ASSESSMENT OF BEARING CAPACITY FOR STRIP FOOTING LOCATED NEAR SLOPING SURFACE CONSIDERING ANN-MODEL

R. Acharyya
Research Scholar, Department of Civil Engineering, Indian Institute of Technology Guwahati, Assam, India.
Email: r.acharyya@iitg.ernet.in, ORCID No.: 0000-0003-4428-532X

A. Dey
Assistant Professor, Department of Civil Engineering, Indian Institute of Technology Guwahati, Assam, India.
Email: arindam.dey@iitg.ernet.in, ORCID No.: 0000-0001-7007-2729

ABSTRACT
There are many circumstances where shallow footings are constructed near sloping ground. When a footing is placed near the crest of sloping surface, the bearing capacity of the soil and the stability of the slope has been decreased remarkably based on the position of the footing with reference to the slope and slope inclination. In this regards, a sequence of finite element analysis has been carried out using Plaxis 2D v2015.02 to investigate the ultimate bearing capacity of strip footings located near $c$-$\phi$ soil slope. The influence of different geo-parameters on the load carrying capacity of the footing has been investigated and the outcomes are appropriately explained. Moreover, large database of numerically simulated ultimate bearing capacity has been considered for developing and verifying the ANN model to establish a predictive model equation and the relative importance of the input parameters. It has been professed that angle of internal friction is the most important input parameter for estimating the bearing capacity of strip footing located on crest of $c$-$\phi$ slope.

Keywords: Ultimate bearing capacity, Slope, Strip footing, Finite element, Plaxis, ANN

1. INTRODUCTION
Bearing capacity ($q_u$) of footing defines the maximum load that the foundation can carry without failure within allowable limits of settlement. The load carrying capacity of foundation depends on geotechnical and geometrical characteristics. Geotechnical characteristics comprise the shear strength and deformation parameters of soil. Geometrical characteristics include the size, depth and shape of the footing. In order to design an
adequate foundation for superstructures, bearing capacity is the key for geotechnical engineers. In this matter, based on several assumptions, Terzaghi [1] had given the first expressions to assess the bearing capacity of a strip footing resting over a semi-infinite horizontal ground surface. Meyerhof [2] further extended the proposition by considering the by assuming that the developed failure surface in the passive zone extends up to the ground surface, thus providing a different set of bearing capacity factors \( N_c, N_q \) and \( N_γ \). Later, on the basis of theory, field and laboratory investigations, Skempton [3] provided modified expressions for \( N_c \) considering footings of different shapes, sizes and embedment depths within a saturated clay medium. Thereafter, based on the work of several researchers [4, 5], a general bearing capacity expression had been formulated including all possible contributions of shape, size, embedment depth, load inclination, and compressibility of the founding medium.

In the hilly regions, as the construction permits, footings are mostly located on the crest of a slope (with some setback distance) or on the slope face itself, for e.g. footings of mobile and electric transmission towers, overhead water tanks, and bridge abutments. The bearing resistance of such footings is necessarily lower than the same placed on horizontal surface, attributed to the curtailed passive resistance zone developed towards the sloping face. For footings located on or near steeper slopes, the bearing resistance gets substantially reduced. In order to address this problem, experimental investigations have been sought to assess the bearing capacity and bearing capacity factors of strip footing positioned on a sandy slope [6, 7]. The formation of plastic zones underneath strip, square or circular footings placed on a sandy slope [8-13] as well as on \( c-\varphi \) soil slope [14, 15] have also been researched. In a similar line, researches have been conducted to investigate the overall performance of footing located at crest of slope, aided with single-sided micro pile or skirt strip footing [16, 17]. Apart from laboratory and numerical investigation, the performance of anchored inclined footings with aid of field experiment, is also reported [18]. However, as earlier classical researches, the above cited cases deal with a single footing resting on a sloping terrain.

North-eastern region of India comprises of hilly areas. As a result, constructions are constructed on or near the slope due to scarcity of plane land. In today’s era the development is increasing in every parts of India and North-eastern area of India is not also staying back. As a result, North-eastern region experiences numerous constructions started from single storey residential buildings to multi storey commercial buildings. In this
regard, an effort has been considered for assessing the impact of different geo-parameters on the ultimate bearing capacity of trip footing placed near $c$-$\phi$ soil-slopes slope with aid of 2-D finite element analysis.

The mechanism associated with the failure of footings located in hilly terrains is understandably complex, and obtaining a classical solution for the same is quite difficult. In this regard, soft computing techniques such as artificial neural networks (ANN), genetic algorithms (GAs), and multi-gene genetic programming (MGGP) can be resorted to for the development of a mapping architecture relating the bearing capacity of such footings and their contributory parameters. The application of ANN in civil engineering applications is quite popular in the assessment of pile load capacity [19-21], development of mathematical constitutive modelling [22, 23], formulation of correlation structures between liquefaction potential and seismic soil parameters [24], and evaluation of hydraulic conductivity and swelling pressure parameters of clayey soil [25-28]. In the purview of foundation engineering, ANN has been used for the prediction of settlement and load carrying capacity of shallow foundations placed on horizontal and sloping surface subjected to centric and eccentric loading [15, 23, 29-32].

The objective of the article is to determine the ultimate bearing capacity and failure mechanism of an isolated strip footing placed at the crest of the slope by considering different geotechnical and geometrical parameters with the aid of Plaxis 2D finite element tool. The outcomes of the simulation have been used in the development of artificial neural network structure. Sensitivity analyses have been done for determining the relative importance of the input parameters. Moreover, a prediction expression has been proposed for evaluation of dimensionless bearing capacity $q_{un} = \frac{q_u}{\gamma H}$ where, $q_u =$ Ultimate bearing capacity, $\gamma =$ Unit weight of soil and $H = $ Height of slope] of footing resting on crest of slope by considering the weights and biases of trained neural network. Although PLAXIS FE simulations are bound to provide the bearing characteristics based on intricate and rigorous mechanisms, it is not always possible to revert to FE simulations for each of the field problems. In this regard, the developed ANN expression for the prediction of bearing capacity would be very benefitting and useful to the consulting and design engineers for determining the ultimate bearing capacity of footing located near the slope considering various important geotechnical and geometrical input parameters. With the aid of simple calculating devices and on the basis of the architecture parameters, the bearing capacity can be easily and quickly computed.
2. NUMERICAL ANALYSIS

The numerical modelling for the present study is achieved with Plaxis 2D v2015.02, a commercially available finite element tool, which incorporates 2-D ground water flow, stability and deformation analyses in geotechnical engineering. It is capable to deal with numerous intricate aspects of geotechnical structures. Plaxis 2D comprises constitutive models for advanced non-linear, anisotropic or time-dependent analyses of geomaterials, and incorporates non-hydrostatic and hydrostatic pore pressures in soil. Literatures reveal that Plaxis 2D has been successfully used in the numerical studies for the assessment of bearing capacity of footing located at crest of sandy slope [8, 9, 13]. In the present research, the same has been used in the numerical modelling of strip footing resting on c-φ soil slope.

2.1 Description of the Modelling

In the current research, the numerical model has been prepared for footings located near the sloping ground as revealed in Fig. 1. For determination of optimum model dimensions, the “0.1q” (q is the failure load) stress contour signifies the furthest important isobar has been considered, beyond which the Impact of the applied stress is insignificant (Boussinesq’s elastic stress theory). The model size has been considered in such a way that the important isobar has not been touched by the edges of the domain.

In the simulation “standard fixity” has been applied on the model. Figure 2 depicts that the bottom edge of the domain is supposed to be rigid, therefore vertical and horizontal fixities have been employed, while horizontal fixity has been specified to vertical edges. In order to permitting free deformation to slope face, no fixities have been provided. In the present numerical investigation, the bearing capacity of the strip footing has been assessed for different location in the domain. Figure 3 portrays the various positions of strip footings located near the sloping surface with or without embedment depth.
In the finite element investigation, the domain of the geometry has been discretized into finite number of elements. In order to determine accurate displacements and stresses, 15-noded triangular elements were considered as it offers more nodes and Gauss points as compared to the 6-noded triangular elements. Plaxis 2D program generates automatic finite element meshes with aid of ‘robust triangulation technique’. Plaxis 2D provides a few basic meshing arrangements, which can be progressively refined with user defined coarseness factors. The adopted mesh should be adequately and optimally fine to attain precise and realistic numerical outcomes. The present investigation has been carried out with ‘fine meshing scheme’, aided with local refinements wherever large stress concentrations are expected to occur.

![Fig. 2 Typical meshing scheme and fixities](image1)

Fig. 2 Typical meshing scheme and fixities

![Fig. 3 positions of footings near the slope](image2)

Fig. 3 positions of footings near the slope

In the present study, elastic-perfectly plastic Mohr-Coulomb (M-C) model has been considered for soil behavior. M-C model requires five material parameter inputs, specifically three strength parameters (cohesion $c$, angle of internal friction $\phi$, and dilatancy angle $\psi$) and two deformation parameters (Elastic modulus $E$ and Poisson’s ratio $\nu$). As per the plasticity theory, the influence of soil dilation is expressed in terms of dilative coefficient, $\eta = \psi / \phi$, which varies in the range of $0 \leq \eta \leq 1$. The case $\eta = 1$ corresponds to the material
following an associated flow rule [33, 34], and the same has been adopted in the current study. The soil properties considered in the present research is given in Fig. 4. It has been perceived from past research that the unit weight of soil ($\gamma$) has limited influence on $q_u$ [13], and thus in the present numerical investigation, a constant magnitude of $\gamma = 16$ kN/m$^3$ has been used.

In the current study, the footing has been considered as rigid rough strip footing modelled for $M_{20}$ concrete and represented by linearly elastic (LE) model, the parameters of which are given in Table 1. An interface element has been given at the boundary of the concrete and soil elements, whose stiffness is obtained using Newton-Cotes algorithm [35]. A strength reduction factor ($R_{\text{inter}}$) has to be chosen for the interface element in order to provide its deformation characteristics. In the present study, $R_{\text{inter}} = 1$ has been adopted to represent a rough strip with no-slip mechanism with respect to the adjacent soil elements.

![Fig. 4 Chart of varying geotechnical and geometrical parameters](image)

**Table 1 Footing properties**

<table>
<thead>
<tr>
<th>Unit weight ($\gamma$) (kN/m$^3$)</th>
<th>Elasticity Modulus $E$ (GPa)</th>
<th>Poisson's ratio ($\nu$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>22</td>
<td>0.15</td>
</tr>
</tbody>
</table>
3. DEVELOPMENT OF ANN MODEL

Neural networks are bio-mimetic data mining structures containing several simple, yet highly interrelated, processing elements (termed as artificial neurons) in a complex architecture, which are used to develop correlation maps between the contributory parameters (inputs) and the model outcomes (outputs). Neural networks can successfully operate on datasets for solving practical problems, even if the datasets do not follow specific mathematical or logical patterns, or bears intricately complex relations difficult to decipher. Such architectures are capable of rapid generation of correlation structures with the aid of supervised or unsupervised training programs.

In the present study, ANN toolbox available with Matlab v2015A [36] has been considered for the development of the correlation map between the bearing resistance of strip footings on slopes and the influential parameters. This tool permits the user to choose various parameters of a neural architecture (the number of hidden layers and its neuronal connections with their connection weights and biases) and its various operator functions used in training, testing and validation of the neural architectures. In the present study, a multilayer feed-forward cum back-propagation network aided by Levenberg-Marquardt’s learning rule [28] has been used. The network structure considered in the present study comprises a 6-9-1 architecture (Fig. 5). The number of neurons in the hidden layer should be optimum to harness optimal network performance; too few neurons can provide under-fitting phenomenon, while too many neurons can lead the system toward remembering even the noisy arrangements in the records [20]. In the current networking, nonlinear tan-sigmoid transfer operator has been considered for all the neural connectors. The maximum iterations have been set to 50000, wherein the improvement in successive learning iterations are measured in terms of the Mean Square Error (MSE), as expressed in Eq. 1.

\[
MSE = \frac{1}{N_d} \sum_{i=1}^{N_d} (O_{\text{Simulation}} - O_{\text{ANN}})^2
\]  

(1)

where, \( O_{\text{Simulation}} \) is the simulated magnitudes, \( O_{\text{ANN}} \) is the predicted magnitudes of the identical unit and \( N_d \) is the total records.
4. VALIDATION OF NUMERICAL MODEL

In the present numerical investigation, the experimental research done by scholars [9] has been taken for validation of numerical modelling. Researchers [9] have estimated $q_u$ and bearing capacity factors for strip footings located at the crest of slope. Investigators have considered two strip footings of width 4 cm and 6 cm. The reinforced test box of size 100 cm long, 45 cm width and 40 cm height has been used in their experimental investigation. The angle of slope was 30°. Researchers have considered Italy sand for constructing the sloping ground. The friction angle ($\phi$), relative density ($D_r$) and dry unit weight ($\gamma_d$) and of soil were 38°, 87% and 17 kN/m$^3$ respectively. Researches have measured the bearing capacity of strip footing for different setback distances specifically for 16 cm, 30 cm and > 30 cm. The vertical load has been provided over the footing up to failure and the load carrying capacity of the footing has been evaluated.

The numerical models are validated with aid of 2D finite element analysis. Before conducting the validation investigation, a convergence analysis has been done for selecting optimum mesh size. The convergence investigation has been done for different footing sizes and various setback distances. Figure 6 describes the results of the convergence analysis in terms of non-dimensional average element size (NAES, ratio of average...
element length and the height of the slope), which shows that the outcomes are nearly similar beyond NAES ≈ 0.04, and the same has been used for the estimation of the numerical results from the validation model.

The validation of numerical model has been done with considering the fine meshing structure. In order to verify the numerical study the same model dimension and similar soil properties have been considered as taken by researchers [9]. In this regard, a rectangular footing of width of 4 cm width located at crest of the slope with setback distance of 30 cm has been modelled. The validation study has been represented in terms of load-settlement pattern. Figure 7 illustrates that there is a significant match between numerical outcomes and results obtained from experimental investigation done by Castelli and Lentini [9].

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**Fig. 6 Typical mesh convergence study**

**Fig. 7 Verification of the finite element model**
5. RESULTS AND DISCUSSION

5.1 Parametric Study

In the present study, various geotechnical and geometrical parameters are varied to realize the impact of parameters over the assessment of $q_u$ of footing placed near slope surface. In the parametric investigation, the influence of different geo-parameters on the bearing capacity of footing has been shown in terms of $q_u$ for different setback ratios ($b/B$).

Before conducting the parametric study, a convergence analysis has been performed. In order to perform the convergence test for meshing different slope angle, footing position, embedment depth of footing has been considered. It has been revealed that the magnitudes of obtained $q_u$ are same beyond fine mesh (NAES of 0.047). It has been perceived from Fig. 8 that fine mesh is the optimum mesh configuration to get finest results. The slope height (6 m) which has not been changed in the simulation, considered to calculate the non-dimensional mesh size. Furthermore, the further investigation has been done with fine mesh.

![Fig. 8 Mesh convergence analysis for footing located at slope](image)

**Fig. 8** Mesh convergence analysis for footing located at slope

5.1.1 Variation of cohesion ($c$)

Figure 9 explicates the influence of cohesion ($c$) of soil over the load carrying capacity of footing positioned over the crest of slope. It has been realized that the $q_u$ enhances with rise in cohesion in soil. It defines the fact that increase in cohesion effects increase in shear strength of foundation soil hence the load carrying capacity of footing increases.
5.1.2 Slope angle (β)

Figure 10 elucidates the impact of slope inclination on the assessment of \( q_u \) of footing positioned at crest of slope. In order to check the influence of slope angle, different slope angles were considered specifically, ranges from 20° to 40°. It has been revealed that \( q_u/\gamma H_s \) reduces with raise in slope inclination. Outcomes ascribe the information that the resistance of outward lateral movement of soil beneath the footing reduces due to rise in slope angle therefore load carrying capacity of footing decreases. It has been perceived that the after the setback back distance of 6\( B \), the impact of slope inclination over the estimation of bearing capacity diminishes. Moreover, after setback ratio (\( b/B \)) of 6, the footing shows same performance as footing behaves on horizontal ground.

![Fig. 9 Influence of cohesion (c) on \( q_u/\gamma H_s \)](image1)

![Fig. 10 Impact of angle of slope (β) on \( q_u/\gamma H_s \)](image2)
5.1.3 Width of footing \((B)\)

Figure 11 illustrates the influence of footing width \((B)\) over estimation of \(q_u\) of footing located near the crest of sloping surface. In this respect, the size of footing width has been varied from 0.5 m to 2 m. It has been perceived that the \(q_u/\gamma H_s\) enriches with enhance in footing width. It explains the fact that increases in footing width, the load on the footing will be spreader over the larger area in the subsoil and hence bearing capacity improved.

\[
\begin{align*}
&\text{Fig. 11 Impact of footing width \((B)\) on } q_u/\gamma H_s \\
&\text{Fig. 12 Impact of friction angle \((\varphi)\) on } q_u/\gamma H_s
\end{align*}
\]

5.1.4 Friction angle \((\varphi)\)

Figure 12 portrays the influence of friction angle \((\varphi)\) over the estimation of \(q_u\) of footing positioned on the crest of sloping surface. It has been revealed that the \(q_u/\gamma H_s\) enhances with raise in angle of internal friction of soil.
The results define the information that the passive resistance of soil increases for rise in friction angle of soil and hence the load carrying capacity of footing increases.

5.1.5 Embedment depth of footing (\(D_f\))

Figure 13 depicts the influence of embedment depth of footing positioned near the slope on estimation of bearing capacity. In this regards, different embedment ratio (\(D_f/B\)) have been taken, ranging from 0 to 1 as it has been seen in Fig. 3. It has been perceived that the \(q_u/\gamma H_s\) improves with raise in embedment depth ratio (\(D_f/B\)). The outcome describes the fact that any increase in embedment depth, the confinement increases in foundation soil and hence load carrying capacity of footing increases. It has also been revealed that the obtained dimensionless bearing capacities are identical for setback ratios which are beyond 6\(B\).

5.2 Study of Failure Mechanism

In the present numerical investigation, the failure mechanism developed beneath the strip footing located near the crest of sloping surface has been investigated for different slope angles and setback distances. Figure 14 exhibits that the passive zone formed beneath the footing deeply affected by the slope face, when footings placed near to slope. The effect is more noticeable for decreasing the setback distance. As a result, the load carrying capacity of the footing reduced due to inadequate generation of passive confinement from sloping face.

In the present research, the failure mechanism has been checked up to setback ratio (\(h/B\)) of 10. It has been professed that the effect of slope on developed failure mechanism remains up to setback ratio (\(h/B\)) of 6. The effect of slope completely disappeared for any further increase of setback ratio. The formed failure mechanism...
underneath the footing was indistinguishable for setback distances which are beyond 6B. Moreover, the developed failure mechanisms beyond 6B are identical to failure mechanism generated beneath the footing for footing positioned on uniform horizontal ground.

Fig. 14 Representative development of plastic zones

5.3 ANN Results

5.3.1 Normalization of input and output

In the present research, the expression has been used for normalization given in Eq. 2. In this investigation, raw data of inputs and outputs has been normalized between 0 and 1.

\[
P_i^* = \frac{P_i^* - P_i^{\text{min}}}{P_i^{\text{max}} - P_i^{\text{min}}} \tag{2}
\]

where \( P_i^* \) and \( P_i^o \) are the \( i \), components of input or output vector before and after normalization respectively.

\( P_i^{\text{max}} \) and \( P_i^{\text{min}} \) are the maximum and minimum magnitudes of all components of input or output vector before normalization.
5.3.2 Number of hidden neuron

In the current study, a convergence analysis has been conducted to identify the optimal number of neurons required in the hidden layer to properly characterize the input-output correlation of the analysed dataset. The tolerance is set over a mean squared error (MSE) value of 0.02, which has been attained when the number of neurons in the hidden layer exceeded 9, as shown in Fig. 15.

![Fig. 15](image_url)

Fig. 15 Estimation of optimum number of hidden neurons during the training phase

To verify the neural network architecture obtained from above convergence analysis, the number of neurons in hidden layer has been varied with the percentage of error found in validation set after analysis. The percentage of error ($E$) has been calculated from the following expression (Eq. 3).

$$E(\%) = \frac{\text{ABS}(O_{\text{Simulation}} - O_{\text{ANN}})}{O_{\text{Simulation}}} \times 100$$  \hspace{1cm} (3)

where $E$ is the error in percentage, $O_{\text{Simulation}}$ is the simulated magnitudes and $O_{\text{ANN}}$ is the predicted magnitudes of the identical unit.

Figure 16 portrays the pattern of error in validation set with increasing the number of neurons in the hidden layer. It has been perceived that the percentage of error reduces with increasing the number of neurons up to 9, beyond this number of neurons in hidden layer the percentage of error increased in validation set. It has been observed that the minimum percentage of error, 0.89\%, has been found for 9 numbers of neurons which is same as that obtained from the training set. Thus, the optimality of the hidden layer in the chosen ANN architecture is confirmed. The same has been considered as an optimal value and has been used in further studies. Hence, based
on the convergence study, a 6–9–1 ANN architecture has been developed with six input nodes (representing $c$, $\varphi$, $B$, $b/B$, $\beta$ and $D_f/B$), a single output node ($q_{\text{un}}$) and nine (9) hidden neurons (as shown in Fig. 5).

![Diagram](image)

Fig. 16 Impact of number of neurons in hidden layer during the validation phase

5.3.3 Generalization and performance of the ANN architecture

Neural network architecture has the ability to modify its performance with respect to a particular problem. ANN has self-adjusting capability to capture the valid pattern of given dataset, and this process is referred as training. In training, the connection weights of neurons changes systematically to produce the desire outcomes. The main purpose of training is to identify the optimal connection weights which will provide tolerable and minimum MSE [37]. In neural networking, the separation of samples for training, testing and validation has been done as per the conventional usage [22], and same has been provided in Table 2. The optimal weights obtained from training step, have been retained for testing and validation stages. The training parameters used in the study has been tabulated in Table 3.

<table>
<thead>
<tr>
<th>Performance</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training</td>
<td>1120</td>
</tr>
<tr>
<td>Testing</td>
<td>480</td>
</tr>
<tr>
<td>Validation</td>
<td>400</td>
</tr>
</tbody>
</table>
Table 3 Training parameters considered in the present study

<table>
<thead>
<tr>
<th>Training parameters</th>
<th>Magnitudes and nomination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training function</td>
<td>‘trainlm’ (Levenberg Marquardt)</td>
</tr>
<tr>
<td>Transfer function</td>
<td>a. Hidden layer</td>
</tr>
<tr>
<td></td>
<td>b. Output layer</td>
</tr>
<tr>
<td></td>
<td>a. ‘tansig’ (non-linear function)</td>
</tr>
<tr>
<td></td>
<td>b. ‘purelin’ (linear function)</td>
</tr>
<tr>
<td>Performance function</td>
<td>‘mse’ (Mean square error)</td>
</tr>
<tr>
<td>Error after learning</td>
<td>0.001</td>
</tr>
<tr>
<td>Divide function</td>
<td>‘dividerand’</td>
</tr>
<tr>
<td>Epochs</td>
<td>50000</td>
</tr>
<tr>
<td>Number of neurons in input layer</td>
<td>06</td>
</tr>
<tr>
<td>Number of hidden layer</td>
<td>01</td>
</tr>
<tr>
<td>Number of neurons in hidden layer</td>
<td>09</td>
</tr>
<tr>
<td>Number of output layer</td>
<td>01</td>
</tr>
</tbody>
</table>

The training has been continued as long as the error decreased in the test data set, and the same has been stopped when the error started growing during the process. The training started considering the default termination values as provided in Table 4. The training is halted when the error is lowest in testing data set (early stopping) although the error on training dataset has been found to decrease further as training is continued [38]. The number of validation checks signifies the number of consecutive iterations that the validation performance fails to decrease. Several termination criteria has been used simultaneously namely epochs, computational time (s), initial and maximum mu (controlling weight perturbations), minimum gradient (Min_grad) and maximum validation fails (Max_fail). The training program is supposed to stop when any of criteria is met. In the present study, the training program terminated when the maximum number of validation fails reached the termination criterion of 10. The magnitudes achieved by rest of the parameters at the event of termination are provided in Table 4.
Table 4 Stopping parameters and training criteria considered for ANN training

<table>
<thead>
<tr>
<th>Stopping parameters</th>
<th>Value considered for termination</th>
<th>Stopping value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epochs</td>
<td>50000</td>
<td>10000</td>
</tr>
<tr>
<td>Time (sec)</td>
<td>1800</td>
<td>18</td>
</tr>
<tr>
<td>Mu (initial)</td>
<td>0.001</td>
<td>0.010</td>
</tr>
<tr>
<td>mu_max</td>
<td>$10^1$</td>
<td></td>
</tr>
<tr>
<td>Min_grad</td>
<td>$10^{-7}$</td>
<td>0.0616</td>
</tr>
<tr>
<td>Max_fail</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 17 depicts the outcome of the training process as an agreement of the magnitudes of $q_{un}$ obtained from Finite Element simulations (simulated $q_{un}$) with those obtained as predictions of the ANN model (predicted $q_{un}$). In the training phase, a coefficient of correlation ($R^2$) has been obtained as 0.998, which signifies a strong agreement between predicted and simulated outcomes, thus highlighting the optimal training of the ANN architecture. Under this condition, the connection weights (input-hidden and hidden-output) obtained as an outcome of the training programme is highlighted in Tables 5 and 6, which are subsequently used in the testing and validation phases. Table 7 highlights the corresponding biases.

Further, the trained ANN structure, with its optimal connection weights, has been considered for the testing phase to check the prediction capacity of the network. Figure 18 portrays that there is excellent match ($R^2 = 0.996$) of $q_{un}$ obtained from numerical investigation (FE simulations) with those predicted by ANN model. The testing phase with a high correlation coefficient provides a confirmatory that the trained neural network has superior prediction ability.
Fig. 17 Capability of neural structure for training phase

Table 5 Input-Hidden weights

<table>
<thead>
<tr>
<th>X</th>
<th>c (X1)</th>
<th>6.08</th>
<th>0.56</th>
<th>0.17</th>
<th>-0.25</th>
<th>-0.18</th>
<th>0.85</th>
<th>-0.17</th>
<th>-0.46</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>φ (X2)</td>
<td>3.18</td>
<td>0.08</td>
<td>0.18</td>
<td>-0.35</td>
<td>-0.88</td>
<td>-0.56</td>
<td>-0.28</td>
<td>0.094</td>
<td>-0.13</td>
</tr>
<tr>
<td></td>
<td>B (X3)</td>
<td>2.64</td>
<td>0.53</td>
<td>0.050</td>
<td>0.23</td>
<td>0.057</td>
<td>0.069</td>
<td>0.41</td>
<td>-0.14</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>b/B (X4)</td>
<td>-0.53</td>
<td>-0.17</td>
<td>0.018</td>
<td>-0.12</td>
<td>-0.0075</td>
<td>0.051</td>
<td>-0.18</td>
<td>0.33</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>β (X5)</td>
<td>3.84</td>
<td>0.086</td>
<td>-0.37</td>
<td>-0.19</td>
<td>-0.15</td>
<td>0.43</td>
<td>-0.16</td>
<td>0.005</td>
<td>-0.033</td>
</tr>
</tbody>
</table>

Table 6 Hidden-Output weights

| Y   | qe/γHs | 5.66 | 1.56 | -1.83 | -4.58 | -0.60 | -0.17 | 5.19 | 1.31 | 1.71 |
Table 7 Biases obtained after training phase

<table>
<thead>
<tr>
<th>Hidden layer biases ($b_h$)</th>
<th>Output layer biases ($b_o$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-19.00</td>
<td>0.40</td>
</tr>
<tr>
<td>0.80</td>
<td>-</td>
</tr>
<tr>
<td>-1.77</td>
<td>-</td>
</tr>
<tr>
<td>2.10</td>
<td>-</td>
</tr>
<tr>
<td>-0.70</td>
<td>-</td>
</tr>
<tr>
<td>0.90</td>
<td>-</td>
</tr>
<tr>
<td>2.03</td>
<td>-</td>
</tr>
<tr>
<td>0.51</td>
<td>-</td>
</tr>
<tr>
<td>0.73</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 18 Capability of neural structure for testing phase

Subsequently, the trained ANN structure has been engaged with the validation dataset which have not been considered to form ANN architecture, neither in training nor in testing phase. The validation step is useful in checking the aptness of the ANN model to generalize the outcome of the physical problem. Figure 19 illustrates excellent match ($R^2 = 0.998$) between simulated and forecasted magnitudes of $q_{un}$ for strip footing placed near sloping surface, thus highlighting the superior generalization capability and performance of the 6-9-1 ANN architecture.
The progress of training was inspected for all the above phases, i.e. training, testing, and validation, by plotting the variation of MSE with the completed number of iterations, as presented in Fig. 20. It is observed that errors obtained from test set and validation sets have identical appearances and hence, there is no overfitting during the training of the proposed architecture [29].

In the present investigation, the optimality of the trained network has been checked based on the training statistics over several restarts. Figure 21 portrays the pattern of coefficient of correlation ($R^2$) for several restarts of the network. In the current study, the network has been restarted for 100 times and for each restart the magnitude of

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**Fig. 19** Capability of neural structure for validation phase

**Fig. 20** Comparison of MSE
$R^2$ has been recorded. It has been observer that the magnitude of $R^2$ varies from 0.996 to 0.998. The pattern of $R^2$ for several restarts confirms the fact that the neural network is optimally trained and the network having excellent capability of prediction.

![Graph of $R^2$ for several training restarts](image)

**Fig. 21 Behaviour of $R^2$ for several training restarts**

### 5.3.4 Sensitivity study

Sensitivity investigation is a necessary research conducted for the ranking of input variables, thus highlighting their sorted influence on the outcome of the physical problem. Such study aids in the ‘design of experiments’ when large number of the numerical or physical simulations are required, while based on the sensitive influence of the contributing parameters, only few of them are selected for the actual modelling. Thus, sensitivity analysis helps in the reduction in the number of model simulations, without jeopardizing the generality of the obtained results. Various methods have been advised in the articles for identifying the vital input parameter [23, 24]. In the present study, Garson’s algorithm [39, 15, 20, 23, 27, 28, 40] has been considered to identify the significance of inputs. In order to apply Garson’s algorithm, firstly the input-hidden (Table 5) and hidden-output (Table 6) weights are segregated, and the absolute magnitudes of weights have been considered to differentiate the rank of input parameters (Eq. 4).

$$\text{Input}_x = \sum_{n} \left\{ \frac{|\text{Hidden}_{xN}|}{\sum_{l} |\text{Hidden}_{lN}|} \right\} \quad \text{(Garson’s algorithm)}$$

(4)

where, $\text{Hidden}_{xN}$ represents the absolute magnitudes of the connection weights between the input nodes and hidden layer nodes (as provided in Table 2), and $\text{Hidden}_{lN}$ represents the absolute magnitudes of the product of
the connection weights between the input-hidden and hidden-output nodes. The sample HiddenZN as obtained for the 6-9-1 ANN model developed for the present study is provided in Table 8, the values of which are obtained from the corresponding products of the parameter magnitudes provided in Table 5 and Table 6.

Table 8 Product of the input-hidden and hidden-output weights

<table>
<thead>
<tr>
<th>X</th>
<th>Hidden N1</th>
<th>Hidden N2</th>
<th>Hidden N3</th>
<th>Hidden N4</th>
<th>Hidden N5</th>
<th>Hidden N6</th>
<th>Hidden N7</th>
<th>Hidden N8</th>
<th>Hidden N9</th>
</tr>
</thead>
<tbody>
<tr>
<td>c (X1)</td>
<td>22.67</td>
<td>0.62</td>
<td>-0.32</td>
<td>1.158</td>
<td>0.11</td>
<td>-0.14</td>
<td>-0.89</td>
<td>-0.61</td>
<td>0.87</td>
</tr>
<tr>
<td>φ (X1)</td>
<td>34.50</td>
<td>0.91</td>
<td>-0.07</td>
<td>5.43</td>
<td>0.27</td>
<td>0.44</td>
<td>-5.06</td>
<td>-0.76</td>
<td>1.10</td>
</tr>
<tr>
<td>B (X1)</td>
<td>18.05</td>
<td>0.12</td>
<td>-0.34</td>
<td>1.61</td>
<td>0.53</td>
<td>0.09</td>
<td>-1.49</td>
<td>0.12</td>
<td>-0.23</td>
</tr>
<tr>
<td>b/B (X1)</td>
<td>14.98</td>
<td>0.83</td>
<td>-0.09</td>
<td>-1.06</td>
<td>-0.03</td>
<td>-0.01</td>
<td>2.13</td>
<td>-0.18</td>
<td>-0.35</td>
</tr>
<tr>
<td>β (X1)</td>
<td>-3.04</td>
<td>-0.27</td>
<td>-0.03</td>
<td>0.55</td>
<td>0.004</td>
<td>-0.008</td>
<td>-0.94</td>
<td>0.44</td>
<td>-0.28</td>
</tr>
<tr>
<td>Df/B (X1)</td>
<td>21.81</td>
<td>0.13</td>
<td>0.68</td>
<td>0.90</td>
<td>0.09</td>
<td>-0.07</td>
<td>-0.85</td>
<td>0.007</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Consequently, Equation 2 indicates the assessment of importance ranking for input predictors X (X = 1-6), utilizing the weights of individual input neurons Z (Z = 1–6) to each of the hidden layer nodes N (N = 1–9), and lastly to a single output node (Y). The outcome of the sensitivity study carried out with the aid of Garson’s method to provide the importance ranking to the input parameters is highlighted in Table 9. It can be observed that the geotechnical parameters (shear strength parameters) are more sensitive and play a dominant role in deciding the magnitude of $q_{uc}$.

Table 9 Rank of input-parameters from Garson’s algorithm

<table>
<thead>
<tr>
<th>Input</th>
<th>Relative importance</th>
<th>Relative importance (%)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>1.68</td>
<td>18.72</td>
<td>2</td>
</tr>
<tr>
<td>φ</td>
<td>3.17</td>
<td>35.27</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1.47</td>
<td>16.39</td>
<td>3</td>
</tr>
<tr>
<td>b/B</td>
<td>1.02</td>
<td>11.37</td>
<td>5</td>
</tr>
<tr>
<td>β</td>
<td>0.60</td>
<td>6.64</td>
<td>6</td>
</tr>
<tr>
<td>Df/B</td>
<td>1.05</td>
<td>11.61</td>
<td>4</td>
</tr>
</tbody>
</table>
According to Garson’s method, the angle of internal friction, $\phi$, has been identified as the utmost significant input parameter lagged by $c$, $B$, $D_f/B$, $b/B$ and $\beta$, according to Garson’s method, as shown in Table 8. Hence, it has been revealed from the sensitivity investigation that the angle of internal friction of soil was most significant input parameters to assess $q_u$ of strip footing placed at the crest of a $c-\phi$ slope.

The ranking of input parameters found from Garson’s algorithm (1991) has been checked through an alternative local perturbation technique with the graceful degradation of the error curves. The basic concept that has been considered is that each of input parameters of the trained network has been altered slightly and the corresponding change in the output has been recorded, while the remaining input parameters are maintained constant. In the sensitivity investigation, input parameters have been varied from -20% to 20% of their mean value in the predictive network to realize the influence of various geotechnical and geometrical input parameters on the output. Figure 22 portrays the percentage change in prediction of $q_u/\gamma H_s$ due to percentage change in input parameters. The graceful degradation technique has been applied to 5 different sets of input parameters, and the results obtained are plotted as error bars in the figure. The small error bars indicate the affirmative consistency of the applied technique. It has been perceived that the angle of internal friction of soil, $\phi$ is the most influential input parameter as it has maximum effect on the changes in prediction, followed by the effects of $c$, $B$, $D_f/B$, $b/B$ and $\beta$. It can be observed that both the local perturbation technique and Garson’s algorithm yields similar results, thus putting confidence on the outcome.

Fig. 22 Relative importance of the input parameters by means of graceful degradation
5.3.5 ANN Prediction equation for $q_u/\gamma H_s$

A predicting equation is prepared with aid of weights obtained from trained neural network model [20, 31, 32, 40, 41]. The mathematical expression connecting input parameters ($c$, $\varphi$, $B$, $b/B$, $\beta$, $D/f$) with output ($q_u/\gamma H_s$) represented in Eq. 5.

$$(q_u/\gamma H_s)_n = f_{Sig}(b_h + \sum_{h=1}^{h} w_{hh}(b_h + \sum_{i=1}^{m} w_{ih} X_i)))$$

(5)

Where $(q_u/\gamma H_s)_n$ is the normalized magnitude of $q_u/\gamma H_s$ in the variation of [-1,1]. $X_i$ is the normalised magnitude of inputs in the variation of [-1,1]. $f_{Sig}$ Sigmoid transfer function, $f_{Lin}$ linear transfer function, $m$ is the number of input present as variables, $h$ is the number of neurons present in the hidden layer, $w_{ih}$ is the connection weight between $i$th layer of input and $N$th neuron of hidden layer, $w_{NN}$ is the connection weight between $N$th neuron of hidden layer and single output neuron, $b_{NN}$ is the bias at the $N$th neuron of hidden layer and $b_{0}$ is the bias at the output layer.

The model equation for $q_{un}$ of strip footing located at the crest of slope, subjected to vertical loading as revealed in Fig. 1, has been framed with aid of weights and biases provided in Table 3, Table 4 and Table 5. The $(q_u/\gamma H_s)_n$ magnitude is in the variation of [-1, 1] and this required to be de-normalized as per following Eq. 5.

$$(q_u/\gamma H_s) = 0.5[(q_u/\gamma H_s)_n + 1][((q_u/\gamma H_s)_{max} - (q_u/\gamma H_s)_{min}) + (q_u/\gamma H_s)_{min}]$$

(5)

where $(q_u/\gamma H_s)_{max}$ and $(q_u/\gamma H_s)_{min}$ are the maximum and minimum magnitudes of $q_u/\gamma H_s$ correspondingly in the records.

6. CONCLUSIONS

On the basis of the finite element numerical simulations and ANN based prediction to obtain the bearing capacity of strip footing located on the crest of a generalized soil slope, the following conclusions are drawn:

- Bearing capacity of a footing, locating near sloping surface, raises with enhance in the shear strength variables of foundation soil, embedment depth and footing width, though it decreases with the raise in angle of slope.

- Bearing capacity of strip footing located at crest of slope improves with rise in setback distance. The footing exhibits similar behaviour to that on horizontal ground after a setback ratio of $(b/B)_{critical} = 6$. 
Based on the high correlation coefficients obtained for training, testing and validation phases of the ANN model, a 6-9-1 ANN structure provided the ‘best capable’ model to predict the $q_{un}$ of strip footings located at crest of a $c$-$\varphi$ slope surface.

From sensitivity study, as per Garson's methods, angle of internal friction of soil, $\varphi$ is established to be the most influential parameters.

Based on the weights and biases obtained from trained neural network, a prediction expression has been provided which can be suitably considered for estimating $q_{un}$ of strip footing placed on a $c$-$\varphi$ slope referring to the prior knowledge of the geometrical and geotechnical parameters. This prediction expression will help as a quick tool for the practising and consulting engineers involved in the design of hill-side urbanizations and constructions.

REFERENCES


