Problem Set 5

Advanced Engineering Mathematics (ME 501)
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1. Longitudinal vibrations of an elastic bar of length L in the axial direction are governed by the wave equation $u_{tt} = c^2 u_{xx},\ c^2 = E/\rho$ where E and ρ are Young's modulus of elasticity and density of the bar, respectively. The rod is fastened at one end and free at the other. If the initial displacement and velocity are $f(x)$ and zero, respectively, then find the motion of the bar.

2. Using Fourier transform find the temperature (u) distribution of an infinitely long bar of thermal diffusivity c^2 if it has the initial distribution e^{-4x^2}. The initial boundary value problem is

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2},\ -\infty < x < \infty,\ t > 0$$
$$u(x,0) = e^{-4x^2},\ -\infty < x < \infty$$

3. Show that if a solid bar of length π and thermal diffusivity 1 is insulated at the ends has the initial temperature distribution $1 - x/\pi$, then at any time $t > 0$ its distribution is given by the infinite series

$$\frac{1}{2} + \frac{4}{\pi^2}\left(e^{-t}\cos x + \frac{1}{9}e^{-9t}\cos 3x + \frac{1}{25}e^{-25t}\cos 5x + \cdots\right)$$

4. An infinite string is pulled locally (around $x = 0$) as $u(x,0) = e^{-|x|}$ and released from rest. Write down the initial boundary value problem and solve it using Fourier transform.

5. Solve the following Laplace equation with prescribed boundary conditions

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ -\infty < x < \infty,\ 0 < y < \pi$$
$$u(x,0) = H(x)e^{-2x},\ \text{where } H(x) \text{ is the unit step function}$$
$$u(x,\pi) = 0,\ -\infty < x < \infty$$

6. The upper and lower sides of a square plate of side a is perfectly insulated while the left and right sides are kept at temperature 0 and $f(y)$, respectively. Find the steady-state temperature distribution in the plate.