Importance of Constitutive Model in assessing 3-D Bearing Capacity of Square Footing on Clayey Soil using Finite Element Analysis

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ABSTRACT

The classical bearing capacity estimation techniques commonly adopts plane-strain condition and mostly confines to Mohr-Coulomb (MC) elastic-perfectly plastic model for representing the soil behaviour. However, being a plane-strain adaptation, MC model cannot be explicitly used to delineate the stress-strain response and bearing capacity of an arbitrarily shaped footing resting on semi-infinite medium. Further MC model cannot represent the post-peak strain-softening behaviour. In this regard, this article addresses the finite-element based bearing capacity estimation of a square footing resting on the surface of a semi-infinite clayey medium. The footing is represented by linear-elastic element, while the clayey foundation soil has been represented by both MC and Modified Cam-Clay (MCC) models. Comparison of the load-displacement response of the uniformly loaded footing, obtained from considering the MC and MCC models, clearly delineates the shortcoming of the MC and necessity of adopting MCC behaviour in a 3-dimensional geotechnical problem.

Keywords: 3-D bearing capacity; Constitutive model; Finite Element Analysis

1. INTRODUCTION

Assessment of bearing capacity has been one of the most popular problems in geotechnical since its early days. The classical works related to the estimation of bearing capacity confined to strip footings resting on semi-infinite medium (Terzaghi 1943; Meyerhof 1951; Vesic 1973). However, it is worth mentioning that in contrast to strip footings, isolated square or rectangular, or combined, footings to be more common for building foundations. As such footings, when loaded, represent 3-dimensional stress distribution, their characteristic response and failure mechanism is supposed to be different from the plane-strain mechanism observed for strip footing. In this regard, there have been modification in the basic theories of bearing capacity for strip footings to accommodate the shape effect. Experimental and numerical approaches have been resorted to estimate the bearing capacity of square footings on cohesionless or cohesive soils (Cerato and Lutenegger 2006; Acharyya and Dey, 2017; Acharyya et al. 2018).

In all the cases, it has been a common practice to utilize Mohr-Coulomb (MC) elastic-perfectly plastic model for representing the soil behavior. In order to cater the shape effect of footings on the bearing capacity, it is a common practice to utilize suitable shape factors governed by the aspect ratio of the footing. However, being a plane-strain adaptation, MC model cannot explicitly be used to delineate the stress-strain response and bearing capacity of an arbitrarily shaped footing resting on semi-infinite medium. Moreover, the MC model fails to address any post-peak strain-softening feature in the load-displacement characteristics of a loaded footing. Owing to the deficiency of M-C model in highlighting the development of intermediate principal stresses, the 3-dimensional response of such foundations cannot be elucidated. This necessitates the application of an advanced generalized 3D-constitutive model that can accommodate both strain hardening and strain softening behaviour of the foundation soil. Application of Modified Cam-Clay (MCC) model, within a numerical
framework, can be an effective means of simulating the 3D bearing capacity problems. Application of MCC model for geotechnical problems has already been into practice (Abed and Vermeer 2004; Lim et al. 2010; Wang and Bienen 2016; Oh and Vanapalli 2018).

In this regard, this paper addresses the estimation of bearing capacity of a square footing resting on the surface of a semi-infinite clayey medium. The analysis has been conducted with the aid of finite element analysis. The footing is represented by linear-elastic element, while the foundation soil, the clayey medium, has been represented by both MC and MCC models. The influence of the chosen constitutive model on the load-deformation response and the developed failure mechanism has been clearly elucidated through the study.

2. DESCRIPTION OF CONSTITUTIVE MODELS

The generalized 3D nonlinear elasto-plastic constitutive relation can be represented by the following incremental form

\[ d\sigma' = C_{ijkl} d\varepsilon_{ij} \]  

where \( \sigma' \) and \( \varepsilon_{ij} \) define the effective stress and strain tensor, respectively and the elasto-plastic tangent stiffness tensor \( C_{ijkl} \) is given by

\[ C_{ijkl} = E_{ijkl} E_{ijkl'} P_{ijkl'} Q_{ijkl'} E_{ijkl'} \]  

where \( E_{ijkl} \) is the isotropic elastic stiffness tensor, \( H \) is the hardening modulus, \( P \) and \( Q \) are the directions of outer normal to the plastic potential (g) and yield surface (f), respectively, \( P_q = \partial g/\partial \sigma' \) and \( Q_q = \partial f/\partial \sigma' \). Simulations have been carried out employing two different constitutive models, Modified Cam-Clay (MCC) and Mohr-Coulomb (MC). For the present work, focus has been restricted only to the associative formulation of these two models enforcing same mathematical expression for the plastic potential and the yield surface \( (g = f) \).

The MCC model is a critical state concept-based hardening (softening) model with following expression for the yield surface

\[ f = \frac{q'}{M} + p'(p' - p'_p) \]  

In the above expression, \( q' \), \( p' \), \( M \), \( p'_p \) are the shear stress, effective mean pressure, shear stress ratio \( (q'/p') \) at critical state and pre-consolidation pressure. In MCC model, a logarithmic relation is assumed between specific volume \( (v) \) and \( p' \) in virgin isotropic compression

\[ v - v_0 = -\lambda \ln \left( \frac{p'}{p_0} \right) \]  

where \( \lambda \) is the isotropic compression index. During unloading or reloading, a different logarithmic relation is followed

\[ v - v_0 = -\kappa \ln \left( \frac{p}{p_0} \right) \]  

where \( \kappa \) is the isotropic swelling index. The isotropic hardening (softening) relation is defined in terms of evolution of \( p'_p \) with plastic volumetric strain \( (e'_p) \)

\[ dp'_p = \frac{vp'_p}{\lambda - \kappa} \]  

The bulk modulus \( (K) \) depends on \( p' \) and \( \kappa \)

\[ K = \frac{vp'_p}{\kappa} \]  

The yield condition of the elastic-perfectly plastic MC model consists of following six yield functions in terms of principal stresses

\[ f_{1a} = \frac{1}{2} \left( \sigma'_2 - \sigma'_3 \right) + \frac{1}{2} \left( \sigma'_2 + \sigma'_3 \right) \sin \phi - c \cos \phi \leq 0 \]

\[ f_{1b} = \frac{1}{2} \left( \sigma'_3 - \sigma'_2 \right) + \frac{1}{2} \left( \sigma'_3 + \sigma'_2 \right) \sin \phi - c \cos \phi \leq 0 \]

\[ f_{2a} = \frac{1}{2} \left( \sigma'_3 - \sigma'_1 \right) + \frac{1}{2} \left( \sigma'_3 + \sigma'_1 \right) \sin \phi - c \cos \phi \leq 0 \]

\[ f_{2b} = \frac{1}{2} \left( \sigma'_1 - \sigma'_3 \right) + \frac{1}{2} \left( \sigma'_1 + \sigma'_3 \right) \sin \phi - c \cos \phi \leq 0 \]

\[ f_{3a} = \frac{1}{2} \left( \sigma'_2 - \sigma'_1 \right) + \frac{1}{2} \left( \sigma'_2 + \sigma'_1 \right) \sin \phi - c \cos \phi \leq 0 \]

\[ f_{3b} = \frac{1}{2} \left( \sigma'_2 - \sigma'_1 \right) + \frac{1}{2} \left( \sigma'_2 + \sigma'_1 \right) \sin \phi - c \cos \phi \leq 0 \]

In case of soil behaviour prediction through MC model, the elastic response is assumed linear...
with constant elastic parameter values. The constitutive model parameters for MC and MCC model and their values used for FEM simulation have been listed in Table 1.

Fig. 1 presents the stress-strain and volumetric predictions of single element triaxial test for both the models at 200 kPa confining pressure with the parameter set given in Table 1. It can be observed from Fig. 1(a) that the predicted maximum shear stress from two models nearly becomes same (400 kPa) at a large axial strain, i.e., at 30%. For the MC model, however, the maximum shear stress is achieved at a very low axial strain level (around 1%) and only a slight variation on this onset strain may arise due to change in the elastic properties. The MCC model aptly captures the compressive volumetric response of the normally consolidated soil at 200 kPa confining pressure [Fig. 1(b)]; whereas, mostly a dilative repose has been predicted by the MC model. It is important to note that the volumetric strain in MCC model nearly becomes constant when the maximum shear stress has been achieved, i.e, it corresponds to a critical state marking the failure of the material. The excessive dilation prediction of the MC model may have significant impact on the manifestation of the failure mechanism in the bearing capacity simulation that is further explored in the successive section. Further, such excessive dilation can only be handled by incorporation of non-associative flow rule in the model (Wood, 2004) which is kept out of the scope of this paper.

Table 1 MCC and MC model parameter values considered for FEM simulation

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCC</td>
<td>( \lambda )</td>
<td>0.077</td>
</tr>
<tr>
<td></td>
<td>( \kappa )</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>( M )</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>( p_p )</td>
<td>200 kPa</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>0.3</td>
</tr>
<tr>
<td>MC</td>
<td>( c )</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( \phi )</td>
<td>30(^\circ)</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>( E )</td>
<td>20 &amp; 60 MPa</td>
</tr>
</tbody>
</table>

Fig. 1 Triaxial test predictions for MCC and MC model at 200 kPa confining pressure with the parameter set given in Table 1

5. FINITE ELEMENT MODELING

In the present study, the bearing capacity of a square footing, resting on normally consolidated clayey foundation soil, has been assessed with the aid of three-dimensional finite element simulation using PLAXIS 3D AE.01. This software package is sufficed by several features to deal with complex geotechnical problems related to deformation, stability and flow through geotechnical media. Additionally, the availability of a large number of varying constitutive models for representing the stress-strain-time behaviour of soil makes it a very useful investigative tool to study the response of geotechnical entities (PLAXIS Material model manual 2015).

Fig. 2 depicts the typical FE model used for the present study. Following the procedure described in Acharyya et al. (2018), the size of the model domain (14 m in length and breadth with a height of 6 m) has been so decided that the existing lateral boundaries do not influence the stress or strains generated beneath the loaded footing. Standard fixity has been applied to the FE model such that the lateral boundaries are allowed only vertical deformation, while the
bottom boundary is considered non-yielding. The model geometry has been discretised with the aid of 10-noded tetrahedral elements providing a second-order interpolation of displacements, accompanied by a 4-point Gaussian integration scheme. Various types of meshing schemes can be adopted in PLAXIS, governed by the ‘mesh coarseness factor’. For the present case, mesh convergence study had been carried out (Acharyya et al. 2018) and, accordingly, an optimal fine mesh has been chosen, associated with local refinements adjacent to the footing. Further details of the FE analysis can be found in PLAXIS 3D Reference Manual (2015).

The rigid square footing of width 2 m, made of M20 concrete, is represented by Linear Elastic (LE) element (unit weight $\gamma = 25$ kN/m$^3$, elastic modulus $E = 22$ GPa, Poisson’s ratio $\nu = 0.15$) comprising 10-noded tetrahedral elements, and accompanied by an interface element (at the boundary of soil and concrete material) ensuring no-slippage condition. In order to comprehend the influence of constitutive behaviour of soil on the bearing characteristics of a square footing, the soil is represented by two types of constitutive behaviour, namely Mohr-Coulomb (M-C) and Modified Cam Clay (MCC) models.

6. RESULTS AND DISCUSSIONS

The load bearing response of the footing resting on the clayey medium has been assessed through load-settlement plots, along with the displacement and stress distributions within the foundation.

Fig. 3 shows the load-settlement response obtained at the mid-point of the footing resting on MC soil. It can be observed that the ultimate bearing capacity ($q_u$) is approximately 308 kPa. As per Terzaghi (1943), the ultimate bearing capacity of a square footing resting on the surface of a cohesionless semi-infinite medium can be ascertained as $q_u = 0.4\gamma BN$. Considering the bearing capacity factor $N_r = 22.5$ for $\phi = 30^\circ$, $q_u$ can be computed as 306 kPa. The excellent match between the theoretical and numerical magnitudes aids in the validation of the developed FE model.

Fig. 4 shows the typical failure deformations of the footing resting on MC soil having stiffness 60 MPa. It can be observed that the deformation pattern closely follows a general shear failure, wherein the soil elements beneath the footing is pushed downwards in the form of a pyramid, which induces an outward and upward radial shear in the adjacent soil elements towards the edge of the footing. This behavioural response is similar to the formation of the slip lines beneath a footing undergoing a general shear failure following a plane-strain condition, conforming to a vertical section passing through the centre of the footing.

Fig. 5 shows the load-settlement response of the footing resting on MCC soil. It can be observed that in comparison to the footing resting on the MC soil, the ultimate bearing capacity enhances to approximately 680 kPa, accompanied by a higher failure settlement. Fig. 6 highlights the deformation pattern of the soil, which exhibits volumetric compression, and is markedly different from the dilative pattern observed for MC soil. It can be noted that the MCC soil does not exhibit heaving at the edges of the footing, which was evident for MC soil. Further scrutiny shows that in the MCC soils, outward radial shears initiates beneath the
footing itself, thus leading to a mixed elastic wedge and radial shear zone beneath the footing, while in the case of MC soil, the radial shear zone was distinctively outside the elastic wedge. Fig. 7 illustrates the deformation iso-surfaces developed in the MC and MCC soils at failure. It can be clearly observed that the deformation pattern for the footing on MC soil is mostly confined to the edges of the footing, while, in the MCC soil, it is distributed to the entire foundation soil beneath the footing. Further, the volume of soil participating in resisting the applied stresses is significantly higher in the case of MCC soil in comparison to the MC soil, thus the bearing capacity of footing on MCC soil is distinctly higher. However, it is worth mentioning that the failure deformation is also significantly higher for footing on MCC soil.

Fig. 4 Failure deformation pattern of MC soil

Fig. 5 Load-settlement response of footing resting on MCC soil

Fig. 8 highlights the distribution of vertical stress beneath the footing resting on MC and MCC soil. Such distribution aids in understanding the volume of foundation soil influenced by the applied stress, similar to the pressure bulb as obtained with the Boussinesq’s theory for elastic conditions. It can be observed that in comparison to the isobars developed in the MCC soil, the isobars span to wider area beneath the footing resting on MC soil. Further, it can be noted that the vertical stress contours are very distinctly developed for MC soil owing to the distinct formation of deformation patterns, as shown in Fig. 4, while the stress contours developed in the MCC soil (Fig. 6) are inadvertently scrambled due to mixed elastic-radial shear zones developed beneath the footing at failure.

Fig. 6 Failure deformation pattern of MCC soil

Fig. 7 Deformation iso-surfaces developed in MC and MCC soils at failure

7. CONCLUSION

This article presents the importance and influence of two different constitutive models,
Mohr-Coulomb and Modified Cam-Clay, on the assessment of bearing capacity of a square footing resting on the surface of a normally consolidated clayey medium. It is vividly evident from the results that the load-deformation and failure mechanisms of the MC and MCC soils are markedly different, which, in turn, largely affects the bearing capacity of the square footing resting on NC clayey soil. The excessive dilative behaviour of the MC model fails to represent the volumetric compression of the NC clayey soil, which is justifiably represented by the MCC model. Such findings call for the adoption of MCC model for finding the response of NC clayey soil and the supported structures.

Fig. 8 Vertical stress contours developed beneath footing resting on MC and MCC soils

REFERENCES


