Probabilistic Assessment of Seismic Response of Toe-Excavated Partially Saturated Hillslopes

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Abstract:

Toe excavation of the hills is a common practice in the highlands of India for construction or extension of roadway projects. Landslides in such cut slopes occurs very often and have dire consequences on human lives, properties and communication network in the hilly regions. Rise in natural ground water level due to torrential rain during monsoon season and seismic activity are two most crucial factors responsible for most of these landslides in cut slopes. The customary stability assessment techniques for such cut slopes is based on deterministic method, where the stability is assessed with reference to a single safety measure commonly known as factor of safety. Nonetheless, due to uncertainty related to various geotechnical parameters, the standard deterministic approach may end up resulting in mismatched design solutions. This paper reports the seismic behaviour of cut slopes in presence of water table for different seismic zones in India utilising a probabilistic approach. The study also exhibits the influence of correlation coefficient, spatial variation of shear strength parameters and the coefficient of variation on the seismic response of the partially saturated cut slope. Furthermore, the study reports the effect of incorporation of uncertainty in water table location and pseudo-static earthquake forces on seismic response of the partially saturated cut slope. A nonlinear time-history analysis is also carried out to estimate the more realistic seismic behaviour of the partially saturated cut slope during earthquake.

Keywords: *Toe-excavation; Partially saturated slopes; Pseudo-static earthquake forces; Factor of safety; Probability of failure; Coefficient of variation; Correlation coefficient.*

1. Introduction

Excavation of hills is often carried out in hilly regions of India to materialize new road constructions or existing road expansions. In the highlands of north-eastern India, roads are the prime means of transportation network. The fast growing population arises further requirement to urbanize the regions that are vulnerable to various natural hazards. Torrential rain and seismicity are the two important factors that triggers different hazards such as landslides and debris flows. Due to heavy rainfall, the rain water infiltrates in the slope increasing the ground water table and thereby attenuating the shear strength of soil. The increase in the positive transient pore pressure results in significant loss of soil suction causing landslides [1]. The situation becomes more critical when the slope is situated in a seismically active region such as in the North Eastern or Himalayan mountainous region of India [2]. Failure of such cut slopes are of dreadful consequences including the damage of various infrastructure, disruption of urban networks and potential fatalities [2, 3, 4]. Several studies have reported about cut slope stability over the years; however, majority of these studies are based on traditional slope stability assessment techniques that ignores the prevailing uncertainty in geotechnical engineering [2, 3, 4]. Different avenues of uncertainties in geotechnical assessments include inherent spatial variation of mechanical properties of soil, error during measurement of in-situ field data or laboratory testing and implicit uncertainty in transformation models utilised for computing design parameters [2, 3]. The inherent spatial variation in mechanical properties of soil emerges from the continuous modification of the in-situ or residual soil mass due to various natural geological phenomenon. The error during measurement of in-situ field data or laboratory testing percolates basically due to operation, tools and other testing effects. The uncertainty inherent to the empirical relations or correlation models (based in-situ or laboratory assessments) utilised for determining the design soil properties also contributes significantly to the overall geotechnical uncertainty.

Very few literature reports the application of a probabilistic framework to decipher the seismic behaviour of slope. Based on probabilistic approach, the seismic behaviour of an embankment slope was efficiently and successfully predicted by Tsompanakis *et al*. [5]. Xiao *et al*. [6] carried out probabilistic seismic analysis of slopes by considering the peak ground acceleration as an uncertain parameter and the spatial variation of shear strength parameters of soil as random field. For a given site and the seismic exposure time, the study illustrated the efficacy of the probabilistic analysis for seismic stability assessment of slopes. Burgess *et al*. [7] reported the seismic behaviour of slope utilising random finite element method (RFEM) and illustrated the effectiveness of probabilistic analysis in terms of probable savings in a project cost if spatial variation of geotechnical parameters is suitably incorporated. The study also presented various seismic slope stability charts using a computer model, *Rslope2d*, based on RFEM [8, 9]. The charts can be utilised by the practicing engineers as a substitute of the computer based analysis. Although the realistic characteristics of an earthquake motion is random, the mentioned studies considered a constant pseudo-static earthquake force, thereby overlooking the randomness in the strong motion. For a heterogeneous slope deposit, Malekpoor *et al*. [10] reported a conservative prediction of the probability of failure after incorporating variability in the earthquake coefficients. Most recently, Chakraborty and Dey [3, 4] reported the probabilistic stability of dry cut slopes under earthquake conditions. However, as per the authors' knowledge, until date, no study reported the seismic stability of a partially saturated cut slope within a probabilistic framework. The existence of water table in a hill slope remarkably affects the shear strength of hillslope material, while its inclination governs the intensity of the seepage forces on the hill slopes. With the aid of deterministic analyses, the significance of the presence of ground water table is already illustrated in earlier literature [11–13]. A deterministic approach of slope stability analysis mostly considers idealized conditions involving a hypothesized geometry, perfectly determined geotechnical and hydrological parameters and a well-defined failure mechanism, thereby leading to a well-sorted deterministic outcome from the analysis. However, it is well understood that each of the aforementioned assumptions can be largely deviating from idealized deterministic condition, as each of these is affected by uncertainties. Conventional local-scale slope stability analysis conveniently assumes a smooth inclined hillslope thereby neglecting the in-situ undulations and geological anomalies. The slope properties are mostly considered uniform, thereby rendering the slope to be a homogeneous one, while completely neglecting the inherent spatial variability in the material properties. Further, the material properties are mostly determined through in-situ and laboratory investigations, in which multiple trials are conducted to identify the mean magnitude of any property, thereby neglecting the uncertainty in the determination of the material property itself. In many cases, the material properties are determined from very limited number of sampling, thereby failing to identify any possible intangible sub-surficial anomalies. Lastly, for conducting a numerical analysis, several simplifying assumptions are adopted to ease out computational efforts and attempting to best reflect the failure mechanism. In this process, the uncertainties and variabilities associated with the real-field failure mechanisms might be substantially overlooked. For example, the consideration of a specific type of slip mechanisms and having a certain chosen geometry might not reflect the in-situ failure geometries. Hence, the problem of slope stability analysis, as dealt conventionally, can be considered as a mean value problem, where the deterministic assertions corroborate to the mean idealizations. Yet, the uncertainties in each parameter, reflected by their spread around the mean value, has an instrumental role on detecting the possibility of failure of a considered slope. Hence, to cater such uncertainties in geometry, material behaviour and failure mechanism, in comparison to a deterministic method, a probabilistic study remains more prudent and sought. The probabilistic approach becomes more important while using a single section of slope to virtually represent locations spread all across a hilly region. In such case, the same representative slope would exhibit different degrees of stability owing to the spatial variations of the geotechnical and hydrological parameters across the hilly terrain.

The present study reports a probabilistic stability investigation of toe-excavated partially saturated hillslope subjected to seismic accelerations. The study also highlights the impact of coefficient of variation (CoV), correlation coefficient (ρ) and spatial variability of shear strength of hillslope material on the seismic response of cut slopes. The influence of uncertainties in water table location and pseudo-static earthquake forces is also reported. Lastly, in order to comprehend the actual dynamic behaviour of the partially saturated cut slope, nonlinear dynamic analysis considering a particular strong motion is also carried out and the results are reported. It would be worth mentioning that the proposed analysis is primarily applicable for landslides, and not inadvertently for debris flows. Debris flows comprise a moving mass of loose mud, sand, soil or rock that remains interacting to the fluid matrix that carries the solid phase under the action of gravitational force. Due to presence of a very wide gradation of the constituent materials, such mass movements do not exhibit a specific failure or slip surface. Although 'debris flows' remain under the umbrella of 'landslides' [14], yet the latter terminology is mostly used for events comprising those constituents whose overall particle size gradation does not include very coarser fractions (> 1 cm). Given the material properties considered in this study, it is not suggested to apply the findings from the current manuscript to 'debris flows'. However, it is also to be remembered that the analysis technique uses limit equilibrium approach, which does not consider any deformation into account, and assesses the stability through force and moment equilibrium. The success of this method depends on the accuracy of geotechnical investigations and the parameters included in the numerical analysis. Hence, the methodology adopted herein might still be restrictively used for stability analysis of hillslopes comprising materials with very wide particle size distributions.

However, given the earlier reasons, the application of the proposed analysis is not recommended for debris flows, and to be restricted only for landslides which offers a more predictive failure surface.

2. Probabilistic Principles and Numerical Model for Present Study

Unlike deterministic approach of identifying a ubiquitous factor of safety (FoS), the probabilistic approach considers the geotechnical uncertainties and reports the probability of failure (P_f) as an indicator of the stability. In this approach, to accommodate the uncertainty in the soil shear strength, the corresponding parameters are expressed as continuous random variables based on the corresponding probability density function (pdf). Following the Central Limit Theorem [15], the 'Normal' pdf is most commonly used to define randomness in geotechnical parameters [2-4, 8, 9]. However, as the 'Normal' distribution fetches negative values during sampling, this distribution is sometimes found unsuitable for characterising random variables in geotechnical engineering, as most of the parameters operate only on positive values. To counteract this limitation, a non-negative distribution, such as a 'Lognormal' distribution can be suitably used. In order to define a random variable, apart from the corresponding pdf and cumulative distribution function (cdf), the statistical descriptors such as 'mean' or 'expectation', 'variance', 'standard deviation' and 'coefficient of variation (CoV)' are also profoundly used; the basic definition of these statistical descriptors could be found in any standard textbook documentation on statistics.

The typical range of CoV for cohesion (c) is 0.05-0.5 and that for angle of internal friction (φ) is 0.02-0.56 [16]. However, because of the site-specific characteristics of the soil properties, there exist differences in the range of CoV suggested in literature. For the present study, a 30 m high hillslope overlying a hard rock bed material is considered. The slope has a crest length of 15 m (i.e. half the height of hill slope, adopted arbitrarily) and a slope inclination of 30°. Following the hillslope cutting practices often adopted in north-east Indian mountainous terrain, a vertical cut generated by excavating 10 m width of the toe is considered in the numerical model (Fig. 1). The stability analyses of the partially saturated slope are carried out with the aid of the Slope/W module of Geo-Studio v2019 [17]. It would be noteworthy that in this study, 'partially saturated slopes' does not mean that the entire slope section is under partial saturation, wherein the saturation state of the slope would be dictated by a specific degree of saturation or its distribution throughout the slope. By 'partially saturated slopes', it is indicated that a portion of the slope is saturated while the rest of the slope is dry. To simulate this scenario of partial saturation of the slope section, an inclined piezometric line is considered passing through the toe of the hillslope and intersecting the left boundary of the hill at a height (h) of 20 m from the base (as shown in Fig. 1). Defining a piezometric line is one of the most common way of defining pore-water pressure conditions in Slope/W module. The soil below the piezometric line remains fully saturated (degree of saturation is 100%), while the soil above the piezometric line is assumed completely dry (degree of saturation is 0). Capillarity induced saturation above the phreatic or piezometric surface is not included in this study.

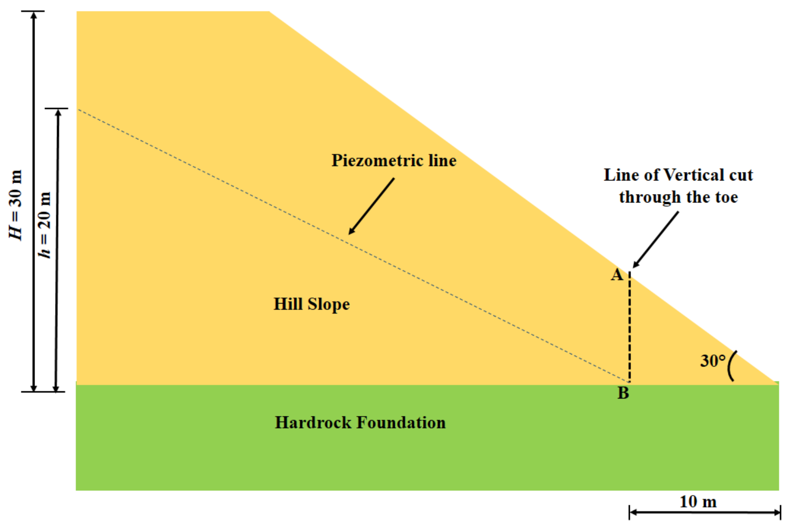


Fig. 1 Numerical model of the slope section adopted for present study

The present study considers a circular failure surface for limit equilibrium method (LEM) based slope stability analysis using Morgenstern Price Method [18]. Although, in the present study, the shear strength parameters are considered as random variables, yet, in each iteration of the Monte-Carlo simulation, the hillslope is considered homogeneous comprising a single combination of c and φ. It has been stated in earlier researches that if the slope material remains homogeneous and the shear strength parameters do not vary within the slope, a circular failure surface can be considered [19, 20]. Deviations from homogeneity can lead to the development of non-circular failure surfaces such as in the presence of soft layer within the hillslope, presence of different types of soil or rockfills, or the cases in which the there is a significant spatial variation of the shear strength properties in the hillslope [18]. However, as homogeneity of the hillslope material is assumed in the present study, the consideration of non-circular failure surface is out of scope of the present study. Morgenstern-Price method of slope stability analysis falls within the class of slope stability analyses in which the active moving mass of soil is bounded by ground surface at the top and the slope surface at the bottom. The potential moving mass is discretized into many vertical slices [21], which is further used to compute overall sliding force and moment integrals [22, 23]. The problem being statically indeterminate from force-equilibrium perspective, an equilibrium equation is developed to establish direction of interslice forces through a functional correlation between the lateral and vertical interslice forces and a multiplicative scaling factor λ [24]. In the process, the factor of safeties (FoSs) are determined from both forced and moment equilibrium expressions through a Newton-Raphson procedure. Further, alike Spencer's method of slope stability [25], Morgenstern-Price method is capable of accounting the presence of water table within a homogeneous hill slope. In such circumstance, the vertical distance from the slice base mid-point up to the piezometric line is estimated and it is multiplied to the unit weight of water to estimate the pore-water pressure at the slice base.

The probabilistic approach considered Monte Carlo Simulation (MCS) for estimating the probability of failure (P_f) of the considered cut. In MCS approach, N numbers of samples are drawn from the adopted pdf, and for each of the drawn sample, the slope stability analysis is conducted to check for its stability. The probability of failure is defined as the ratio of number of draws that results in the slope failure (n) to the total number of draws, i.e. N, alternatively described as $P_f = \frac{n}{N}$. US Army Corps of Engineers [26] provides the performance level of a geotechnical structure in terms of the (P_f) and reliability index (β). In general, β varies within 1 to 5, with the P_f ranging from 0.16 to 3×10^{-7}. A relatively small P_f value (i.e., P_f < 2.3%) is desired by geotechnical professionals, that corresponds to $\beta > 2$ and thereby leading to a performance level better than 'poor'.

To consider uncertainty related to geotechnical properties, it is important to first decide the input parameters that need to be considered as random variables. Considering Mohr-Coulomb material for the hillslope soil, apart from c and φ, the soil unit weight (γ) needs to be provided as input. Past literatures have suggested that the variation in γ is very less and, hence, has marginal influence in probabilistic study of geotechnical structures [2, 3, 4]. Therefore, only the shear strength parameters of soil, c and φ, are considered as random variables and are defined by a log-normal pdf as suggested in various literature [2, 3, 4]. Hence, for this study, the unit weigh of soil is adopted deterministically as 18 kN/m^3. Based on several reported literatures, c and φ of soils commonly encountered in North-eastern hilly terrains ranges between 10 – 70 kPa and 15° – 35° [27-40]. In this study, for the probabilistic analysis, the mean values of shear strength parameters are suitably adopted within the designated ranges of the parameters, i.e. c = 40 kPa and φ = 27.5°, and as reported in literature [2, 3, 4], the CoV of both c and φ are varied from 0.2 to 0.4. For a probabilistic study, in order to decide the optimum number of MCS required, a convergence study is carried out for every case (as reported in

Section 3 of the paper). For all the cases, 2000 numbers of Monte-Carlo simulations are found sufficient. Figure 2 shows a typical convergence plot for a partially saturated cut slope located in seismic Zone V and comprising shear strength parameters having a CoV value of 0.4. Similarly, convergence is investigated for all other cases presented in this study; however, the same has not been reported in the paper for brevity.

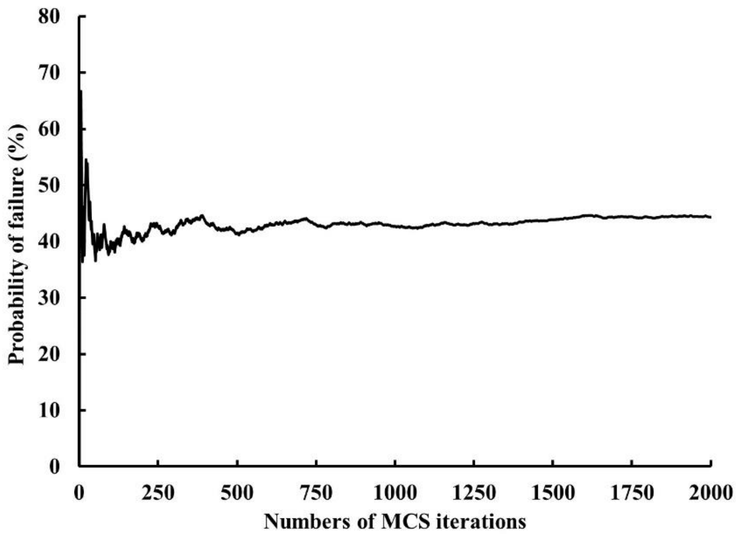


Fig. 2 Typical convergence plot to manifest optimum number of MCS

To introduce the seismicity effects, pseudo-static earthquake condition is initially adopted where the earthquake forces are assigned by the means of equivalent horizontal and vertical inertia forces. In Slope/W, the equivalent inertia forces are assigned by coefficients of horizontal and vertical pseudo-static earthquake accelerations i.e. k_h and k_v, respectively, and are expressed as $k_h = \frac{a_h}{g}$ and $k_v = \frac{a_v}{g}$, where, a_h and a_v are the horizontal and vertical peak ground accelerations, respectively. Furthermore, with the aid of Quake/W module of Geo-Studio v2019 [41], nonlinear dynamic time-history analysis is also carried out. In this part of the study, the model domain need to be discretized in a set of mesh elements. There are different mesh element patterns available in Geostudio, such as (a) Quads and Triangles (b) Triangles only (c) Rectangular grid of Quads, and (d) Triangular grid of Quads/Triangles. In the present study, the use of mixed 'quad and triangle' unstructured mesh with 4-noded quadrilateral and 3-noded triangle element types is utilized. In order to arrive at a convergent mesh, the global element sizes are varied to assess the response from NTHA. A stable solution is achieved for meshes with global element sizes less than 0.5 m. Hence, for the present study, a convergent mesh comprising global element size of 0.25 m with 27652 elements and 27962 nodes was used. Boundary conditions are necessary to implement the restrictions on the load-deformation scenarios at the boundaries of the model. In the present study utilising non-linear dynamic analysis, the base of the model is restrained from displacement in both the directions, and the far lateral boundaries are restrained from vertical displacement but are kept free from any horizontal restraint. The time-dependent stresses are transpired into the Slope/W module to assess the temporal probability of failure, which is further discussed in detail in Section 7 of this paper.

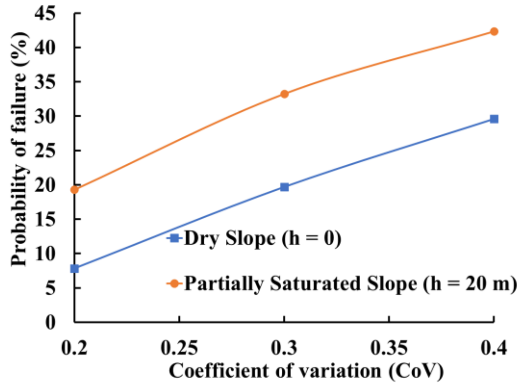
3. Effect of Coefficient of Variation and Correlation Coefficient

This section reports the probabilistic analyses conducted for the cut slope under dry as well as partially saturated condition utilising pseudo-static approach for different CoV and correlation coefficient ($\rho_{c\varphi}$) between cohesion (c) and angle of internal friction (φ). Most of the geotechnical parameters exhibit correlations among them and are not rendered to be completely independent of one another. Several authors have studied the correlation between various soil properties [9,15, 42-47]. In geotechnical engineering, the influence of the correlation between the shear strength parameters cohesion (c) and angle of internal friction (φ) is well established and is found to be quite significant, denoted through their correlation coefficient ($\rho_{c\varphi}$). While some researchers ignored all possible correlations between the soil shear strength parameters for mathematical convenience [42, 48-52], some studies considered the correlation among geotechnical parameters in their numerical studies to be an important factor [53-57]. The correlation coefficient between c and φ is considered to be negative in several studies [2, 3, 4], and is considered the most common. In general, soils comprise both coarser (gravels and sands) and finer (silts and clays) fractions, as governed by their particle size. The engineering characteristics of coarser fractions are primarily dominated by their weights and surface roughness, while the behaviour of finer fractions are more governed by their surface charges and electrochemical bonds. Hence, in general, the engineering behaviour of coarser fractions are mostly governed by surface texture and friction, while for the finer fractions, the cohesion or adhesion plays an influential role. Hence, except some very special geomaterial, soils with higher friction angle exhibits lesser cohesion, and vice-versa, thereby advocating for a negative correlation between the shear strength parameters. A negative correlation means that at any specific point, a higher value of c is accompanied by a smaller value of φ, or vice-versa. As the shear strength of soil being governed by both the parameters, c and φ, the lower contribution from one parameter is usually compensated by the higher contribution from the other. It is reported by several authors that the negative correlation between c and φ increases the structural reliability as compared to adopting no correlation between c and φ [15]. However, a slope with zero or positive correlation between c and φ is likely to be associated with higher risk of failure (when both the shear strength parameters exhibit lesser values), and should be studied as well [2]. The present study considers $\rho_{c\varphi}$ in the range of -0.5 to +0.5 as suggested in various literatures [2, 3, 4].

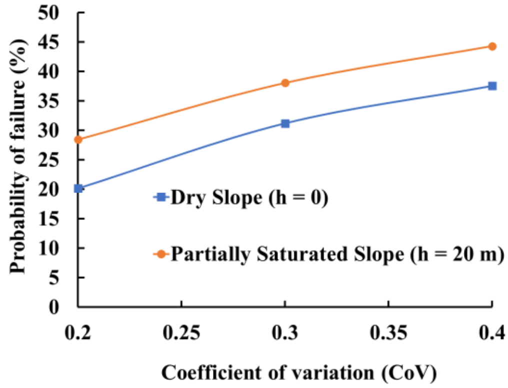
The pseudo-static earthquake coefficients are adopted as per different earthquake zones of India [58]. The analyses are conducted as described in Section 2 and the outcomes of the deterministic analyses are presented in Table 1. The outcomes of the probabilistic analyses are shown in Figs. 3-7. It is seen that the deterministic FoS decreases marginally due to the rise in the water table considered in this study, while it increases for lesser magnitude of the seismic acceleration coefficients. However, deterministic analysis is incapable of addressing the uncertainty related to geotechnical parameters and, hence, might lead to erroneous design of cut slopes. For example, under static condition, the deterministic FoS for dry and partially saturated condition are 1.62 and 1.55 respectively. However, the P_f values of the same slope (comprising shear strength parameters with CoV = 0.3 and ρ = 0) are as high as 1.60% and 2.65%, respectively, for dry and partially saturated conditions, thereby representing a performance level of 'Below Average' and 'Poor', respectively, as per [26]. Therefore, a cut slope adjudged deterministically safe might exhibit failure due to uncertainty related to the soil parameters that need to be catered through a probabilistic framework. Moreover, an increase in seismicity (marked by higher seismic coefficients) leads to a significant increase in the probability of failure. For example, the P_f value of the same slope (comprising shear strength parameters with CoV = 0.2 and ρ = 0) increases from 1.10% to 20.15% for an increase in seismicity coefficient from Zone I to Zone V (as given in Table 1), thereby indicating a degradation in performance level from 'Below Average' to 'Hazardous', as per [26]. These observations from probabilistic analyses reveals that seismicity and rise in water table during monsoon season significantly increases the chances of slope failure. The analyses also show that the values of CoV and ρ of input soil parameters significantly influences the P_f. A larger value of CoV indicates a larger deviation of the assigned soil parameter from its mean value, thereby leading to the increase in P_f. It is observed that the P_f decreases as the correlation coefficient between c and φ (i.e.) attains negative values. A negative $\rho_{c\varphi}$ implies that there are more chances if c being assigned with higher values than its mean, while the φ is assigned smaller values than its mean, or vice versa.

Table 1: Deterministic FoS for a typical slope located in different seismic zones in India

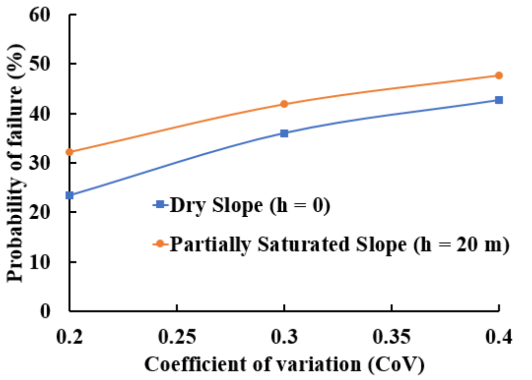
	Zone V k_h **= 0.18** k_v **= 0.09**	**Zone IV** k_h **= 0.12** k_v **= 0.06**	**Zone III** k_h **= 0.08** k_v **= 0.04**	**Zone II** k_h **= 0.05** k_v **= 0.025**	**Static** k_h **= 0** k_v **= 0**
Dry Slope	1.16	1.28	1.37	1.46	1.62
Partially Saturated Slope	1.12	1.23	1.32	1.39	1.55



(a)

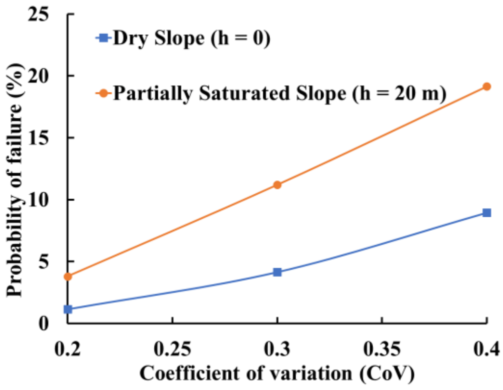


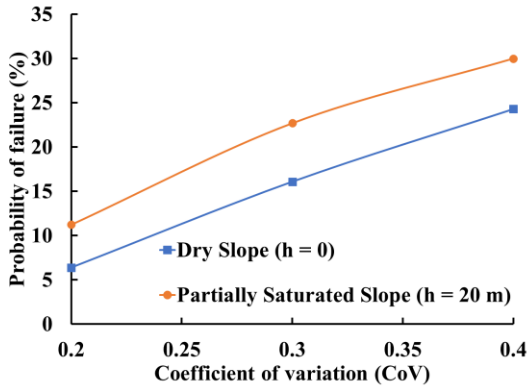
(b)



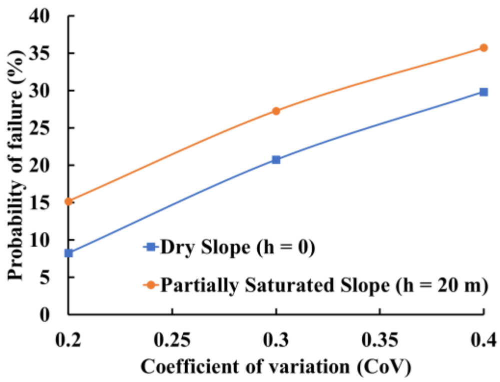
(c)

Fig. 3 Variation of P_f with CoV for a typical slope located at seismic zone V for (a) $\rho_{c\varphi}$ = - 0.5 (b) $\rho_{c\varphi}$ = 0 (c) $\rho_{c\varphi}$ = + 0.5



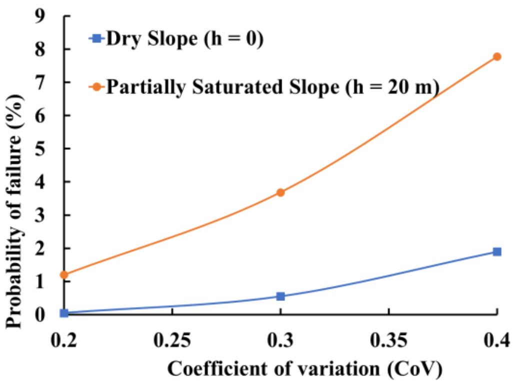


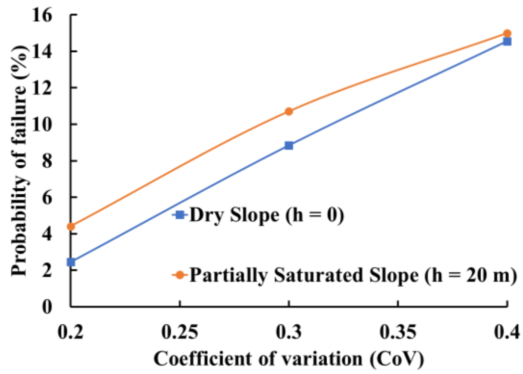
(a) **(b)**



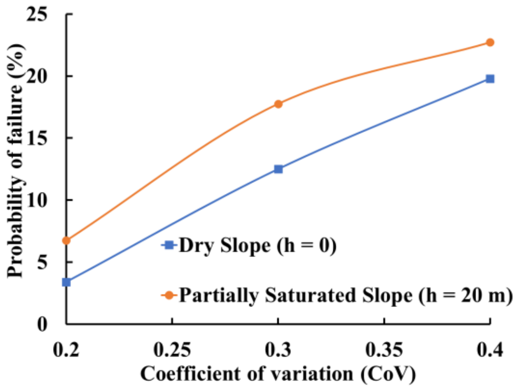
(c)

Fig. 4 Variation of P_f with CoV for a typical slope located at seismic zone IV for (a) $\rho_{c\varphi}$ = - 0.5 (b) $\rho_{c\varphi}$ = 0 (c) $\rho_{c\varphi}$ = + 0.5



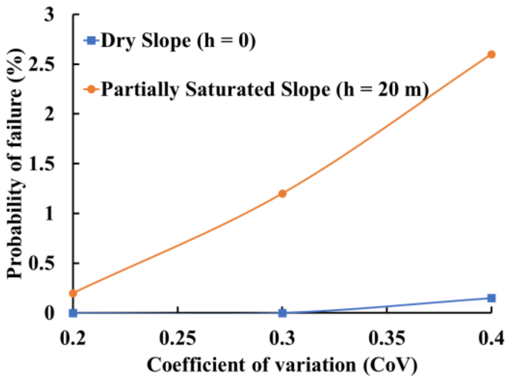


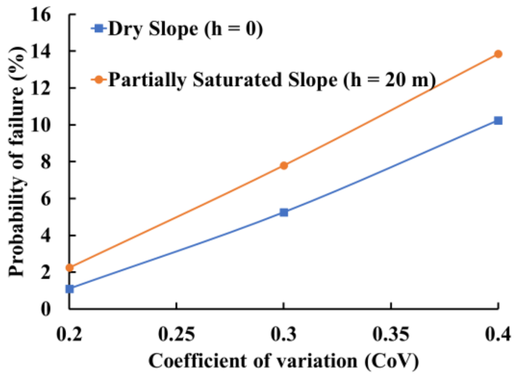
(a) **(b)**



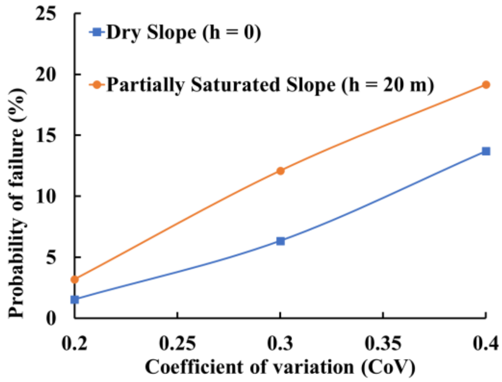
(c)

Fig. 5 Variation of P_f with CoV for a typical slope located at seismic zone III for (a) $\rho_{c\varphi}$ = - 0.5 (b) $\rho_{c\varphi}$ = 0 (c) $\rho_{c\varphi}$ = + 0.5



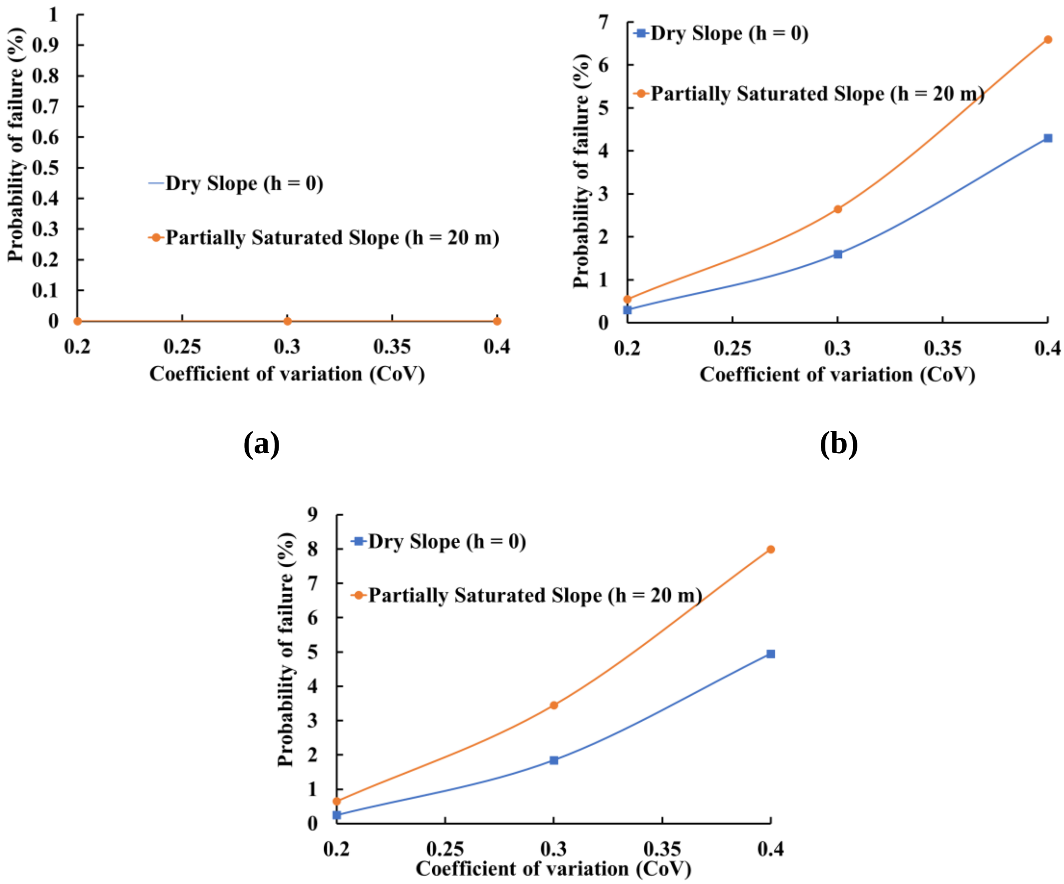


(a) **(b)**



(c)

Fig. 6 Variation of P_f with CoV for a typical slope located at seismic zone II for (a) $\rho_{c\varphi}$ = - 0.5 (b) $\rho_{c\varphi}$ = 0 (c) $\rho_{c\varphi}$ = + 0.5



(c)

Fig. 7 Variation of P_f with CoV for a typical slope under static condition for (a) $\rho_{c\varphi}$ = - 0.5 (b) $\rho_{c\varphi}$ = 0 (c) $\rho_{c\varphi}$ = + 0.5

4. Effect of Inherent Spatial Variation of Shear Strength Parameters of Hillslope Soil

In the previous section, to incorporate the uncertainty in parametric magnitudes, the analyses were conducted adopting c and φ as random variables. However, in such a scheme, each of the MCS iterations considers the soil domain to be homogeneous comprising particular values of c and φ value. Hence, this approach of considering random variable does not induce the spatial variability of shear strength parameters. In order to overcome this limitation, one dimensional (1-D) random fields (RF) of c and φ are assigned in each simulation using the 'correlation length', θ, for simulating the horizontal spatial variability of the shear strength parameters [59], wherein the correlation length governs the spatial distance over which the corresponding soil properties is highly correlated. In a slope stability problem, a 1-D RF is incorporated by sampling the soil domain along the critical slip surface as per an predefined sampling distance. In case the predefined sampling distance exceeds the length of the critical slip surface, the domain is sampled once, while otherwise the sampling is done multiple times. The correlation coefficient of the soil sections [60] is estimated as per Eqn. 1:

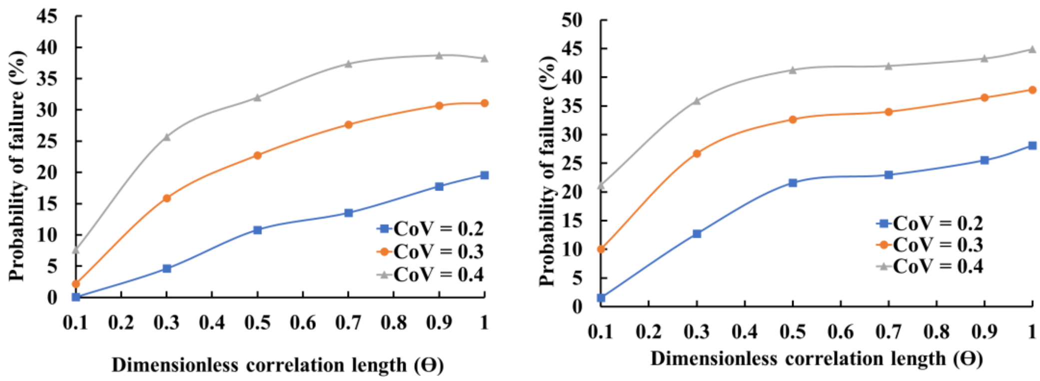
$$\rho(\Delta\Psi,\Delta\Psi') = \frac{\Psi_0^2\Gamma(\Psi_0) - \Psi_1^2\Gamma(\Psi_1) + \Psi_2^2\Gamma(\Psi_2) - \Psi_3^2\Gamma(\Psi_3)}{2\Delta\Psi\Delta\Psi[\Gamma(\Delta\Psi)\Gamma(\Delta\Psi')]^{0.5}} \tag{1}$$

Where,
$$\begin{aligned} \Psi_1 &= \Delta\Psi + \Psi_0 \\ \Psi_2 &= \Delta\Psi + \Psi_0 + \Delta\Psi' \\ \Psi_3 &= \Delta\Psi' + \Psi_0 \end{aligned} \tag{1a}$$

$\Delta\Psi, \Delta\Psi'$ are the length of the two sections, Ψ_0 is the distance between the two sections, and $\Gamma(.)$ is a dimensionless variance function that is approximated as:

$$\Gamma(\Psi) = \begin{cases} 1.0 \text{ when } \Psi \le \theta \\ \dfrac{\theta}{\Psi}, \text{ when } \Psi > \theta \end{cases} \tag{1b}$$

Figure 8 presents the variation of the P_f with Θ (where, $\Theta = \theta / L$, L is width of cut slope and Θ is the dimensionless correlation length). A small value of Θ essentially represents highly erratic random field, whereas, a high value of Θ represents a homogeneous random field, both of which are of less practical importance. The intermediate range of Θ is more practical and is of primary interest to geotechnical practitioners [2, 3, 4]. This study considers the Θ to be varying within a range 0.1-1. It is observed that the P_f is very small or negligible when Θ is very small, while it increases noticeably with the increase in Θ. For very small Θ, the local averaging gets maximised, thereby leading to a high variance reduction which tends the shear strength parameters converge towards their corresponding mean values. As the slope section comprising mean values of the shear strength parameters is deterministically safe (or, stable), the P_f is very small or negligible. However, for very large magnitudes of Θ, minimum local averaging leads to negligible variance reduction, thereby leading to essentially homogeneous medium with different strength parameters for each of the MCS iterations. Hence, it is understood that disregarding the existing spatial variation of geotechnical properties (governed by the correlation length of strength parameters in this case) would result in overestimated or underestimated failure probability, and would not reflect the realistic picture of the on-field instabilities.



(a) **(b)**

Fig. 8 Variation of P_f with Θ for (a) Dry Slope (b) Partially saturated slope comprising shear strength properties with varying CoVs

The application of 1-D RF for simulating spatial variability of geotechnical parameters finds its application in various problems such as braced excavations [61], rainfall-induced landslides [62-64], safety assessment of tailing dykes [65] and dynamic analysis of zoned earthen dams [66]. It is noteworthy to mention that based on the concepts established in earlier literatures [59, 60, 67], this study utilizes only 1-D RF to emulate the spatial variability of shear strength parameters only in horizontal direction. Such application of 1-D RF is conservatively appropriate for the residual slopes that are mostly characterized by a soil layer that reflects spatial variability mostly in the horizontal direction, while the variability in the vertical direction is negligible. However, in most real scenarios, the spatial variability is commonly two-dimensional (2-D), i.e. the variability is reflected in both vertical and horizontal directions. To address the bi-directional spatial variability, a 2-D RF is more appropriate that can be simulated using random finite element theories (RFEM) [68].

6. Effect of Uncertainty in Water Table Location and Pseudo-static Coefficients

In the previous sections, the analyses conducted for partially saturated slopes subjected to pseudo-static conditions considered deterministic or constant location of water table (WT) and pseudo-static coefficients. In this section, the influence of the uncertainty of both these entities are investigated. For this purpose, the height of the piezometric line (*h*), representing the water table, is considered as random variable defined by a 'Normal' pdf. The height '*h*' is varied between 15-25 m, considering 20 m as mean value with a standard deviation of 4 m. Further, k_h and k_v are considered as random variables defined using a 'Log-normal' distribution. For this part of the study, analyses are conducted only with coefficients pertaining to seismic zone V (i.e. with the mean values of k_h = 0.18 and k_v = 0.09). Therefore, random variables for k_h and k_v are defined within the ranges of 0-0.2 and 0-0.1, respectively, with a standard deviation of 0.02 for both the coefficients. Three cases are considered in the subsequent analyses:

Case I: All the variables i.e. *h*, k_h and k_v are considered as deterministic.

Case II: *h* is considered as deterministic while k_h and k_v are considered as random variables.

Case III: All the variables i.e. *h*, k_h and k_v are considered as random variables.

Figure 9 exhibits the outcome of the analyses. It is seen that in comparison to Case I, introduction of uncertainty in k_h and k_v (Case II) causes a slight decrease in the P_f. This is due to the fact that when these coefficients are assigned as random variables, there are chances that in some of the MCS iterations, the random variable is sampling values less than its mean value, thereby resulting in a slight decrease in P_f value. However, along with the uncertainties in the pseudo-static coefficients, the incorporation of uncertainty in water table location (Case III) significantly increases the P_f. Moreover, for Case-III, a slight decrease in the P_f is also observed with increase in the CoV of soil shear strength parameters.

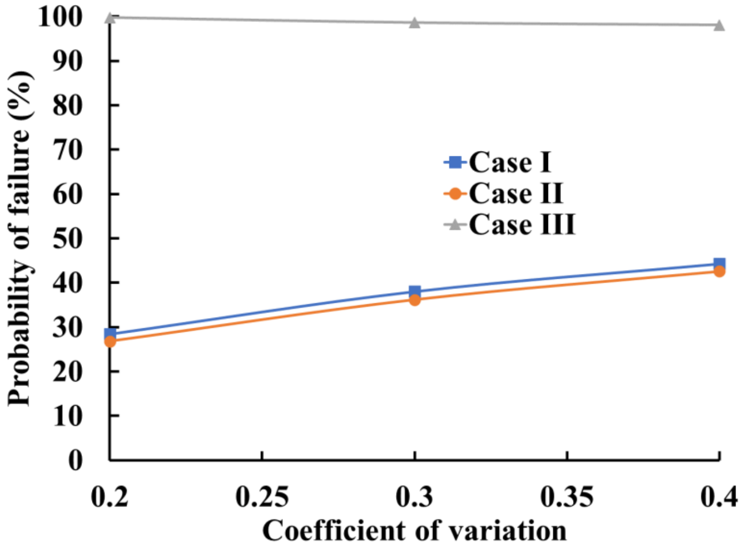


Fig. 9 Variation of P_f with CoV for different cases of incorporating uncertainties in WT location and pseudo-static coefficients

7. Nonlinear Time-History Analysis (NTHA) of Partially Saturated Slope

Soil mass experiences loading-unloading-reloading cycles during dynamic loading, resulting in stress and strain reversal sequences which is represented by hysteresis loops [69]. The initial slope of the 1st cycle of the hysteresis loops is known as maximum shear modulus (G_{max}) [69]. In the Quake/W module of GeoStudio v2019, it is possible to incorporate G_{max} either as a constant or an effective-stress dependent parameter. Earthquake motion comprises several stress reversals and is random in nature. The frequent changes of motion during the entire duration of the significant (or, bracketed) time history leads to complex response of any geotechnical structure, which can be captured through incrementally small time-steps in a dynamic analysis. Majority of the strong motions exist only for a few seconds; thus, the time steps in a dynamic analysis must be several fractions of a second. Generally, a time step of two hundredths (0.02) of a second, or lesser, is adopted. However, it is also to be noted that consideration of considering extremely small time interval makes the nonlinear dynamic analysis computationally exhaustive. In Quake/W module, the nonlinear dynamic analysis scheme makes a single cruise through the entire time history; however, to attain a converged solution, the scheme uses a number of iterations at each time interval.

In this part of the study, for the same partially saturated cut slope section presented in Fig. 1, the nonlinear dynamic analysis conducted using the Quake/W module is coupled with the Slope/W module. The seismic stresses generated in the nonlinear time-history analysis is utilized for assessing the finite element based time-dependent FoS and the P_f of the slope. As shown in Fig. 10, the 1971 San Fernando strong motion (recorded 5 km east of San Fernando dam at the abutment of concrete Pacoima Dam) is scaled-down to 0.18 g [70] and applied to the base of the numerical model. In compliance to the hillslope material chosen for the present study and based on suggestions from earlier literatures [71, 72], the damping ratio (ξ), Poisson's ratio (v) and G_{max} of the soil are appropriately considered as 0.02, 0.334 and 100 MPa, respectively. Analyses are conducted for two cases by considering the location of the WT as deterministic and random, respectively.

The outcomes of the NTHA are shown in Fig. 11. It is observed that for the case with deterministic location of WT, the temporal P_f varies from 0.05% to 12.80% with a mean value approximately equal to 1.34%. However, by considering the WT location as random, the temporal P_f varies from 0.35% to 79.10% with an approximate mean value as 32.79% . It can be recapitulated from Section 6 that the same slope section under pseudo-static condition (for k_h = 0.18, k_v = 0.09) resulted in a high probability of failure value of 42.60% and 98.05% when water table is considered deterministic and random, respectively. Hence, as compared to the pseudo-static analyses, it is seen that the NTHA results in a much lesser magnitude of temporal P_f. Therefore, it clearly reveals that a pseudo-static analysis highly overestimates the P_f that might lead to an uneconomical design. The results exhibit the shortcoming of the pseudo-static analysis that consider the peak seismic force to be time-invariant, thereby imparting more energy into the slope section as compared to actual time-variant strong motion. The P_f fluctuates significantly during the strong motion and might even attain momentarily high magnitudes, yet owing to the subsequent stress reversals, it might not lead to complete collapse of the slope section. Moreover, in dynamic analysis as well, it can be noted that the incorporation of uncertainty in the location of WT significantly increases the P_f of the slope section.

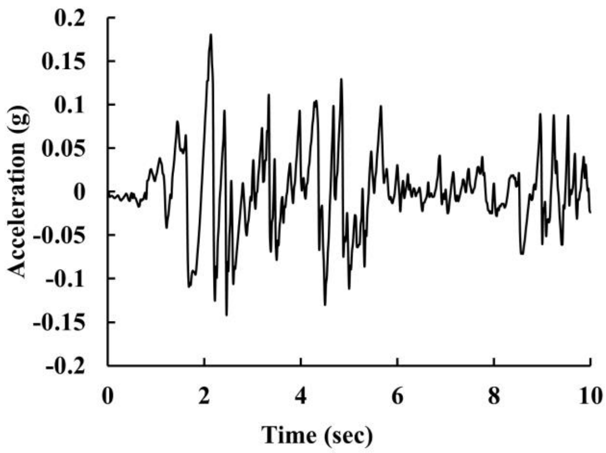


Fig. 10 Scaled down 1971 San Fernando strong motion used in the present study

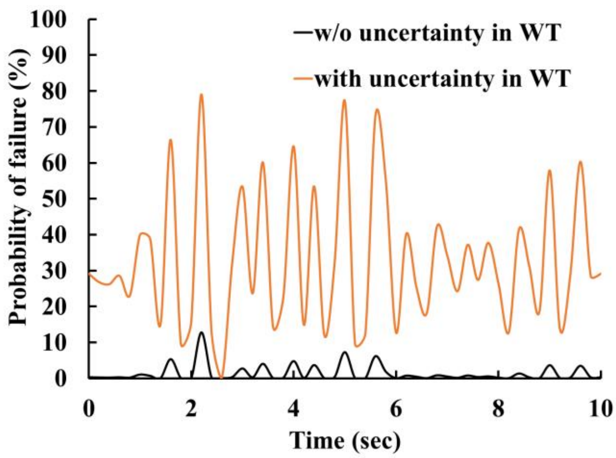


Fig. 11 Temporal variation in the maximum P_f for partially saturated slope

8. Conclusions

This study presents the usage of a probabilistic approach to illustrate the seismic response of partially saturated cut slope symbolically located in different seismic region of India. The study highlights the influence of CoV, correlation coefficient and the spatial variability of shear strength parameters on seismic response of the cut slope under dry as well as partially saturated conditions. It is seen that under pseudo-static earthquake condition, the probability of failure of the slope, comprising shear strength parameters with CoV in the range of 0.2-0.4 and located in seismic zone V, varies over a wide range of 0.05% - 38.25% and 1.56% - 44.90% for dry and partially saturated conditions, respectively. For a realistic assessment of seismic behaviour of cut slope sections, the outcomes recognize the utmost importance to conduct an extensive field study for computing the statistical parameters for simulating the spatial variability of geotechnical parameters to the best possible extent.

The study also reports the influence of uncertainty related to the WT location and pseudo-static earthquake coefficients on the seismic response of cut slopes. In comparison to the case where k_h and k_v are considered deterministic, introducing uncertainty in coefficients resulted in a marginal decrease in P_f. Moreover, along with uncertain k_h and k_v, incorporation of uncertainty in WT location is found to significantly increase the P_f in the tune of 64%.

Furthermore, based on a non-linear time-history analysis, the temporal seismic response of the cut slope is illustrated. The outcomes reveal that the pseudo-static analysis overestimated the peak value of P_f approximately by 30% and 19% for the WT location being considered as deterministic and random, respectively. Unlike the pseudo-static approach with overestimate time-invariant seismic energy, the NTHA considers the temporal variation of seismic forces and hence lead to a more realistic assessment P_f of the cut slope subjected to a strong motion.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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