Evolution of Temperature Field Developed in Arc Welded Steel Butt Joints and its Effect on Cooling Rate: An Experimental and Mathematical Approach

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Abstract

In present work, an integrative comprehensive study has been portrayed to determine the temperature distribution along the longitudinal direction of rectangular butt joint from the weld bead. A mathematical model incorporating temperature dependent thermal conductivity along with constant heat generation, is developed using Adomian Decomposition method (ADM) to analyse temperature distribution. Experimentation has been carried out by mounting K-type thermocouple in the rectangular plate of AISI 1040 at predefined locations. The experimental readings and mathematical model has been validated by empirical correlation of peak temperature equation. To investigate the cooling rate Rosenthal’s 3D model has been utilized for derivation of cooling rate along the longitudinal direction of rectangular workpiece. The study of microstructure also has been carried out for uniform and non-uniform cooling rate. Experimentally it was found out that temperature loss is very severe near the surface of weld pool (800°C - 1400°C) whereas it remains steady on the rear end of the plate due to transient nature. The analytical model as derived by Adomian decomposition method, entails that the temperature distribution is very gradual along the longitudinal direction from the weld bead.

Keywords: Adomian decomposition method, cooling rate, transient temperature field, butt joint

1. Introduction

Fusion welding is one of the important manufacturing process for joining the structural elements with a myriad applications such as shipbuilding industries, railways components, building structures, automobiles, bridges and even in nuclear industries. It is a rapid process of fusion and solidification between two metals by generating the heat produced from the electric arc between the electrode and weldments. The investigation of fusion welding process covers various important aspects like metallurgical properties of structures (phase transformation, microstructural changes, microhardness etc.), effect of arc temperature on residual stress development, distortion developed due to formation of thermal cycle, physics and behaviour of weld pool on base metal. As thermal cycle provides all the information related to any structural changes occurred in joint along with information related to penetration depth and heat affected zone (HAZ). It is very important to control the temperature distribution in weld pool [1]. Cooling rate is another one major property to be considered because it has major impact on strength and hardness of the joint. The analytical method for investigating the heat flow applied in moving point heat source was first successfully developed by Rosenthal [2] and still it is one of the most popular basic mathematical model for carrying out research work on heat flow phenomenon on welding. Quigley et al. [3] demonstrated heat flow from the workpiece of a TIG welding arc and described cooling mechanism with the help of vaporization and radiation. Sevesson et al. [4] presented the experimental and analytical study of cooling curves influenced by thermal field, along the fusion boundary of steel weld deposits. Some recent notable work represented by the researchers such as Murthy Y. V. L. N. et al. [5] carried out a comprehensive work based on numerical simulation of welding using transient thermal and thermo-elasto-plastic formulations; Ushio M. and Wu C. S. [6] investigated the 3-dimensional heat and fluid flow in a moving gas metal arc welding (GMAW) with
mathematical modeling; Little G. H. and Kamtekar A. G. [7] studied the effect of thermal properties in welding efficiency during transient temperature; Zhu X. K. and Chao Y. J. [8] presented welding simulation code using FEM to explore the temperature dependent material properties during detailed non-linear thermo-mechanical analysis of arc welding; Komanduri and Hou [9] prescribed analytical and numerical model to develop the expression of cooling rate as well as temperature distribution by using the popular Carslaw and Jeager’s mathematical model; Gery D. et al. [10] used Goldak’s double ellipsoidal model to find out the effect of welding speed, energy input and heat source distribution on the temperature variation; a thorough study on estimation of cooling rate in welding of plates with intermediate thickness accounting heat loss factor has been carried out by Poorhaydari et al. [11]; Arora et al. [12] proposed a generalised correlation developed by dimensional analysis based on theory of rapid solidification; analysis of thermal cycle during multipass arc welding has been investigated by Pathak et al. [13] and simulated cooling rate has been compared with experimental results; Manikandan [14] calculated cooling curves of Inconel 718 fusion zone using argon and helium gas shielded GTAW with a filler metal.

Motivated by the literature survey summarized above, in this present analysis a parametric study has been carried out to establish an analytical correlation of non-dimensional temperature distribution based on Adomian Decomposition Method (ADM) to represent thermal cycle from fusion boundary towards the longitudinal direction of the butt joint. In popular Rosenthal’s 3D model the cooling rate has been demonstrated along the fusion boundary (weld pool) [15] but in this study the expression of cooling rate at the longitudinal (opposite direction of the fusion zone) direction has been derived as experiment has been conducted by mounting thermocouples at the pre-defined locations in the metal plate towards the same direction. Also empirical peak temperature and cooling rate based on critical plate thickness has been involved to represent comparison between experimental and theoretical results. Successful application of ADM in recent years reported such as Chiu and Chen [16] portrayed ADM for solving the convective longitudinal fins with variable thermal conductivity; Chang [17] demonstrated the thermal characteristics developed due to temperature dependent surface heat flux in porous media; Pamuk [18] provided useful information about solving method of non-linear heat transfer equations by decomposition method; Alizadeh et al. [19] produced analytical expression for cooling rate based on ADM with thermal boundary layer.

2. Analytical Study

In this work analytical study is included of construction of temperature distribution model based on adomian decomposition method and expression of cooling rate based on Rosenthal’s 3D equation in longitudinal direction of weld pool.

2.1. Temperature distribution based on Adomian Decomposition Method

Conduction heat transfer through a slab in \(0 \leq x \leq L\) with heat generation at a constant rate of \(g_0\) is investigated in this section. The thermal conductivity depends on temperature in the form of \(K(T) = K_0(1 + \beta T)\). The governing equation is as

\[
\frac{d}{dx}[K_0(1 + \beta T) \frac{dT}{dx}] + g_0 = 0 \quad (1)
\]

Where \(K_0\) is the thermal conductivity of the weld pool at the ambient temperature and \(\beta\) is the parameter explaining the variation of thermal conductivity in fraction. The boundary conditions are:

\[
\frac{dT}{dx} \bigg|_{x=0} = 0 \quad \text{and} \quad T(x = L) = 0 \quad (2)
\]

The exact solution of equation (1) is:

\[
T(x) = \frac{1}{\beta} \sqrt{\frac{g_0}{K_0} \left( L^2 - x^2 \right)} \quad (3)
\]
Let, $\alpha = \frac{g_{0\gamma}}{\beta_{0\gamma}}$ and equation (1) reduces to

$$\frac{d^2T}{dx^2} = -\alpha - \beta(T, \frac{d^2T}{dx^2}) - \beta \left(\frac{dT}{dx}\right)^2$$

(4)

Applying Adomian decomposition method in equation (4), equation (4) can be written as:

$$L_xT = -\alpha - \beta NA - \beta NB$$

(5)

Here $L = \frac{\partial^2}{\partial x^2}$ is a 2nd order differential operator.

The non-linear terms in equation (5) can be written as:

$$NA = T \frac{d^2T}{dx^2} = \sum_{m=0}^{\infty} A_m$$

(6) and

$$NB = \left(\frac{dT}{dx}\right)^2 = \sum_{m=0}^{\infty} B_m$$

(7)

Which can be introduced as:

$$A_0 = T_0 \frac{d^2T_0}{dx^2}$$

$$A_1 = T_1 \frac{d^2T_0}{dx^2} + T_0 \frac{d^2T_1}{dx^2}$$

$$A_2 = T_2 \frac{d^2T_0}{dx^2} + T_1 \frac{d^2T_1}{dx^2} + T_0 \frac{d^2T_2}{dx^2}$$

......

$$B_0 = \left(\frac{dT_0}{dx}\right)^2$$

$$B_1 = 2 \frac{dT_0}{dx} \frac{dT_1}{dx}$$

$$B_2 = \left(\frac{dT_1}{dx}\right)^2 + 2 \frac{dT_0}{dx} \frac{dT_2}{dx}$$

......

Applying the inverse operator $L_x^{-1}$ to both sides of equation (5),

$$T = T_0 - \beta L_x^{-1}NA - \beta L_x^{-1}NB$$

(8)

Applying the boundary conditions as mentioned in equation (2) and $T(x = 0) = C$, we can obtain from equation (8):

$$T_0 = C - \alpha \frac{x^2}{2!}$$

$$T_1 = \beta \alpha C \frac{x^2}{2!} - 3\beta \alpha^2 \frac{x^4}{4!}$$

$$T_2 = -\beta^2 \alpha^2 C \frac{x^2}{2!} + 9\beta^2 \alpha^2 C \frac{x^4}{4!} - 45\beta^2 \alpha^3 \frac{x^6}{6!}$$

$$T_3 = \beta^3 \alpha^3 C \frac{x^2}{2!} - 18\beta^3 \alpha^3 C \frac{x^4}{4!} + 225\beta^3 \alpha^4 C \frac{x^6}{6!}$$

$$-1575\beta^3 \alpha^4 \frac{x^8}{8!}$$

......

The final solution of ADM in terms of $\alpha$ and $\beta$ is:

$$T = \sum_{m=0}^{\infty} T_m = T_0 + T_1 + T_2 + ... + T_n$$

(9)

Implementation of Newton-Raphson method provides solution of C [19] is 0.5 for $\alpha = 10$ and $\beta = 0.5$.

2.2. Proposed cooling rate based on Rosenthal's analytical model

The analytical 3D solution developed by Rosenthal can be given as follows [14, 16]:

$$\frac{2\pi kR(T - T_0)}{H} = \exp[-\frac{V(R - x)}{2\alpha}]$$

(10)

Cooling rate in longitudinal direction (refer fig. 1) is denoted as y direction and along the weld pool is given as x direction (transverse direction in fig. 1). To find out the cooling rate along the longitudinal direction starting from fusion boundary to rear end of the plate, the steps are as follows:

The predefined assumptions for the present analysis are:

- Heat loss along the width (x direction) and along the thickness (Z direction) has been neglected.
- All thermo-physical properties such as thermal conductivity, thermal diffusivity, density of base metal remains constant.
- Heat loss due to convection and radiation are neglected.

Rearranging equation (10), we get
\[ T - T_0 = \frac{Q}{2\pi KR} \exp\left\{ -\frac{V(R - x)}{2\alpha} \right\} \] (11)

Now radial distance, \( R = \sqrt{x^2 + y^2 + z^2} \) (12)

As heat loss in x and z direction have been neglected, thus \( x = 0 \) and \( z = 0 \).

So, from equation (12), \( R = y \) and equation (11) becomes:

\[ T - T_0 = \frac{Q}{2\pi KR} \exp\left\{ -\frac{Vy}{2\alpha} \right\} \] (13)

Differentiating equation (13) with respect to y at a constant time t,

\[ \left( \frac{\partial T}{\partial y} \right)_t = -\frac{Q}{2\pi KR} \frac{V}{2\alpha} \exp\left\{ -\frac{Vy}{2\alpha} \right\} \] (14)

Replacing equation (14) with equation (11), we get

\[ \left( \frac{\partial T}{\partial y} \right)_t = -\frac{V}{2\alpha} (T - T_0) \] (15)

Again differential displacement with respect to time t for a constant temperature \( T \) is:

\[ \left( \frac{\partial y}{\partial t} \right)_T = V \] (16)

Now combining equation (15) and (16), the final expression of cooling rate (°C/s) is:

\[ \left( \frac{\partial T}{\partial t} \right)_y = \left( \frac{\partial T}{\partial y} \right)_t \left( \frac{\partial y}{\partial t} \right)_T = -\frac{V^2}{2\alpha} (T - T_0) \] (17)

### 2.3. Empirical correlations

The Adam’s peak temperature equation can be expressed as follows [15]:

\[ \frac{1}{T_p - T_0} = \frac{1}{T_m - T_0} + \frac{4.312\rho C_p d \times y}{H_{net}} \] (18)

### 3. Experimentation

As referred from fig. 2 five k-type thermocouples are mounted in specimen along the longitudinal direction of specimen maintaining equal distance each. The rear end of the thermocouple wires are inserted to the channels of Data Acquisition System (DAQ) as shown in fig. 3. and fig. 3. The DAQ is in turn connected to the computer for analyzing the data in LABVIEW software. Fig. 5 denotes the formation of weld bead after the completion of single pass.

### Table 1: Specification of instruments used in experiment

| Base metal | Size: 180x60x6 mm |
| Material:  | AISI 1040         |
4. Results and discussion

Table 2: Welding parameters for analysis [10, 11, 20]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho$ (kg/m$^3$)</td>
<td>7.845 x 10$^3$</td>
</tr>
<tr>
<td>Thermal coefficient $\beta$ (/°C)</td>
<td>13.6 x 10$^{-6}$</td>
</tr>
<tr>
<td>Thermal conductivity $K$ (w/m-K)</td>
<td>51.9</td>
</tr>
<tr>
<td>Specific heat $C_p$ (kJ/kg-K)</td>
<td>0.486</td>
</tr>
<tr>
<td>Melting point $T_m$ (°C)</td>
<td>1482</td>
</tr>
<tr>
<td>Emissivity $\varepsilon$</td>
<td>0.35</td>
</tr>
<tr>
<td>Thermal diffusivity $\alpha$ (m$^2$/s)</td>
<td>9.1 x 10$^{-6}$</td>
</tr>
<tr>
<td>Voltage (volt ) $E$</td>
<td>40</td>
</tr>
<tr>
<td>Current (Amps) $I$</td>
<td>150, 175, 200, 225</td>
</tr>
<tr>
<td>Welding time (sec) $t$</td>
<td>35</td>
</tr>
<tr>
<td>Welding/Electrode velocity (mm/s) $V$</td>
<td>1.714</td>
</tr>
<tr>
<td>Heat transfer efficiency $\eta$</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Experimental data acquired from DAQ has been distribution based on ADM from fusion boundary to longitudinal direction of the plate

From fig. 5, it is clear that presently derived non-dimensional temperature distribution (refer equation 9) based on ADM follows the conventional concept of temperature distribution from weld pool towards the longitudinal direction (refer fig. 1) of the plate butt joint. The temperature distribution produced by ADM in present investigation has matched with the results produced by Chang [17] and Pamuk [18] with same value of convergence.
shown as peak temperature for five different heat inputs i.e. Four different amperages (refer table 2) has been demonstrated in fig. 6. It can be tallied with the variation of fig. 5. As in ADM, welding velocity and other thermophysical properties are not involved the temperature distribution (fig. 5) is steady state, whereas in experimental reading are in transient nature so at higher temperature region (700ºC-1000ºC) temperature field is varying much compared to rear end of the plate lying in lower temperature.

This analysis has shown good agreement with Pathak et al. [13]. Figure 7 denotes the theoretical temperature field as found out from Adam’s peak temperature correlation (refer equation 18).

For determination of cooling rate minimum and maximum input current has been selected (refer table 5). Figure 8 denotes the proposed technique of cooling rate based on Rosenthal’s 3D model (refer equation 17) with theoretical peak temperature (refer equation 18). It is indicating higher cooling rate near the fusion boundary and gradually decreasing towards the longitudinal direction of the plate and it exactly matches with the results produced by Komanduri and Hou [9]. Figure 9 shows the proposed cooling rate of maximum and minimum input current based on experimental peak temperature. Though it is matching exactly with fig. 8, but the sudden decrement of cooling rate represents the transient nature of thermo-physical properties near the fusion boundary (500ºC-800ºC).

Fig. 8. Cooling rate based on proposed method of theoretical peak temperature for 150 amps and 225 amps

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Fig. 10. SEM image of crack formation due to non-uniform cooling rate
Fig. 11. SEM image of fine grain boundaries due to uniform cooling rate

Figure 10 and fig.11 portrayed the influence of cooling rate developed due to thermal cycle of fusion boundary. It is evident that in welded joints cooling rate must be uniform to produce sound structure without any defect.

5. Conclusion

The following remarks can be drawn from the current analysis:

- The adomian decomposition method has been validated in current analysis for prediction of temperature in non-dimensional form (fig. 5). Though it is not included the welding velocity and other thermo-physical properties of base metal but still it can be useful alternative for establishing the analytical expression of temperature cycle.

- Theoretical Peak temperature study (Fig. 7) has shown good agreement only in high temperature range compared to fig. 6. But at lower temperature (rear end of the plate) it shows higher temperature compared with experimental data. This is due to the assumption of constant thermo-physical properties.

- To determine the cooling rate radiation and convection neglected, but proposed technique can be useful for demonstrating the effect of cooling rate along longitudinal direction both experimentally and theoretically. Rate of vaporization also can be utilized for determining thermal history (fig. 8 and fig. 9).

References

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