Problem Set 4

Advanced Engineering Mathematics (ME 501)

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1. Find the Fourier series expansion of the following functions

$$f(x) = \begin{cases} \pi, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \\ & f(x+2\pi) = f(x) \end{cases} \qquad f(x) = \begin{cases} 2+x, & -2 \le x \le 0 \\ 2-x, & 0 < x \le 2 \\ & f(x+4) = f(x) \end{cases}$$

$$f(x) = \begin{cases} x^3, & -\pi < x < \pi \\ & f(x+2\pi) = f(x) \end{cases} \qquad f(x) = \begin{cases} 1, & -\pi/2 < x < \pi/2 \\ -1, & \pi/2 < x < 3\pi/2 \\ & f(x+2\pi) = f(x) \end{cases}$$

2. Find the Fourier series expansion of the following functions and find the sum of the series

$f(x) = \pi + x,\ -\pi < x < \pi,\ f(x+2\pi) = f(x)$

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 \le x < \pi \end{cases},\ f(x+2\pi) = f(x)$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \text{ and } 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

$$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ \sin x & 0 \le x \le \pi \end{cases}$$

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots$$

$f(x) = e^x, -\pi < x < \pi,\ f(x+2\pi) = f(x)$

$$\frac{1}{1+2^2} - \frac{1}{1+3^2} + \frac{1}{1+4^2} - \cdots + \frac{(-1)^n}{1+n^2} + \cdots$$

3. Check for the existence and subsequently find the Fourier (or inverse Fourier) transform of the following functions

$$f(x) = \exp(-a|x|),\ -\infty < x < \infty,\ a > 0 \qquad f(x) = 1/(5+ix)$$

$$\hat{f}(\kappa) = (\sqrt{\pi}\kappa e^{-\kappa^2/8})/(4\sqrt{2}\,i) \qquad f(x) = 1/(1+x^2)$$

4. Using the Fourier integral representation of the function

$$f(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases} \qquad f(x) = \begin{cases} 0, & x < 0 \\ e^{-x}, & x > 0 \end{cases}$$

$$f(x) = \begin{cases} |x|, & -\pi \le x \le \pi \\ 0, & |x| > \pi \end{cases} \qquad f(x) = \begin{cases} -(1+x), & -1 < x < 0 \\ 1-x, & 0 \le x \le 1 \\ 0, & |x| > 1 \end{cases}$$

5. Let $f(x) = 0$ for $x < 0$ and $x > a$ while $f(x) = 1$ for $0 < x < a$. show that

$$\int_0^\infty \frac{\sin ax}{x}\,dx = \frac{\pi}{2},\ a > 0.$$

6. Find the complex form of Fourier integral representation for the function

$$f(x) = \begin{cases} |x|, & -\pi < x < \pi \\ 0, & |x| \geq \pi \end{cases} \qquad f(x) = \begin{cases} 0, & x < 0 \\ e^{-kx}, & x > 0,\ k > 0 \end{cases}$$

$$f(x) = \begin{cases} 1 + x, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \qquad f(x) = \begin{cases} \sin(\pi x), & -2 \leq x \leq 2 \\ 0, & |x| > 2 \end{cases}$$

7. Show the following

$$\int_0^\infty \frac{\sin w \cos xw}{w}\,dw = \begin{cases} \pi/2 & 0 \leq x < 1 \\ \pi/4, & x = 1 \\ 0, & x > 1 \end{cases}$$

$$\int_0^\infty \frac{w^3 \sin xw}{w^4 + 4}\,dw = \frac{\pi}{2} e^{-x} \cos x,\ x > 0$$

$$\int_0^\infty \frac{1 - \cos \pi w}{w} \sin xw\,dw = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

8. Using Fourier transforms, find the solution of the following differential equations.

$$y' - 4y = H(t)e^{-4t}, -\infty < t < \infty \qquad y'' + 3y' + 2y = \delta(t - 3)$$