Linear Collider searches of beyond the Standard Model Physics

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Introduction

The theory of space-time geometry (Riemannian) known as the classical theory of gravity. The unification of the Strong, Electromagnetic and weak field theory called as the theory of Standard Model (SM).

The Standard Model (SM) and GTR describe very well, all physical phenomena from cosmological process to the properties of subnuclear structures.
Why we need Noncommutative space-time?

Figure: Rotating Black hole
Motivation for non-commutative space-time

1. Near the Black hole, the time moves differently than in normal space. Because of its enormous mass.

2. If we probe physics at Planck scale \( l = l_p = \sqrt{\frac{\hbar G}{C^3}} \), the Compton length will be less than or equal to Planck scale.

3. This creates a large mass \( m \geq \frac{\hbar}{l_p C} \) in a tiny volume \( l_p^3 \).

4. This is large enough to form a black hole with a huge event horizon, which hides the information sent out by the probe.

5. How to prevent a gravitational collapse from vacuum fluctuations near the Planck length?

Noncommutative structure implies, time is a derivative of space.
Non-Commutative space-time

1. Below Planck scales it is natural to conceive a more general spacetime structure. A noncommutative one where (as with quantum mechanics phase-space) uncertainty relations are arise naturally.

2. The space-time co-ordinates become operators and can be expressed as

\[
\left[ \hat{X}_\mu, \hat{X}_\nu \right] = i \Theta_{\mu\nu} = \frac{i}{\Lambda_{NC}^2} C_{\mu\nu}
\]

\( \Lambda \) is the NC scale and \( C_{\mu\nu} \) is the antisymmetric constants.

3. In the case of \( \Theta_{\mu\nu} \) constant, the commutator defines a Heisenberg algebra and imply spacetime uncertainty:

\[
\Delta \hat{X}_\mu \Delta \hat{X}_\nu \geq \frac{1}{2} | \Theta_{\mu\nu} |
\]

4. The field theory in non-commutative spacetime based on star products and Seiberg-Witten (SW) maps allows the generalization of the SM to the case of non-commutative space-time, keeping the original gauge group and particle content.
Moyal-Weyl ★ products and Seiberg-Witten map

Moyal-Weyl (WM) products

\[(f \circ g)(x) = \exp \left( \frac{1}{2} \Theta_{\mu \nu} \partial_x \partial_y \right) f(x)g(y) \big|_{y=x} \]

SW map:
In this approach, both the gauge field and transformation (gauge) parameter is expanded as a power series in \( \Theta_{\mu \nu} \) as follows

\[\hat{\psi}(x, \Theta) = \psi(x) + \Theta \psi^{(1)} + \Theta^2 \psi^{(2)} + \ldots.\]

\[\hat{A}_\mu(x, \Theta) = A_\mu(x) + \Theta A_\mu^{(1)} + \Theta^2 A_\mu^{(2)} + \ldots.\]

The advantage in the SW approach is that this construction can be applied to any gauge theory (including the standard model) in which matter can be in an arbitrary representation.

For example, We can write the NCQED lagrangian as

\[\mathcal{L} = \frac{1}{2} i \left( \hat{\psi} \gamma^\mu D_\mu \hat{\psi} - (D_\mu \hat{\psi}) \gamma^\mu \hat{\psi} \right) - m \hat{\psi} \hat{\psi} - \frac{1}{4} \hat{F}_{\mu \nu} \hat{F}^{\mu \nu}\]

Where \( D_\mu \hat{\psi} = \partial_\mu \hat{\psi} - ie \hat{A}_\mu \hat{\psi} \) and \( \hat{F}_{\mu \nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - ie \left( \hat{A}_\mu \hat{A}_\nu - \hat{A}_\nu \hat{A}_\mu \right) \)

\[\hat{\psi}(x, \Theta) \rightarrow \hat{\psi}'(x, \Theta) = U \circ \hat{\psi}(x, \Theta) \text{ and} \]

\[\hat{A}_\mu(x, \Theta) \rightarrow \hat{A}'_\mu(x, \Theta) = U \circ \hat{A}_\mu(x, \Theta) \circ U^{-1} + \frac{i}{e} U \circ \partial_\mu U^{-1}\]
1. With respect to gauge sector, there are two versions of noncommutative standard model (NCSM): one is **minimal version** and another one is **non-minimal version**.

2. The difference between two models is due to the freedom of the choice of traces in the kinetic terms for gauge fields.

3. The fermion sector of the action $S_{NCSM}$ is not affected by choosing different traces over the representation in the gauge part of the action and remains the same in both models.

4. **mNCSM**: The representation that yields a model as close as possible to the SM without new triple gauge boson couplings.

5. **nmNCSM**: The trace is chosen over all particles on which covariant derivatives act and which have different quantum numbers.
Higgstralung process \( (e^- e^+ \rightarrow ZH) \) in mNCSM

\[
\frac{-ie}{\sin 2\theta_W} \gamma^\mu \left\{ \left( \frac{-1}{2} + 2 \sin^2 \theta_W \right) + \frac{1}{2} \gamma_5 \right\} e^{i \left( \frac{p_1 \Theta p_2}{2} \right)}
\]

\[
\frac{iM_Z^2}{\nu} \left\{ 2 \cos \left( \frac{p_3 \Theta p_4}{2} \right) \eta^{\mu \nu} + \left[ \cos \left( \frac{p_3 \Theta p_4}{2} \right) - 1 \right] \left[ \left( \Theta p_4 \right)^{\mu \nu} p_3^\nu + \left( \Theta p_4 \right)^{\nu \mu} k^\mu \right] \right\}
\]
Differential cross section

\[ d\sigma = \frac{\lambda^{1/2}(S,M_Z^2,M_H^2)}{64\pi^2 S^2} |M|^{2NC_{SM}} d(cos \theta) d\phi \]

For Standard Model

\[ |M|^2_{SM} = \left( \frac{16\pi \alpha M_Z^2}{Z} \left[ \frac{1}{4} + \left( \frac{1}{2} + 2 \sin^2 \theta_W \right)^2 \right] \right) \cdot \left\{ \frac{S}{2} + \frac{S}{2M_Z^2} \left( M_Z^2 + \frac{\lambda(S,M_Z^2,M_H^2)}{4S} \right) \right\} \]

For Non-Commutative Standard Model

\[ |M|^2_{NC_{SM}} = \cos^2 \left( \frac{P_3 \Theta P_4}{2} \right) |M|^2_{SM} \]

We decompose \( C_{\mu\nu} \) into electric like parts \( \vec{\Theta}_E = (\theta_{01}, \theta_{02}, \theta_{03}) \) and magnetic like parts \( \vec{\Theta}_B = (\theta_{23}, \theta_{31}, \theta_{12}) \). i.e \( \vec{\Theta}_E = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k}) \) and \( \vec{\Theta}_B = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k}) \)

Non-commutative tensor

\[ \Theta_{\mu\nu} = \frac{1}{\Lambda_{NC}^2} \left( \begin{array}{ccc} 0 & \Theta_{EX} & \Theta_{EY} & \Theta_{EZ} \\ -\Theta_{EX} & 0 & -\Theta_{BZ} & \Theta_{BY} \\ -\Theta_{EY} & \Theta_{BZ} & 0 & -\Theta_{BX} \\ -\Theta_{EZ} & -\Theta_{BY} & \Theta_{BX} & 0 \end{array} \right) \]

\[ P_3 \Theta P_4 = \frac{1}{\sqrt{3} \Lambda_{NC}^2} \left( \begin{array}{cccc} 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & 1 \\ -1 & 1 & 0 & -1 \\ -1 & -1 & 1 & 0 \end{array} \right) \]

\[ (P_3 \Theta P_4) = \frac{1}{2\Lambda_{NC}^2} \sqrt{\frac{\lambda(S,M_Z^2,M_H^2)}{3}} (\cos \theta + \sin \theta (\sin \phi + \cos \phi)) \]

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Total cross section and Azhimuthal distribution

\[(\Delta \sigma)_{NC} = \sigma_{NC} - \sigma_{SM}\]

\[\delta_r = (\Delta \sigma)_{NC} / \sigma_{SM}\]
Azhimuthal distribution

\[ \frac{d\sigma}{d\phi} \text{ (fb/radian)} \]

\( \sqrt{S} = 0.5\text{TeV} \)
- \( \Lambda = 0.5\text{TeV} \)
- \( \Lambda = 0.6\text{TeV} \)
- \( \Lambda = 0.7\text{TeV} \)
- \( \Lambda = 1.0\text{TeV} \)
- \( \Lambda = 1.5\text{TeV} \)
- SM case

\( \sqrt{S} = 3.0\text{TeV} \)
- \( \Lambda = 0.5\text{TeV} \)
- \( \Lambda = 0.6\text{TeV} \)
- \( \Lambda = 0.7\text{TeV} \)
- \( \Lambda = 1.0\text{TeV} \)
- \( \Lambda = 1.5\text{TeV} \)
- SM case

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NC effect: Earth rotation

The NC electric vector $\vec{\Theta}_E$ (in the primary($\hat{i}_X$, ...) frame and earth based lab($\hat{i}$, ...) frame)

$$\vec{\Theta}_E(t) = \Theta_E \left( s_\eta c_\xi \hat{i}_X + s_\eta s_\xi \hat{j}_Y + c_\eta \hat{k}_Z \right) = \Theta_{Ex} \hat{i} + \Theta_{Ey} \hat{j} + \Theta_{Ez} \hat{k}$$

where $\Theta_E = |\vec{\Theta}_E| = 1/\Lambda^2$. Here $s_\eta = \sin \eta$, $s_\xi = \sin \xi$, $c_\xi = \cos \xi$.

$\eta$, $\xi$ specifies the direction of $\vec{\Theta}_E$ in the primary coordinate system.
Connecting lab bases and primary bases

The transformation rules between the primary bases \( \hat{i}_X, \hat{j}_Y, \hat{k}_Z \) and lab bases \( \hat{i}, \hat{j}, \hat{k} \) used above are

\[
\begin{align*}
\hat{i}_X &= (c_\alpha s_\zeta + s_\delta s_\alpha c_\zeta) \hat{i} + c_\delta c_\zeta \hat{j} + (s_\alpha s_\zeta - s_\delta c_\alpha c_\zeta) \hat{k} \\
\hat{j}_Y &= (-c_\alpha c_\zeta + s_\delta s_\alpha s_\zeta) \hat{i} + c_\delta s_\zeta \hat{j} + (-s_\alpha c_\zeta - s_\delta c_\alpha s_\zeta) \hat{k} \\
\hat{k}_Z &= -c_\delta s_\alpha \hat{i} + s_\delta \hat{j} + c_\delta c_\alpha \hat{k}
\end{align*}
\]

Note that \( \zeta = \omega t \). In our analysis we set \( \alpha = \pi/4 \) and \( \delta = \pi/4 \).
NC effect due to earth rotation

In the Lab frame,

\[
\begin{align*}
\Theta_{E_x}^{lab} &= \Theta_E \left( s_\eta c_\xi (c_\alpha s_\zeta + s_\delta s_\alpha c_\zeta) + s_\eta s_\xi (-c_\alpha c_\zeta + s_\delta s_\alpha s_\zeta) - c_\eta c_\delta s_\alpha \right) \\
\Theta_{E_y}^{lab} &= \Theta_E \left( s_\eta c_\xi c_\delta c_\zeta + s_\eta s_\xi c_\delta s_\zeta + c_\eta s_\delta \right) \\
\Theta_{E_z}^{lab} &= \Theta_E \left( s_\eta c_\xi (s_\alpha s_\zeta - s_\delta c_\alpha c_\zeta) - s_\eta s_\xi (s_\alpha c_\zeta + s_\delta c_\alpha s_\zeta) + c_\eta c_\delta c_\alpha \right)
\end{align*}
\]

The NC contribution

\[
\begin{align*}
p_2 \Theta p_1 &= -\frac{s}{2} \Theta_{E_z}^{lab} \\
p_4 \Theta p_3 &= -\frac{s}{2} \left( s_\theta c_\phi \Theta_{E_x}^{lab} + s_\theta s_\phi \Theta_{E_y}^{lab} + c_\theta \Theta_{E_z}^{lab} \right)
\end{align*}
\]

\((\delta, a)\) defines the location of the laboratory.\((\delta, a) = (\frac{\pi}{4}, \frac{\pi}{4})\) which is the OPAL experiment at LEP.
Time-averaged cross-section and distributions

The time-averaged observables are

\[
\langle \frac{d\sigma}{d\cos\theta} \rangle_T = \frac{1}{T_{\text{day}}} \int_{0}^{T_{\text{day}}} \frac{d\sigma}{d\cos\theta} dt,
\]

\[
\langle \frac{d\sigma}{d\phi} \rangle_T = \frac{1}{T_{\text{day}}} \int_{0}^{T_{\text{day}}} \frac{d\sigma}{d\phi} dt,
\]

\[
\langle \sigma \rangle_T = \frac{1}{T_{\text{day}}} \int_{0}^{T_{\text{day}}} \sigma dt,
\]

with \(T_{\text{day}}(= 2\pi/\omega) = 23h 56m 4.09053s\)
Azimuthal distribution due earth rotation

\[\langle \frac{d\sigma}{d\phi}\rangle (\text{fb/radian})\]

\(\eta = 0\)
\(\eta = \frac{\pi}{4}\)
\(\eta = \frac{\pi}{2}\)
\(\eta = \frac{3\pi}{4}\)
\(\eta = \pi\)

SM case

\(\sqrt{S} = 0.5\text{TeV}\)
\(\sqrt{S} = 1.0\text{TeV}\)
\(\Lambda = 0.6\text{TeV}\)
\(\Lambda = 0.5\text{TeV}\)
\(\Lambda = 0.6\text{TeV}\)
\(\Lambda = 0.7\text{TeV}\)
\(\Lambda = 0.8\text{TeV}\)
\(\Lambda = 1.0\text{TeV}\)
\(\Lambda = 1.5\text{TeV}\)

SM case

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Azimuthal distribution due to earth rotation

![Graphs showing azimuthal distribution](image)

**With out Earth rotation**

![Graphs showing azimuthal distribution without Earth rotation](image)
Conclusion

1. The impact of the space-time noncommutativity (with and without earth rotation) on the Higgstralug process in the TeV energy linear collider have been studied.

2. One can see from azimuthal distributions that \( \frac{d\sigma}{d\phi} \) and \( \left\langle \frac{d\sigma}{d\phi} \right\rangle_T \) are anisotropic.

3. So far we have considered one experiment with \( (\delta, a) = (\frac{\pi}{4}, \frac{\pi}{4}) \).

4. If there are several detector sites in the \( e^- e^+ \) collider experiment and the direction of \( e^- \) beam in each site is set to be along to the different direction, then the angular distributions and the time variation of observables should behave differently in each experiment.
References

Thank you

For your attention