT(r) = -r\omega^2 dm; \quad dm = \rho dr = \frac{M}{L} dr$$

$$dT = -\frac{M\omega^2}{L} r dr \quad \int_{T_0}^{T(r)} dT = -\int_0^r \frac{M\omega^2}{L} r dr, \quad T(r) - T_0 = -\frac{M\omega^2}{L} \frac{r^2}{2}$$

$$T(r) = T_0 - \frac{M\omega^2}{2L} r^2. \quad T(L) = 0 = T_0 - \frac{1}{2}M\omega^2 L. \quad T_0 = \frac{1}{2}M\omega^2 L,$$

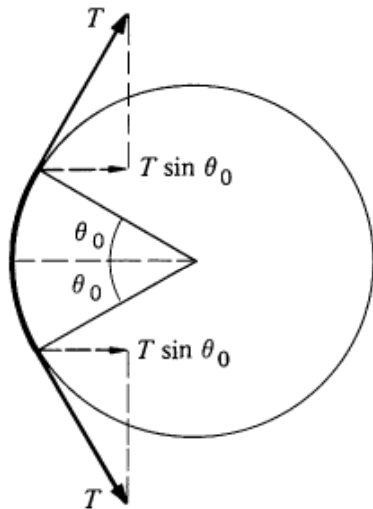
$$T(r) = \frac{M\omega^2}{2L} (L^2 - r^2).$$



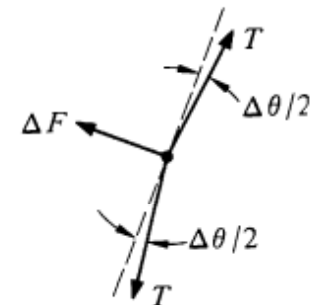
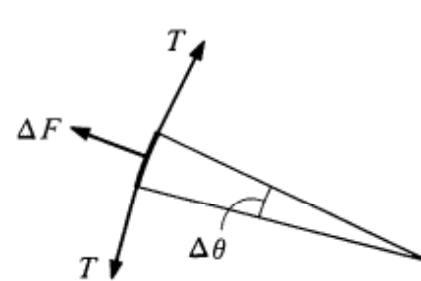
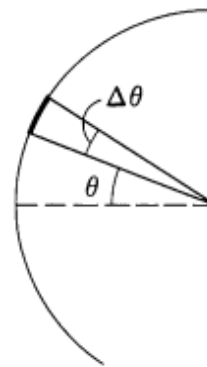
Pulleys:

When a pulley is used to change the direction of a rope under tension, there is a reaction force on the pulley. The force on the pulley depends on the tension and the angle through which the rope is deflected.

A string with constant tension T is deflected through angle $2\theta_0$ by a smooth fixed pulley. What is the force on the pulley?



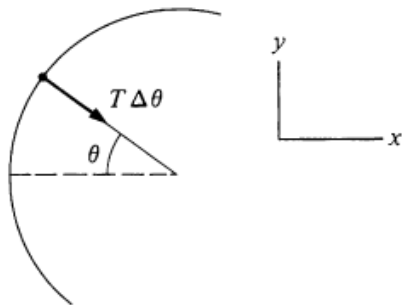
$$2T \sin \theta_0$$



$$\Delta F - 2T \sin \frac{\Delta\theta}{2} = 0$$

$$\text{For small } \Delta\theta, \sin(\Delta\theta/2) \approx \Delta\theta/2 \text{ and } \Delta F = 2T \frac{\Delta\theta}{2} = T\Delta\theta.$$

Thus the element exerts an inward radial force of magnitude $T\Delta\theta$ on the pulley.



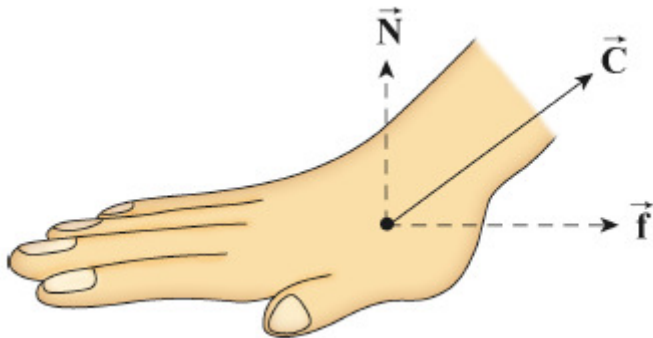
$$\int_{-\theta_0}^{\theta_0} T \cos \theta d\theta = 2T \sin \theta_0.$$

The normal force and friction:

Rest your hand on a table; the atoms that form the molecules that make up the table and your hand are in contact with each other. If you press harder, the atoms are also pressed closer together. The electrons in the atoms begin to repel each other and your hand is pushed in the opposite direction by the table.

According to Newton's Third Law, the force of your hand on the table is equal in magnitude and opposite in direction to the force of the table on your hand.

The total force of the table acting on your hand, $\vec{F}_{\text{contact}} \equiv \vec{C}$, is called the *contact force*. This force has both a normal component to the surface, \vec{N} , called the **normal force**, and a tangential component to the surface, \vec{f} , called the **friction force**.

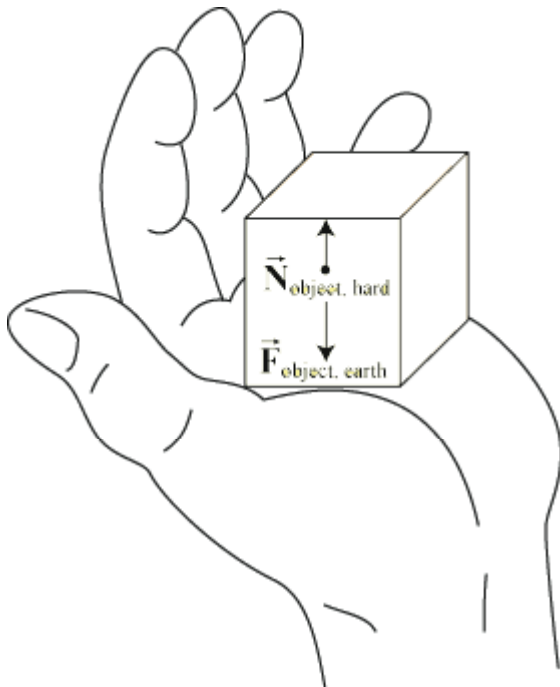


$$\vec{C} \equiv \vec{N} + \vec{f}$$

\vec{N} and \vec{f} then are not independent forces .

Normal force and weight:

Hold an object in your hand.



There are two forces acting on the object. One force is the gravitation force between the earth and the object, and is given by $\vec{F}_{\text{object, earth}} = \vec{F}_{\text{gravity}} \equiv m\vec{g}$, where \vec{g} is the gravitational acceleration.

The other force on the object is the contact force between your hand and the object. Since we are not pushing the block horizontally, this contact force on your hand points perpendicular to the surface, and hence has only a normal component, \vec{N} .

By Newton's 2nd law: $\vec{F}_{\text{gravity}} + \vec{N} = \vec{0}$, $N - mg = 0$, $N = mg$.

Though in this particular example, the normal force is equal in magnitude to the gravitational force and opposite in direction, they are NOT third Law pair. The normal force $\vec{N}_{\text{object, hand}}$ and the gravitation force $\vec{F}_{\text{object, earth}}$ are two completely different forces.

Friction:

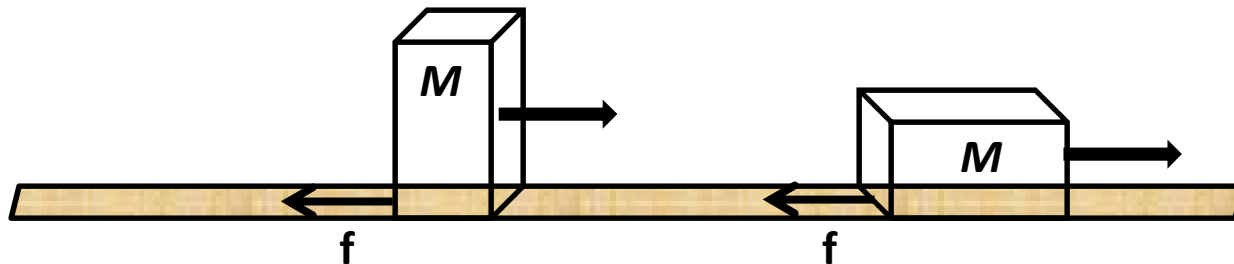
- Friction arises when the surface of one body moves, or tries to move, along the surface of a second body.

- The maximum value of the friction is found to be

$$f_{\text{friction}} = \mu N$$

where N is the normal force and μ is the coefficient of friction.

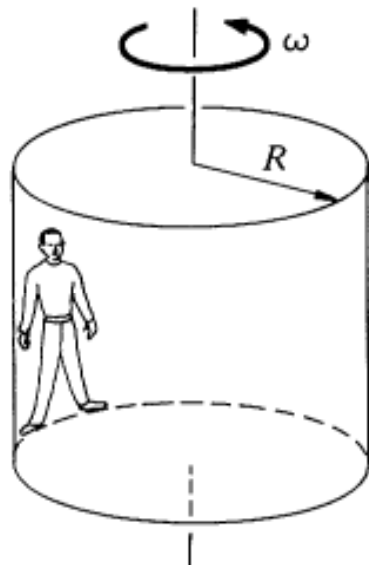
- When a body slides across a surface, the friction force is directed opposite to the instantaneous velocity and has magnitude μN . The force of sliding friction is slightly less than the force of static friction, but for the most part we shall neglect this effect.
- For two given surfaces, the force of sliding friction is independent of the area of contact.



- Friction is independent of the area of contact because the actual area of contact on an atomic scale is a minute fraction of the total surface area.
- Friction occurs because of the interatomic forces at these minute regions of atomic contact.
- Non rigid bodies, like automobile tires, are more complicated. A wide tire is generally better than a narrow one for good acceleration and braking.
- Frictional force is also independent of relative velocity between two surfaces.
- This is approximately true for a wide range of low speeds, as the speed increases and air friction is encountered, it is found that the friction not only depends on the speed, but upon the square and sometimes higher powers of the speed.

The spinning terror:

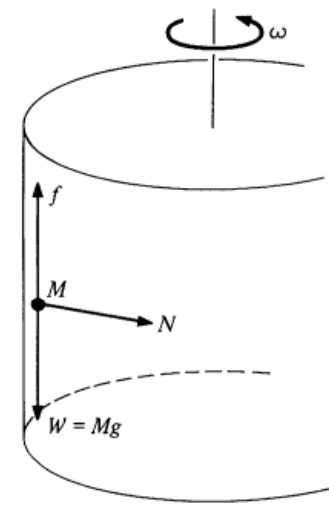
The Spinning Terror is an amusement park ride—a large vertical drum which spins so fast that everyone inside stays pinned against the wall when the floor drops away. What is the minimum steady angular velocity ω which allows the floor to be dropped away safely?



$$N = MR\omega^2.$$

By the law of static friction,

$$f \leq \mu N = \mu MR\omega^2.$$

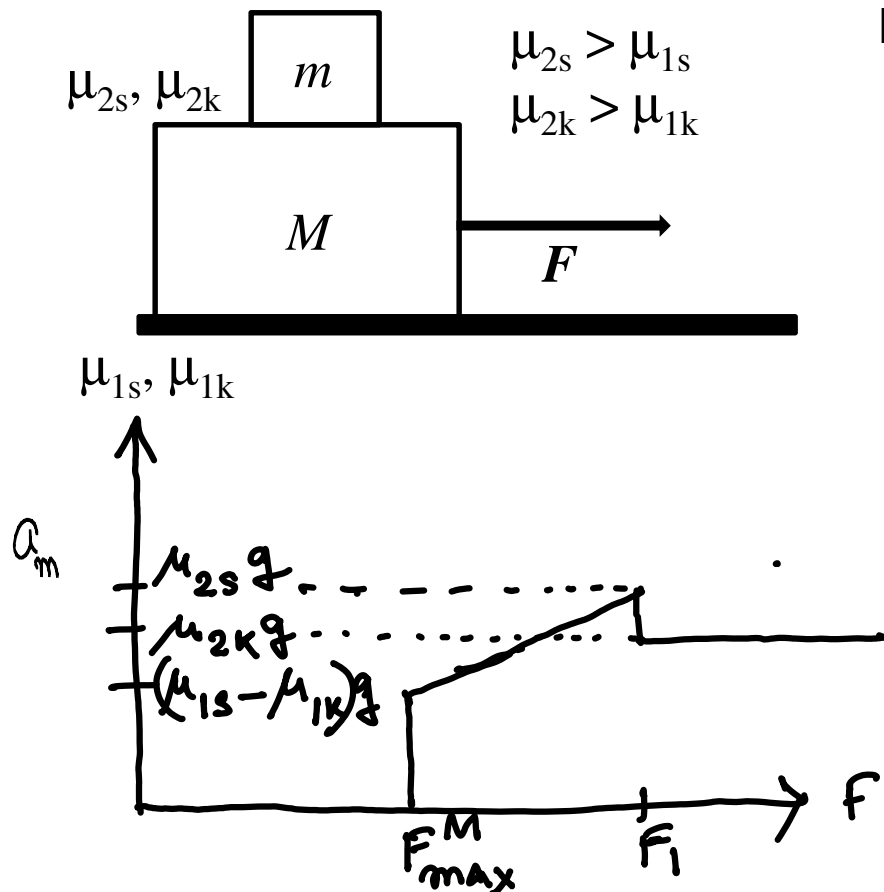


Since we require M to be in vertical equilibrium, $f = Mg$,

$$\text{Therefore, } Mg \leq \mu MR\omega^2 \quad \text{or} \quad \omega^2 \geq \frac{g}{\mu R} \quad \text{or} \quad \omega_{\min} = \sqrt{\frac{g}{\mu R}}.$$

Block on Block

A block of mass m is kept on another block of mass M ($M > m$) and the assembly is placed on a surface. The coefficients of static and kinetic frictions between the lower block and the surface are μ_{1s} , μ_{1k} respectively and those between the surface of the two blocks are μ_{2s} , μ_{2k} respectively. The lower block is pulled by a force F which is increased from zero till the blocks are in contact. Plot the accelerations of both the blocks as functions of F .



No motion: $F = f_{\max}^M = \mu_{1s} (M + m) g$

Starts sliding:

$$a(M + m) = F - \mu_{1k} (M + m) g$$

$$a = \frac{F}{(M + m)} - \mu_{1k} g, \text{ with } a_{f_{\max}^M} = (\mu_{1s} - \mu_{1k}) g$$

For m : $f_{\max}^m = \mu_{2s} mg$, or $a_m = \mu_{2s} g$

The corresponding force is:

$$\mu_{2s} g (M + m) = F - \mu_{1k} (M + m) g,$$

$$\text{or } F = (\mu_{2s} + \mu_{1k}) (M + m) g = F_1$$

$$ma_m = \mu_{2k} mg, \text{ or } a_m = \mu_{2k} g$$

For M: $a_M M = F - \mu_{1k} (M + m) g - \mu_{2k} m g$

or $a_M = \frac{F}{M} - \frac{g}{M} [\mu_{1k} (M + m) + \mu_{2k} m]$

with $a_{M, F_1} = \frac{g}{M} [\mu_{2s} (M + m) - \mu_{2k} m]$

