THERMAL BUCKLING OF TEMPERATURE DEPENDENT
FUNCTIONALLY GRADED CYLINDRICAL PANEL

V. R. Kar* and S. K. Panda

*Dept. of Mechanical Engg., NIT, Rourkela, 769008, Email:visheshkar@gmail.com
2Dept. of Mechanical Engg., NIT, Rourkela, 769008, Email:call2subrat@gmail.com

Abstract

The buckling behaviour of functionally graded cylindrical panels under thermal loading is investigated in this article. In functionally graded material, material properties vary smoothly from metal phase to ceramic phase. In this study, the effective material properties of the functionally graded panels are considered as temperature dependent and the gradation is taken in the transverse direction according to the power-law distribution of volume fractions of each constituent. Thermal buckling behaviour of cylindrical panel has been obtained numerically through ANSYS based on the ANSYS parametric design language code. The model has been discretised using an eight node serendipity element with six degrees of freedom per node (SHELL281) from the ANSYS library. The solutions are obtained by solving the eigenvalue type buckling using Block Lanczos method. The accuracy of the model has been checked through corresponding convergence and comparison study with those available literatures. Finally, the simulation model has been extended to study the effect of different parameters such as power-law index, thickness ratio, curvature ratio and aspect ratio on buckling strength for both temperature independent and dependent material properties of each constituent.

Keywords: FGM, Thermal buckling, Finite element method, Temperature dependent

1 Introduction

Functionally graded material (FGM) is a new kind of advanced in-homogenous material developed by the continuous variation of material properties in one or more direction. The constituents of FGM are metal/alloy and ceramic which are very distinct from each other. The benefit of using these materials in FGM is to retain high fracture toughness as well as high heat resistant. This new kind of material provides an edge over conventional composite materials by eliminating inter-laminar thermal stress concentration as well as delamination. Due to these novel characteristics, FGM made structures are suitable for thermal barrier applications as in aerospace, energy, defence industries etc. The exposure of FGM structures to high temperature field results loss of their structural integrity due to the in-plane thermal load and which causes the geometrical instability. Therefore, many researchers are interested to find the buckling behaviour of FGM structures under thermal environment. To make the article self-contained, some available articles are discussed in brief.

Thai and Choi (2012) employed an efficient refined theory to examine the stability of functionally graded (FG) plates for different boundary conditions. Ghannadpour et al. (2012) analysed the buckling responses of FG flat panels by applying a finite strip method under different thermal loadings such as uniform, linear and nonlinear temperature distribution across the thickness. Zhao et al. (2009) used the first order shear deformation theory (FSDT) and the element-free kp-Ritz method for mechanical and thermal buckling analyses of FG plates. Tung and Duc (2010) investigated the buckling and post-buckling behaviours of FG flat panels under thermo-mechanical loadings based on the classical plate theory and von Karman nonlinearity. Lee et al. (2010) applied the FSDT mid-plane kinematics and the element-free kp-Ritz method to obtain the post-buckling behaviour of FG plates under thermo-mechanical loading. Woo et al. (2005) examined the post-buckling behaviour of moderately thick FG flat and cylindrical panels under in-plane compressive load. Na and Kim (2004, 2006) investigated thermo-mechanical buckling responses under uniform and non-uniform temperature rise across the thickness direction. Zhao and Liew (2010) examined the buckling response of FG cylindrical panels under axial compression and thermal load using the FSDT and the element-free kp-Ritz method.

Based on the brief review and available published literature it is observed that the thermal buckling behaviour of FGM panel with temperature dependent (TD) material properties are not huge in number.
Hence, in this study author’s aim to predict thermal buckling strength of FGM cylindrical panel under uniform temperature with/without TD material properties. It is well known that many commercial finite element (FE) tools are available to analyse different structural responses with ease and less computational cost. ANSYS is one of the known and reliable FE tool for this type of application and well accepted by industries. In order to address the above, the present study aims to develop a FG panel, discretised and solved in ANSYS through an APDL code. Finally, influences of different parameters such as power-law index, aspect ratio, thickness ratio and curvature ratio on thermal buckling load of FG cylindrical panel are examined and discussed in detail.

2 Effective material properties of FGM

FGMs are inhomogeneous in character in which material properties are varying gradually from the one surface to the other along with the thickness direction. The bottom of panel is considered as metal rich whereas top as ceramic rich. The FGM constituents are temperature dependent and can be expressed as (Reddy and Chin, 1998)

\[ P(T) = P_c(T) + P_m(T) \]

The effective material properties of FGM \( P(T) \) can be evaluated by using rule of mixture and power-law distribution.

\[ P(T, z) = P_c(T)V_c(z) + P_m(T)V_m(z) \]

where, subscript ‘c’ and ‘m’ denote ceramic and metal, respectively and \( V(z) \) is the volume fraction follows power-law distribution and expressed as

\[ V_c(z) = \left( \frac{z}{h} + \frac{1}{2} \right)^{-n} \]

\[ V_m(z) = 1 - V_c(z) = 1 - \left( \frac{z}{h} + \frac{1}{2} \right)^{-n} \]

where, \( n(0 \leq n \leq \infty) \) is the power-law index. The variation of volume fraction of the ceramic phase through the dimensionless thickness \( (Z=z/h) \) is plotted in Figure 1 for different values of power law indices.

Finally, the Young’s modulus \( E(T, z) \) and the thermal expansion coefficient \( \alpha(T, z) \) can be expressed as

\[ E(T, z) = \left[ \frac{E_c(T) - E_m(T)}{h} \right] \left( \frac{z}{h} + \frac{1}{2} \right) + E_m(T) \]  
\[ \alpha(T, z) = \left[ \frac{\alpha_c(T) - \alpha_m(T)}{h} \right] \left( \frac{z}{h} + \frac{1}{2} \right) + \alpha_m(T) \]

The Poisson’s ratio \( \nu \) is assumed to be constant throughout the thickness of the FG panel. The TD and Temperature independent (TID) material properties of ceramic and metal are shown in Table 1.

3 Finite element modelling

In the present work, the FG cylindrical shell panel is modelled layer wise using APDL and then discretise by utilizing an eight node serendipity shell element (SHELL281), defined in the ANSYS environment. This element is suitable for analyzing thin to moderately-thick shell structures. The element has six degrees of freedom at each node i.e. translations and rotations in the \( x, y \) and \( z \) directions (ANSYS,2010). An FG cylindrical panel of uniform thickness ‘h’ with rectangular base of sides ‘a’ and ‘b’ is considered in the analyses as shown in the Figure 3.

<table>
<thead>
<tr>
<th>Material</th>
<th>Properties</th>
<th>( P_0 )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>TID (at 300 °K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si₃N₄</td>
<td>E(Pa)</td>
<td>3.48E+11</td>
<td>0</td>
<td>-3.07E-04</td>
<td>2.16E-07</td>
<td>-8.95E-11</td>
</tr>
<tr>
<td></td>
<td>( \alpha ) (K⁻¹)</td>
<td>5.87E-06</td>
<td>0</td>
<td>9.10E-04</td>
<td>0</td>
<td>7.47E-06</td>
</tr>
<tr>
<td>Ti₆Al₄V</td>
<td>E(Pa)</td>
<td>1.23E+11</td>
<td>0</td>
<td>-4.59E-04</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( \alpha ) (K⁻¹)</td>
<td>7.58E-06</td>
<td>0</td>
<td>6.64E-04</td>
<td>-3.15E-06</td>
<td>0</td>
</tr>
</tbody>
</table>
4 Results and discussions

The buckling behaviour of FG cylindrical panel is examined in thermal environment for both TD and TID material properties. A simply-supported boundary condition is used throughout in the section to constrain the all four edges of FG panel. The reference temperature of FG cylindrical panel is considered here is $T_0$ (300ºK) and uniformly raised to a final value of $T$ (400, 500, 600 and 700 ºK). Finite element solutions are obtained using ANSYS with Block Lanczos steps as discussed earlier. The buckling responses of present FG cylindrical panel are validated through the previously published results. Subsequently, the effect of power-law indices ($n$), thickness ratios ($ah$), curvature ratios ($Ra$) and aspect ratios ($ab$) on the critical buckling temperature load ($\Delta T_{cr} = \lambda \times (T - T_0)$, where $\lambda$ is the scaling factor) of FG cylindrical panel are examined.

4.1 Convergence and comparison

Figure 3 shows the convergence and comparison of simply-supported FG cylindrical panel ($ab=0.2, R=1$ and $h = 0.002$) for different power-law indices ($n$=0.5, 2, 5 and 10). Aluminum (Al) and zirconia (ZrO$_2$) are taken as metal and ceramic materials, respectively. The material properties are taken as same as in Zhao and Liew (2010). It is found from the mesh refinement that the values are well converging at a (18x18) mesh. The present results are showing well agreement with the published results and the differences are within the expected line.

4.2 Numerical illustrations

In this section, few problems have been carried out of FG panels under uniform thermal loading. Titanium alloy (Ti–6Al–4V) and silicon nitrated ($Si3N4$) are considered as FGM constituents, throughout in the section. The TID and TD material properties are mentioned in Table 1. Poisson’s ratio is taken constant as 0.28.

Figure 4 shows the variation of critical buckling temperature load of simply-supported FG cylindrical panel ($ab=1$, $ah=100$, $Ra=50$) along with the power-law index for different temperature values. It is observed that the critical buckling temperature load increases with the increase in power-law indices because as $n$ value increases the FG panel turns to metal.

Figure 5 shows the variation of critical buckling temperature load of simply-supported FG cylindrical panel ($ab=1$, $ah=100$, $Ra=50$) along with the thickness ratio for different temperature values. It is seen that the critical buckling temperature load increases with the decrease in thickness ratios, because as the thickness ratio increases the panel geometry changes from thick to thin and the thin structures have lower structural stiffness.

Figure 6 shows the variation of critical buckling temperature load of simply-supported FG cylindrical panel ($ab=1$, $n=2$, $Ra=50$) along with the curvature ratios for different temperature values. It is clear that the critical buckling temperature load decreases with the increase in curvature ratios. It is because of the fact that as the curvature ratio increases the membrane stiffness of the panel reduces.

Figure 7 shows the variation of critical buckling temperature load of simply-supported FG cylindrical panel ($n=2$, $ah=100$, $Ra=50$) along with the aspect ratios for different temperature values. It is observed that the critical buckling temperature load increases with the increase in aspect ratios due to curved panel with large aspect ratio exhibits higher stiffness value.

5 Conclusions

The buckling responses of simply-supported FG cylindrical panel under uniform temperature loading are examined. The material properties of FGM constituents are temperature dependent and the
effective material property is evaluated using the power-law distribution of the volume fractions. The present model is discretised and then solved in ANSYS based on APDL code. The present results are compared with the published results. The effects of geometrical and material parameters on the thermal buckling of FG cylindrical panel are illustrated. It is found that the critical buckling temperature load increases with the increase in power-law indices and aspect ratios and decreases with the increase in thickness ratios and curvature ratios. It is also concluded that the higher value of temperature rise results the larger critical buckling temperature load.

![Figure 4](image1.png)

**Figure 4** Variation of critical buckling temperature load with the power-law indices for simply-supported FG cylindrical panel

![Figure 5](image2.png)

**Figure 5** Variation of critical buckling temperature load with the thickness ratios for simply-supported FG cylindrical panel

![Figure 6](image3.png)

**Figure 6** Variation of critical buckling temperature load with the curvature ratios for simply-supported FG cylindrical panel

![Figure 7](image4.png)

**Figure 7** Variation of critical buckling temperature load with the aspect ratios for simply-supported FG cylindrical panel

References


Lee, Y.Y, Zhao, X. and Reddy, J.N. (2010), Post-buckling analysis of functionally graded plates...


