NOTE: Answer **all six** questions. In case of any doubt, write your assumption and answer the question. Write indices of a tensor clearly. i, j, k refer to standard basis.

1. Let P_1 and P_2 be points at distance s_1 and s_2 from the origin. Find the work done by the gravitational force field given by $\boldsymbol{F} = -GmM(x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k})/(x^2 + y^2 + z^2)^{3/2}$ in moving a particle from P_1 to P_2 where G, m, M are constants. (10 marks)

2. In a children's playroom of size $(a \times a \times a)$, a strong rope has been fixed by nails at the centers of two adjacent walls. From the top corner between these walls to the farthest corner of the room, a narrow wooden plank, with grooves to climb, is placed. A child, climbing along the plank, wants to jump and get hold of the rope. Determine the minimum distance the child has to scale between the plank and the rope. (10 marks)

3. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then find A^{50}. (10 marks)

4. Let the temperature T in a solid at the point $P : (1, -1, 0)$ is given by the expression $T(x, y, z) = x^2y - y^2z - xyz$. Find the rate of change of T at P in the direction of $\mathbf{a} = (1, -1, 2)^T$. (10 marks)

5. Find the line integral of a vector function $\mathbf{F} = (y^2 \cos x + z^3, 2y \sin x - 4, 3xz^2 + 2)^T$ along the curve $C = (\sin^{-1} t, -2t + 1, t^2 - 1)^T$ where $0 \le t \le 1$. (10 marks)

6. Given a vector function $\mathbf{F} = \Phi\mathbf{u}$ where Φ is the scalar field, and $\mathbf{u}$ is the vector field. Prove the following: $\oint_C \mathbf{F}.d\mathbf{r} = \iint_S (\Phi\nabla \times \mathbf{u} + \nabla\Phi \times \mathbf{u}).\hat{n}dA$ where S is the piecewise smooth oriented surface in space and the boundary of S be a piecewise smooth closed curves C. (10 marks)