Computing assignment

Advanced Engineering Mathematics (ME 501)

1. Write a program for the Gauss elimination with pivoting to solve the linear system $Ax = b$. The program should have the following capabilities

- It will read the size of the matrix (n), the coefficient matrix (A) and the right hand side vector (b) from a input file.
- It will write the triangular form of A and the solution vector x in a output file.
- It should be able to handle a situation where in an intermediate step diagonal element turns out to be zero (pivoting becomes essential).

Test the program with $n = 100$, matrix A given below, and b generated by multiplying A with $x = [1\ \ 1\ \ 1\ \ \cdots\ \ 1]^T$.

$$A = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}$$

2. Solve the following ordinary differential equation for $y(x)$ which is a classical example of "boundary value problem"

$$y''' + \frac{1}{2}yy'' = 0, \quad 0 \leq x \leq 10, \qquad y(0) = 0,\ y'(0) = 0,\ y'(10) = 1$$

- Reduce the equation to a set of first-order ODEs.
- Use 4^{th}-order Runge-Kutta method to solve the initial value problem.
- Use the bi-section method to satisfy the third boundary condition.
- Plot $y(x), y'(x), y''(x)$ with x.

3. Consider the integral $I = \int_0^2 x^4 dx$. This may be easily evaluated by hand. Write a program to calculate the integral numerically using Gauss–Legendre quadrature with 1,2 and 3 quadrature points. Determine the error between the numerical estimate and the exact value in each of the three cases. Based on your results, what number of quadrature points would you recommend to evaluate the given integral and why ?

4. It is desired to determine the point of intersection of the curve $y = e^x$ with the straight line y =3x which lies in [0,1]. This may be cast as a root–finding problem, which can then be solved using an iterative approach that starts with an initial guess and terminates when some measure of error becomes lesser than a pre–defined tolerance. One way of defining this error is the relative error $\epsilon = |\frac{x^{k+1} - x^k}{x^k}|$ where k denotes the iteration.

- Using bisection method with a tolerance of 10^{-3}, determine the point of intersection. How many iterations are needed to achieve the result ?
- If the Newton–Raphson method were used instead, with the same initial guess and tolerance as in (a), what would be the number of iterations and point of intersection ?
- Study the effect of initial guess on the result for the Newton–Raphson method, by considering starting values outside of [0,1].
- Based on the results obtained using your programs, which of the two methods would you recommend and why ?