END-SEMESTER EXAMINATION 2017

Course: Advanced Engineering Mathematics (ME501)
Department of Mechanical Engineering
IIT Guwahati

Total Time: 3 Hrs | **25-11-2017, 9AM–12PM** | **Total Marks: 100**

NOTE: Answer **all ten** questions. In case of any doubt, write your assumption and answer the question. Consider Fourier transform: $F(\omega) = \hat{f}(\omega) = (1/\sqrt{2\pi}) \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$.

1. Evaluate the following limit at $x = 1$

$$\lim_{\omega \to \infty} \int_{-\infty}^{\infty} H(t) e^{-t} \, \frac{\sin(\omega(t - x))}{t - x} dt$$

where $H(t)$ is the Heaviside or unit step function. (10 marks)

2. The column space of matrix A is formed by $[1 \;\; 2 \;\; 2 \;\; -1]^T$, $[4 \;\; 9 \;\; 9 \;\; -4]^T$ and $[5 \;\; 8 \;\; 9 \;\; -5]^T$ vectors. The nullspace of A is formed by $[3 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T$, $[14 \;\; 0 \;\; -3 \;\; 1 \;\; 0 \;\; 0]^T$ and $[37 \;\; 0 \;\; -4 \;\; 0 \;\; -5 \;\; 1]^T$ vectors. Find the original matrix A. (10 marks)

3. $f(t)$ is a signal with finite energy whose Fourier transform $F(\omega)$ contains the same amount of energy in the frequency (ω) domain. Suppose Fourier transform of a *characteristic function* $g(t)$, known as the *transfer function* and given by $G(\omega)$, cuts $F(\omega)$ off all the energy outside the band $-\omega_0 \le \omega \le \omega_0$, i.e.,

$$F_{\omega_0}(\omega) = G(\omega) F(\omega) = \begin{cases} F(\omega) & -\omega_0 \le \omega \le \omega_0 \\ 0 & |\omega| > \omega_0 \end{cases}$$

Find the characteristic function. (10 marks)

4. Solve the following initial value problem: $y_1' = 4y_1 + 8y_2 + 2\cos t - 16 \sin t$, $y_2' = 6y_1 + 2y_2 + \cos t - 14 \sin t$, $y_1(0) = 15$, and $y_2(0) = 13$.

5. Find out the temperature distribution $u(x, t)$ of a thin bar (thermal diffusivity c^2) of length l whose initial temperature is $u(x, 0) = 3/2, 0 < x < l$ and is maintained at $u(0, t) = 1$, $u(l, t) = 2$ for $t > 0$. Such non-homogeneous conditions would not allow separation of variable to succeed. Assume a distribution $u(x, t) = U(x, t) + \phi(x)$, insert it in the heat equation, by choosing $\phi'' = 0$ and suitable boundary conditions for ϕ, $U(x, t)$ can be solved using the separation of variable method. (10 marks)

6. Use appropriate integral theorem and find the area of the region in the first quadrant by the curves $y = x$, $y = 1/x$, and $y = x/4$. (10 marks)

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7. If f is continuous, f' is piecewise continuous on $[-l, l]$ and $f(l) = f(-l)$, then Fourier coefficients of f on $[-l, l]$ satisfy the *Parseval's Theorem*

$$\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2) = \frac{1}{l}\int_{-l}^{l} f(x)^2 dx.$$

Find the Fourier coefficients of $\cos(x/2)$ in $[-\pi, \pi]$ to determine the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{(4n^2 - 1)^2}$$

(10 marks)

8. Using Fourier transform, solve the ODE: $y'' - 10y' + 9y = e^{-2ix}\delta(x-2)$. (10 marks)

9. Find the displacement of an elastic string fastened at $x = 0$ and π which is released from its horizontal position with initial velocity $\sin x$. Use the speed of propagation of wave as c^2. (10 marks)

10. Solve the following: $(xy^5 + y)dx - dy = 0$. (10 marks)