GRADE MIXING ANALYSIS IN STEELMAKING TUNDISH USING DIFFERENT TURBULENCE MODELS

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Abstract
Tundish plays an important role in steelmaking process. It acts as a metallurgical reactor and designed and operated as to ensure maximum performance and quality. For advance research, physical models of tundish are fabricated for water modeling study to validate the numerical models. Tundish modeling compromises of complex interaction of physical, chemical and thermal interactions among steel, slag, gas and refractory. In recent years, researchers are seen focussing on multiphase, hydrodynamic modeling of tundish system. In present work, an assessment of RANS equations based standard k-ε, RNG k-ε, Realizable k-ε standard k-ω, and SST k-ω turbulence models have been carried out on steelmaking tundish. A Coupled Level-Set Volume of Fluid (CLSVOF) method was used for solving three dimensional, multi-phase numerical model. The predictions compared against the experimental values reveal that RNG k-ε model gives good approximation of F-curves and swirling of fluid at inlet-plane. Prediction made by all models except SST k-ω model have shown a satisfactory match with the experimental values.

Keywords: Tundish, Turbulence models, Steelmaking, Grade mixing, Continuous casting.

1 Introduction
Advances in continuous casting process made tundish a metallurgical reactor where temperature control, mixing optimization and inclusion removal are done (Mazumdar 2013). Generally, research on tundish technology are carried using water models or numerical models. Numerical modeling of tundish operation based on Navier-Stokes equations have been done by various researchers. Most of the author validated numerical codes with results obtained through in-house built reduced scale water models. It is well known that flow inside tundish is a turbulent flow and hence each numerical investigation reported in literature, has used one of the available turbulence models to precisely approximate the physical phenomenon. To approximate the physics inside tundish by numerical modeling is a complex work because several phenomenon are simultaneously happening. This means that flow phenomenon in tundish is inherently multi-dimensional, multi-phase, reacting and turbulent. Fluid flow modeling, specially in a tundish necessitate significant efforts. There are various turbulence models available in pre-coded CFD softwares for characterization of fluid flow in different state of conditions. Few researchers working on tundish technology have made assessment of turbulence models and compared numerical investigations output with experimental results. Most of the previous numerical investigations, authors (Jha & Dash 2004, Jha et al. 2003) have assumed single-phase and constant bath height of molten metal in tundish during operation. In recent time, there has been few work by employing multi-phase model i.e., considering the phases of molten steel, slag and air (Battaglia et al. 2013). However, a detailed assessment of turbulence models with multi-phase system is required. Since majority of numerical investigations have been carried out on RANS equation models, therefore present work includes only RANS equation based turbulence models for assessment.

In present study a detailed assessment of five different Reynolds-averaged Navier-Stokes (RANS) two equation turbulence models have been carried out. The turbulence models have been compared on certain basis and suitability of each model has been discussed. For this purpose, a reduced scale boat shape billet caster has been used for water modeling experiments in laboratory. Apart from physical investigations, numerical investigations hasalso been carried out on transient, 3-d, multiphase, iso-thermal, turbulent flow. It is indeed a practical phenomenon in tundish operation that phase volume fractions continuously change during teeming period. Multiphase, turbulent flow calculations are complex, time intensive and needs considerable computational efforts. Coupled level-set volume of fluid (CLSVOF)(Sussman & Puckett 2000), a combination of volume of fluid (VOF)(Hirt & Nichols 1981) method with Level-set (LS)(Osher & Sethian 1988) method has been employed to predict the free surface level during the ladle change over process. CLSVOF has combines the advantage of mass
conservation property of the VOF method and the capability of predicting accurate and sharp interface of LS method. During the coupling of Volume of fluid and Level-set, interface is advected by VOF function which is mass conserved and interface normal is calculated by LS function that provide sharp interface. In the present work, the VOF is coupled to the Level-set method to enhance numerical model results.

2 Physical description

A 1/4th reduced scale six strand boat shape (rectangular) billet caster tundish has been considered for physical modeling. Geometric similarity has been maintained by keeping the dimensions of the model and prototype tundish in the same ratio and dynamic similarity is achieved by considering the inertial, viscous and gravitational forces. The dimensional analysis requires similarity of different dimensionless numbers such as Re, Fr, Ri and We(Sahai & Emi 2008). The flow behaviour inside tundish is largely affected by these dimensionless numbers. However, for the quasi steady-state, isothermal, and single-phase flow of water in a reduced scale tundish model, only the Re and Fr number similitude suffice(Chattopadhyay et al. 2011). It is impossible to respect both Reynolds and Froude similarity simultaneously in a reduced scale modeling studies using fluid of similar kinematic viscosity. In many earlier studies therefore, it has been assumed that flow phenomena in tundish are largely dominated by the inertial and gravitational forces (i.e., Froude number) rather than the viscous forces (Reynolds number). Water was used as fluid medium (at room temperature) as it has equivalent kinematic viscosity to molten steel. Figure 1 shows dimensions of tundish. Concentrated water was used as tracer and its injection was done as continuous input to the tundish. Conductivity meter was used for recording concentration at outlets of tundish.

Figure 1. Dimensions of reduced scale tundish

The time vs. concentration curve, known as F curve, was used to compute the formation of intermixed amount because of mixing inside the tundish. At the initial time of experiment, tundish was half filled. At that moment a new ladle was opened to the tundish, which was actually coloured concentrated water. The inflow rate to the tundish was kept maintained at 1.5 times of steady state. The pre-calibrated steady state inflow & outflow rate of this tundish is 12 lt/min. Due to increase in inflow rate, bath height of tundish raised with respect to time. After attaining the maximum free-surface height, the inflow rate was set to the steady state flow rate value.

3 Mathematical Formulation

3.1 Governing equations

The volume-averaged continuity equation and momentum conservation equation describing the fluids flow are respectively:

\[ \nabla \cdot u = 0 \]  

\[ \rho \frac{\partial u}{\partial t} + \rho (u \cdot \nabla)u = -\nabla p + \nabla \cdot \tau + f \]  

\[ \tau = \mu (\nabla u + \nabla^T u) \]  

where \( \tau \) is the viscous stress, \( P \) the pressure, \( f \) the body force. Equation for species concentration (C) given as:

\[ \frac{\partial \rho C}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i C) = \frac{\partial}{\partial x_i} (\sigma_i \frac{\partial C}{\partial x_i}) \]  

The flow front is advanced by solving the following transport equation of the fluid:

\[ \frac{\partial F}{\partial t} + u \cdot \nabla F = 0 \]  

where \( F \) is the volume fraction of the fluid in a cell and \( u \) is the flow velocity vector.

The Level Set function \( \phi \) is a signed distance function from the interface and satisfies \( \phi = 0 \). The interface is defined by \( \phi = 0 \), with \( \phi > 0 \) representing liquid and \( \phi \leq 0 \) representing air. \( \phi \) is evolved by the simple advection equation using the resolved velocity field:

\[ \frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = 0 \]  

3.2 Turbulence models

Standard k-\( \varepsilon \):

Equation for turbulent kinetic energy:

\[ \frac{\partial}{\partial t} (\rho k) + \text{div} (\rho k u) = \text{div} \left[ \left( \frac{\mu}{\sigma_k} \right) \text{grad} k \right] + 2\mu S_{ij} S_{ij} - \rho \varepsilon \]  

Equation for rate of dissipation of turbulent kinetic energy

\[ \frac{\partial}{\partial t} (\rho \varepsilon) + \text{div} (\rho \varepsilon u) = \text{div} \left[ \left( \frac{\mu}{\sigma_\varepsilon} \right) \text{grad} \varepsilon \right] \]

\[ + C_{\varepsilon 1} \varepsilon \frac{e^2}{k} - C_{\varepsilon 2} \]
The turbulent (or eddy) viscosity, $\mu_t$, is computed by combining $k$ and $\varepsilon$ as follows:

$$\mu_t = \rho C_p \frac{k^2}{\varepsilon}$$  \hspace{1cm} (9)

where $C_p$ is dimensionless constant.

$C_1 = 1.44$, $C_2 = 1.92$, $C_p = 0.09$, $\sigma_t = 1$ and $\sigma_k = 1.30$

Reynolds stresses are computed by following Boussinesq relationship:

$$-\rho \overline{\omega u_i} = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho \overline{k \delta_{ij}} + 2 \mu_s \overline{S_{ij}} - \frac{2}{3} \rho \kappa \delta_{ij}$$  \hspace{1cm} (9)

RNG k-$\varepsilon$: The equations for $k$ and $\varepsilon$ are given as follows:

$$\frac{\partial}{\partial t} (\rho k) + \text{div}(\rho ku_i) = \text{div}(\rho \mu_{eff} \nabla k) + \tau_{k\varepsilon} S_{ij} - \rho \varepsilon$$  \hspace{1cm} (10)

$$\frac{\partial}{\partial t} (\rho \varepsilon) + \text{div}(\rho u_i \varepsilon) = \text{div}(\rho \mu_{eff} \nabla \varepsilon)$$  \hspace{1cm} (11)

$$+ C_{1s} \frac{\varepsilon}{k} \tau_{k\varepsilon} - C_{2s} \rho \frac{\varepsilon^2}{k}$$

$$\mu_{eff} = \rho C_p \frac{k^2}{\varepsilon}$$  \hspace{1cm} (12)

where, $C_{1s} = 0.0845$, $C_{2s} = 1.39$, $C_{1s} = 1.42$, $C_{2s} = 1.68$, $C_{1s} = \frac{\eta(1 - \frac{3}{8})}{1 + \beta \eta}$, $\eta = \frac{k}{\varepsilon}$

where $\eta \equiv \frac{S k}{\varepsilon}$, $\eta_0 = 4.38$, $\beta = 0.012$

**Realizable k-$\varepsilon$ model:** The transport equations for $k$ is similar to equation (7) and transport equations for $\varepsilon$ in the realizable model is written as:

$$\frac{\partial}{\partial t} (\rho \varepsilon) + \text{div}(\rho u_i \varepsilon) = \text{div}(\rho \mu_{eff} \nabla \varepsilon)$$  \hspace{1cm} (14)

Where,

$$C_1 = \max \left[ 0.43, \frac{\eta}{\eta + S} \right], \eta = \frac{k}{\varepsilon}, S = \sqrt{2S_{ij} S_{ij}}, G_k: \text{generation of turbulence kinetic energy due to the mean velocity gradients}, G_\omega: \text{generation of turbulence kinetic energy due to buoyancy}, Y_M: \text{represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate}, C_{1s} = 1.44, C_{2s} = 1.9, \sigma_t = 1.0, \sigma_k = 1.2$.

**Standard k-\omega model:** The turbulence kinetic energy, $k$, and the specific dissipation rate ($\omega$), are obtained from the following transport equations:

$$\frac{\partial}{\partial t} (\rho k) + \text{div}(\rho ku_i) = \text{div}(\rho \mu_{eff} \nabla (k)) + P_k$$  \hspace{1cm} (15)

and

$$\frac{\partial}{\partial t} (\rho \omega) + \text{div}(\rho u_i \omega) = \text{div} \left[ \frac{\mu}{\sigma_\omega} \nabla \omega \right] + \frac{Y_1}{2} \rho S_{ij} - \frac{3}{3} \frac{\rho \omega}{\sigma_\omega} \frac{\partial u_i}{\partial x_j} - \frac{1}{2} \rho \omega$$  \hspace{1cm} (16)

Where

$$P = \left( 2 \mu S_{ij} - \frac{2}{3} \rho k \frac{\partial u_i}{\partial x_j} \right)$$

The models constant are as follows: $C_1 = 2.0$, $C_2 = 2.0$, $C_3 = 0.553$, $\sigma_1 = 0.075$, $\sigma_2 = 0.09$

**Shear-Stress Transport (SST) k-\omega model:**

The Reynolds stress computation and k-equation are the same as in Wilcox’s original k-\omega model (eqn. 15), but the $\varepsilon$-equation is transformed into an $\omega$-equation by substituting $\varepsilon = k \frac{\omega}{\omega}$. The $\omega$-equations are provided as:

$$\frac{\partial}{\partial t} (\rho \omega) + \text{div}(\rho u_i \omega) = \text{div} \left[ \frac{\mu}{\sigma_\omega} \nabla \omega \right]$$

$$+ \frac{Y_1}{2} \rho S_{ij} - \frac{3}{3} \frac{\rho \omega}{\sigma_\omega} \frac{\partial u_i}{\partial x_j}$$

$$- \beta_2 \rho \frac{\omega^2}{\omega}$$  \hspace{1cm} (17)

The models constant are as follows: $C_1 = 1.0$, $C_2 = 1.0$, $\sigma_1 = 1.0$, $\sigma_2 = 1.68$, $\beta_1 = 0.44$, $\beta_2 = 0.0828$, $\beta_3 = 0.09$

**3. Numerical detail**

A three dimensional unstructured tetrahedral mesh was used for numerical simulations. Computation was carried out for half of the tundish because of prevalence of symmetry at the centre plane. A control volume based technique has been used to convert the governing equations to algebraic equations. Second-order upwind discretization scheme was used to discretize the transport equations. The SIMPLE algorithm was used for pressure-velocity coupling and body force (due to gravity) has been considered. The species equation was solved in the complete flow domain.

**3.4 Boundary Conditions**

Fluid was assumed to be incompressible. The symmetry boundary condition has been implied at the symmetry plane. The walls were set to no slip condition with zero velocity. At the inlet, turbulence intensity value was taken as 2% and at the outlets, atmospheric pressure condition was assumed. The bottom of the tundish was treated like a wall where no slip conditions were used for the velocity. Tundish outlet nozzles were maintained to give constant throughput rate. Solutions through implicit discretization schemes are said to be stable and less time consuming than explicit scheme solutions. Hence, Implicit discretization scheme with modified-HRIC (High Resolution Interface Capturing) interface interpolation scheme has been used. Academic version of CFD software ANSYS FLUENT 13.0 was used for solving the set of equations.
4 Results and Discussion

During the process of ladle change over, there is mixing of the old and the new grade steel in the tundish. The extent of intermixing is represented by a curve, known as F-curve. This curve is nothing but the temporal variation of tracer concentration at the tundish outlet when tracer (representing new grade steel from the new ladle) starts flowing into the tundish. The concentration of the tracer which is initially zero starts increasing with time and a tracer concentration value of 1 is representation of the tundish completely filled with new grade steel i.e. tracer. A value of F in between 0 to 1 represents the mixture of old and new grade steels. As the value of the concentration is dependent upon the existing flow vector inside the tundish, which is further influenced by the selection of turbulence model, the assessment of different turbulence models namely Standard $k$-$\varepsilon$, Realizable $k$-$\varepsilon$, Standard $k$-$\omega$ and SST $k$-$\omega$ have been done in the present study to see its effect on various tundish parameters like F-curve, filling rate, interface prediction etc by comparing the quantities numerically obtained versus the experimentally measured ones. The F-curves obtained from experiment & by using five different turbulence models have been shown in the Fig 2(a-c) at near outlet, middle outlet and far outlet respectively. These characteristic graphs are used to evaluate the intermixed amount formed in a tundish during ladle change over process. The abscissa of the graph is the time during which the steel coming out through the outlet is of mixed grade quality not acceptable to the steelmaker. This time, multiplied with the volumetric flow rate through a particular outlet will give the volume of such intermixed steel formed. Hence the time is taken as the parameter representing the intermixed amount. The ordinate value gives the concentration of new grade steel coming out of the tundish. Depending upon the specified grade specification limits, the intermixed amount is computed for a particular curve. In Fig 2(a), F curves are drawn at the near outlet for experimental as well as different turbulence model cases. It is seen that all the curves show the concentration value of the tracer gradually increasing from 0 at initial time (representing only old grade steel from tundish outlet) and approaching 1 (representing completely new grade steel) as the time progresses. The curve obtained by the experiment is well matched by most of the turbulence models during the initial time up to 100 seconds except the SST$k$-$\omega$ and Realizable $k$-$\varepsilon$. After 100 seconds, these two turbulence models predict closer to the experimental curve whereas the prediction by two other turbulence models namely standard$k$-$\omega$ and standard$\varepsilon$ start to deviate from the experimentally obtained value. RNG$k$-$\varepsilon$ is seen to match very much closer to the experimental curve. It can be seen that RNG $k$-$\varepsilon$ model predicts physical phenomenon in better way and satisfactorily matches with the experimental curve of grade mixing.

In Fig 2(b) and Fig 2(c), F curves are drawn at the middle and far outlet respectively for experimental as well as different turbulence model cases. In these curves, a somewhat more visible difference in the concentration value predicted even during the initial time (of upto 100 sec) can be noticed. RNG model seems to be predicting the concentration values closer to the experimentally observed one in these cases too. It is difficult to quantify the extent of match between the curves obtained by experimental and the turbulence model predicted ones in these curves. Hence intermixed amount has been calculated for three different grade specification values at these outlets and a comparison has been made among the turbulence models so as to find which of the turbulence models predicts intermixed amount close to the experiment. Table 1 to 3 gives the intermixed amount obtained from the three outlets for three grade specification values and a variation of the predicted
value for individual turbulence model as compared to the experimental curve is presented in Fig. 3(a-c) for near, middle and far outlets respectively.

### Table 1 Intermixed amount for near outlet

<table>
<thead>
<tr>
<th>Near Outlet</th>
<th>20:60</th>
<th>40:80</th>
<th>20:80</th>
</tr>
</thead>
<tbody>
<tr>
<td>RNG k-ε</td>
<td>37.6</td>
<td>80.9</td>
<td>97.5</td>
</tr>
<tr>
<td>Realizable k-ε</td>
<td>64.25</td>
<td>114.3</td>
<td>140.05</td>
</tr>
<tr>
<td>SST k-ω</td>
<td>64.2</td>
<td>202.6</td>
<td>227.1</td>
</tr>
<tr>
<td>Standard k-ε</td>
<td>43.11</td>
<td>73.3</td>
<td>92.54</td>
</tr>
<tr>
<td>Standard k-ω</td>
<td>33.45</td>
<td>59.08</td>
<td>74.28</td>
</tr>
<tr>
<td>Experiment</td>
<td>37.86</td>
<td>94.3</td>
<td>107.5</td>
</tr>
</tbody>
</table>

### Table 2 Intermixed amount for middle outlet

<table>
<thead>
<tr>
<th>Middle Outlet</th>
<th>20:60</th>
<th>40:80</th>
<th>20:80</th>
</tr>
</thead>
<tbody>
<tr>
<td>RNG k-ε</td>
<td>79.3</td>
<td>133.7</td>
<td>163.9</td>
</tr>
<tr>
<td>Realizable k-ε</td>
<td>94.2</td>
<td>219.7</td>
<td>259.4</td>
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<tr>
<td>SST k-ω</td>
<td>129.2</td>
<td>220.3</td>
<td>264.7</td>
</tr>
<tr>
<td>Standard k-ε</td>
<td>59.3</td>
<td>101.2</td>
<td>126.6</td>
</tr>
<tr>
<td>Standard k-ω</td>
<td>54</td>
<td>81.3</td>
<td>107.3</td>
</tr>
<tr>
<td>Experiment</td>
<td>59.5</td>
<td>154.3</td>
<td>168.5</td>
</tr>
</tbody>
</table>

### Table 3 Intermixed amount for far outlet

<table>
<thead>
<tr>
<th>Far Outlet</th>
<th>10:40</th>
<th>20:40</th>
<th>10:50</th>
</tr>
</thead>
<tbody>
<tr>
<td>RNG k-ε</td>
<td>66.5</td>
<td>43.3</td>
<td>93</td>
</tr>
<tr>
<td>Realizable k-ε</td>
<td>98.8</td>
<td>72.9</td>
<td>158.9</td>
</tr>
<tr>
<td>SST k-ω</td>
<td>174</td>
<td>115</td>
<td>265</td>
</tr>
<tr>
<td>Standard k-ε</td>
<td>56.5</td>
<td>39.2</td>
<td>70.3</td>
</tr>
<tr>
<td>Standard k-ω</td>
<td>132</td>
<td>92</td>
<td>203</td>
</tr>
<tr>
<td>Experiment</td>
<td>82.7</td>
<td>60.5</td>
<td>119.2</td>
</tr>
</tbody>
</table>

The variation of intermixed amount predicted is both negative and positive. A negative value means under-prediction of the intermixed amount whereas a positive variation means an over-prediction, as represented on the ordinate of the bar graphs. The standard k-ε model has good prediction of mixing model at the initial 100 sec of experiment but at later steps of time Realizable k-ε model predicts it precisely. Since near outlet is closely located to inlet, the flow velocity is high here and consequently a more turbulent flow with larger eddies. Turbulence production at inlet zone has increased with time due to the constant increase of free surface height in tundish. One possible reason for deviation of standard k-ε model curve may be due to transport of turbulence by eddy motions in adjacent of near outlet. In Fig 2(a-b) the F curve obtained through standard k-ε model have same characteristics but in Fig 2(c) it lacks poorly at far outlet. Because the flow field at far zone is more stable and have less fluid velocity magnitude. The turbulence is generated and maintained by shear in the mean flow. Turbulence quantities are generally anisotropic in nature & directly affected by shear. These models lacks in prediction of anisotropic diffusion of new grade steel in tundish due to Boussinesq’s analogy of isotropic assumption. It is seen that Realizable k-ε model has predicted F-curve fairly but not precisely. It has fairly predicted the initial 100 seconds and last 150 seconds of experiments. It may be said that this model has some substantial improvements against the standard k-ε model as stated in literatures. It is noted that RNG k-ε model has fairly predicted the F-curve at all three outlets. This model has advantage of special treatment of statistical mechanics. The fluid flows in tundish have some swirling behaviour and this is widely reported in literatures on mathematical modeling of tundish operations. Such a capability of this model enhances the approximation of swirling flows in tundish. Another possible reason for good approximation may be due to RNG theory. The tundish have different types of flow zones, some regions has shown better mixing capability and high turbulence production while some regions have low velocity magnitude field. RNG model contains the analytically derived differential formulas for effective viscosity, which includes the low Reynolds number effect. A k-ω model turbulence model proposed by Wilcox has been also employed in present case of study. In k-ω model length scale is calculated along with another second variable called turbulence frequency ω. This model has similar benefits like k-ε models. It can be noted from Fig. 2(a&b) those curves obtained by both models are nearly overlapping each other. However in case of far outlet (Fig. 2(c)) k-ω model have not performed well.

![Figure 3(a) Near outlet](image-url)
In case of SST k-ω model, it is noted from Fig 2(a-c) that F curve are under predicted. SST k-ω turbulence model is hybrid of standard k-ε model and standard k-ω model. This combines the Wilcox k-ω model for near-wall region and standard k-ε model in the fully turbulent region, away from the wall. However due to sensitive nature of this model, mixing curves are slightly offset from experimental curves.

5 Conclusion

An assessment of RANS equations based k-ε and k-ω turbulence models have been carried out on steelmaking tundish. Following conclusions can be drawn:

1) Multiphase models are more appropriate for numerical modeling. It can be used for modeling of different phase materials like air, slag, molten steel in tundish operations. CLSVOF method represent better understanding of free-surface height of melt, grade mixing and fluid flow behaviour during ladle change operation.

2) RNG k-ε turbulence model was able to reproduce swirl and recirculation of fluid. This model has shown better approximation of F-curve as compared to other turbulence models. All turbulence models except SST k-ω model have fairly predicted interface of water and air.

3) k-ω turbulence models perform with poor results as compared to other turbulence models.

References


