Muon anomalous magnetic moment and positron excess at AMS-02 in a gauged horizontal symmetric model

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Introduction
  1. Standard Model contribution to muon g-2
  2. Main Point

Model
  1. Muon g-2
  2. Dark matter

Summary
There exist two interesting signals called:

- Muon anomalous magnetic moment \((g-2)\) measured at BNL. \(^1\)
- The positron excess over cosmic-ray background measured by AMS-02. \(^2\)
- These two signals can have a common beyond standard model explanation.

\(^1\) Muon Collab., PRD 73, 072003
\(^2\) AMS Collab., PRL 110, 141102
In interacting quantum field theory $g$ gets correction,

\[ \Gamma_\mu(q^2) = \left( \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} q^\nu F_2(q^2) \right) \]

When muon is on mass-shell,

\[ F_2(0) = \frac{g - 2}{2} \equiv a_\mu \quad \text{(anomalous magnetic moment)} \]
The standard model contribution to the muon magnetic moment is

$$\Delta a_{\mu}^{SM} = \Delta a_{\mu}^{QED} + \Delta a_{\mu}^{Weak} + \Delta a_{\mu}^{Had}$$

Loops in the standard model that contribute to muon magnetic moment.
<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>116 584 718.853</td>
<td>0.036</td>
</tr>
<tr>
<td>Weak</td>
<td>153.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Leading order HVP</td>
<td>6 907.5</td>
<td>47.2</td>
</tr>
<tr>
<td>Higher order HVP</td>
<td>-100.3</td>
<td>2.2</td>
</tr>
<tr>
<td>HLbL</td>
<td>116</td>
<td>40</td>
</tr>
<tr>
<td>Theory (total)</td>
<td>116 591 796</td>
<td>62</td>
</tr>
<tr>
<td>Experiment</td>
<td>116 592 089</td>
<td>63</td>
</tr>
<tr>
<td>Experiment - Theory (3.3σ)</td>
<td>293</td>
<td>88</td>
</tr>
</tbody>
</table>

*Standard model contribution to $\Delta a_\mu \times 10^{-11}$ and comparison of theory and experiment.*

**Main Point**

**In SM, contribution of weak interaction to muon g-2**

\[ a_{\mu}^{\text{W}} \propto \frac{m_{\mu}^2}{M_W^2} \]

**In MSSM, contribution to muon g-2**

\[ a_{\mu}^{\text{MSSM}} \propto \frac{m_{\mu}^2}{M_{\text{SUSY}}^2} \]
Introduce 4th generation of quarks and leptons,

\[
\begin{pmatrix}
\mathcal{C}'_L, & \mathcal{C}'_R, & s'_R, & \left( \begin{array}{c}
\nu'_L \, \nu'_R \, \mu'_L \, \mu'_R
\end{array} \right)
\end{pmatrix}
\]

We also add three right handed neutrinos \((\nu_iR)\) in the model.

We introduce a \(SU(2)_H\) horizontal gauge symmetry between the 4th generation leptons and muon family.

To give masses to the particles and to evade the bounds on the 4th generation from the higgs production at LHC, we introduce three new scalars \((\phi_i, \eta_{i\alpha}, \chi_{\alpha})\).
<table>
<thead>
<tr>
<th>Particles</th>
<th>$G_{STD} \times SU(2)_{HV}$ Quantum numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{eLi} \equiv (\nu_e, e)$</td>
<td>(1, 2, $-$1, 1)</td>
</tr>
<tr>
<td>$\psi_{Li\alpha} \equiv (\psi_\mu, \psi_\mu')$</td>
<td>(1, 2, $-$1, 2)</td>
</tr>
<tr>
<td>$\psi_{\tau Li} \equiv (\nu_\tau, \tau)$</td>
<td>(1, 2, $-$1, 1)</td>
</tr>
<tr>
<td>$E_{R\alpha} \equiv (\mu_R, \mu'_R)$</td>
<td>(1, 1, $-$2, 2)</td>
</tr>
<tr>
<td>$N_{R\alpha} \equiv (\nu_{\mu R}, \nu_{\mu' R})$</td>
<td>(1, 1, 0, 2)</td>
</tr>
<tr>
<td>$e_R, \tau_R$</td>
<td>(1, 1, $-$2, 1)</td>
</tr>
<tr>
<td>$\nu_{eR}, \nu_{\tau R}$</td>
<td>(1, 1, 0, 1)</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>(1, 2, 1, 1)</td>
</tr>
<tr>
<td>$\eta_\beta$</td>
<td>(1, 2, 1, 3)</td>
</tr>
<tr>
<td>$\eta_{i\alpha}$</td>
<td>(1, 1, 0, 2)</td>
</tr>
<tr>
<td>$\chi_\alpha$</td>
<td>(1, 1, 0, 2)</td>
</tr>
</tbody>
</table>

Representation of the various fields in the model under the gauge group $G_{STD} \times SU(2)_{HV}$. 
The Masses of the gauge bosons are,

\[ M^2_W = \frac{g^2}{2} (2 \langle \eta \rangle^2 + \langle \phi \rangle^2), \quad M^2_Z = \frac{g^2}{2} \sec^2 \theta_W (2 \langle \eta \rangle^2 + \langle \phi \rangle^2), \quad M^2_A = 0, \]

\[ M^2_{\theta^+} = g_H^2 (4 \langle \eta \rangle^2 + \frac{1}{2} \langle \chi \rangle^2), \quad M^2_{\theta^3} = \frac{1}{2} g_H^2 \langle \chi \rangle^2 \]

The masses of the leptons are,

\[ m_e = h_1 \langle \phi \rangle, \quad m_\tau = h_3 \langle \phi \rangle, \quad m_{\nu_e} = \tilde{h}_1 \langle \phi \rangle, \quad m_{\nu_\tau} = \tilde{h}_3 \langle \phi \rangle \]

\[ m_\mu = h_2 \langle \phi \rangle + k_2 \langle \eta \rangle, \quad m_{\nu_\mu} = \tilde{h}_2 \langle \phi \rangle + \tilde{k}_2 \langle \eta \rangle, \]

\[ m_{\mu'} = h_2 \langle \phi \rangle - k_2 \langle \eta \rangle, \quad m_{\nu_{\mu'}} = \tilde{h}_2 \langle \phi \rangle - \tilde{k}_2 \langle \eta \rangle, \]
Contributions to muon $g-2$

Feynman diagrams of scalar $\eta_{i\alpha}^\beta$ and $SU(2)_{HV}$ gauge boson $\theta^+$, which give contributions to muon ($g-2$).
- In the limit of $M_{\theta^+}^2 \gg m_{\mu'}^2$, the muon magnetic moment comes,

$$[\Delta a_\mu]_{\theta^+} = \frac{g_H^2}{8\pi^2} \left( \frac{m_\mu m_{\mu'} - 2/3 m_\mu^2}{M_{\theta^+}^2} \right)$$

- In the limits $m_{\mu'}^2 \gg m_h^2$, $m_{\mu'}^2 \gg m_A^2$,

$$[\Delta a_\mu]_{h,A} = \frac{1}{8\pi^2} \left( \frac{3m_\mu m_{\mu'}(y_h^2 - y_A^2) + m_{\mu'}^2(y_h^2 + y_A^2)}{6m_{\mu'}^2} \right)$$

- In the limit $m_{H^\pm}^2 \gg m_{\nu_{\mu'}}^2$,

$$[\Delta a_\mu]_{H^\pm} = -\frac{y_{H^\pm}^2}{8\pi^2} \left( \frac{3m_\mu m_{\nu_{\mu'}} + m_\mu^2}{6m_{H^\pm}^2} \right)$$
\[ \Delta a_\mu = [\Delta a_\mu]_{\theta^+} + [\Delta a_\mu]_{hA} + [\Delta a_\mu]_{H^\pm} \]

- We use \( g_H = 0.087 \), \( y_h = 0.037 \), \( y_A = 0.020 \), \( y_{H^\pm} = 0.10 \), \( m_{\mu'} = 740 \text{ GeV} \), \( M_{\theta^+} = 1400 \text{ GeV} \), \( m_{H^\pm} = 1700 \text{ GeV} \), \( m_H = 125 \text{ GeV} \), \( m_A = 150 \text{ GeV} \), and find the muon anomalous magnetic moment,

\[ \Delta a_\mu \sim 2.9 \times 10^{-9} \]
We identify the 4th generation neutral lepton \( (\nu'_\mu \equiv \chi) \) as the dark matter.

The possible annihilation channels are,

\[ \chi \rightarrow \mu^-, \nu_\mu, \mu^+, \nu^c_\mu \]

Feynman diagram of dark matter annihilation with corresponding vertex factor.
The thermal average of annihilation rate is given as \(^4\),

\[
\langle \sigma v \rangle (x) = \frac{g_H^4}{512 m^2_\chi} \frac{x^{3/2}}{\pi^{3/2}} \int_0^\infty \frac{\sqrt{z} \exp[-xz/4]}{(\delta + z/4)^2 + \gamma^2} dz
\]

where \( z \equiv v^2 \) and \( \delta, \gamma \) defined as,

\[
M_{\theta_3}^2 \equiv 4m^2_\chi(1 - \delta), \text{ and } \gamma^2 \equiv \Gamma_{\theta_3}^2 (1 - \delta)/4m^2_\chi
\]

We use \( g_H = 0.087, \delta \sim 10^{-3} \) and \( \gamma \sim 10^{-4} \) to get,

\[
\Omega h^2 = 0.1199 \pm 0.0027
\]

This fix the masses,

\[
M_{\theta_3} \sim 1400 \text{ GeV}, \ m_\chi \sim 700 \text{ GeV}
\]

\(^4\text{Murayama, arXiv:0812.0072}\)
AMS-02

- AMS-02 is a particle physics experiment mounted on the International Space Station.

**Magnet** bends in opposite directions charged particles/antiparticles

**Transition Radiation Detector (TRD)** identifies electrons and positrons among other cosmic-rays

**Time-of-Flight System (ToF)** warns the sub-detectors of incoming cosmic-rays

**Silicon Tracker (Tracker)** detects the particle charge sign, separating matter from antimatter

**Ring-Imaging Cherenkov Detector (RICH)** measures with high precision the velocity of cosmic-rays

**Electromagnetic Calorimeter (ECAL)** measures energy of incoming electrons, positrons and γ-rays

**Anti-Coincidence Counter (ACC)** rejects cosmic rays traversing the magnet walls

**Tracker Alignment System (TAS)** checks the Tracker alignment stability

**Star Tracker and GPS** defines the position and orientation of the AMS-02 experiment

**Electronics** transform the signals detected by the various particle detectors into digital information to be analyzed by computers
We use PPPC4DMID code for getting the positron spectrum $dN_{e^+}/dE$ and propagate it through GALPROP code.

The positron flux spectrum compared with data from AMS-02.\textsuperscript{5}

\textsuperscript{5}AMS colla.,PRL 110,141102
Compare with Fermi-LAT

The $\gamma$-ray spectrum compared with data from Fermi-LAT.  

$^6$Fermi-LAT colla., PRL 104, 101101
We studied the 4th generation extension of SM and introduced a $SU(2)_{HV}$ symmetry between 4th generation and muon family.

The 4th generation neutrino ($\nu'$) is identified as the dark matter.

We proposed a common explanation to the AMS-02 positron excess and anomalous muon magnetic moment.

There is no gauge interactions with quarks at tree level, so we also evade the bounds from direct detection experiments.
Thank You!
Magnetic moment of a magnet is a measure of the strength and the direction of its magnetism.

The muon magnetic moment $\mu$ is proportional to its spin ($c = \hbar = 1$).

$$\vec{\mu} = g \left( \frac{e}{2m} \right) \vec{S}$$

Lande $g$-factor is predicted from free Dirac Eq. to be

$$g = 2$$

for elementary fermions.
• Relic density

\[
\sigma_v = \frac{1}{16\pi} \frac{g_H^4 m_\chi^2}{(s - M_{\theta_3}^2)^2 + \Gamma_{\theta_3}^2 M_{\theta_3}^2}
\]

\[
\Gamma_{\theta_3} = \frac{2g_H^2}{48\pi} M_{\theta_3}
\]

• The cross-section is,

\[
\langle \sigma \nu \rangle(x) = \frac{1}{n_{EQ}^2 \frac{m_\chi}{64\pi^4 x}} \int_{4m_\chi^2}^{\infty} \hat{\sigma}(s) \sqrt{s} K_1 \left( \frac{x\sqrt{s}}{m_\chi} \right) ds,
\]
where,

\[ n_{EQ}^2 = \frac{g_i m_X^3}{2\pi^2 x} K_2(x), \]

\[ \hat{\sigma}(s) = 2g_i^2 m_X \sqrt{s - 4m_X^2} \sigma v, \]

\[ \sigma v = \frac{2}{256\pi m_X^2} \frac{g_H^4}{(\delta + v^2/4)^2 + \gamma^2} \]
Boltzmann equation for $Y_\chi = n_\chi/s$,

$$\frac{dY_\chi}{dx} = -\frac{\lambda(x)}{x^2} \left( Y^2_\chi(x) - Y^2_{\chi\text{eq}}(x) \right)$$

where

$$\lambda(x) \equiv \left( \frac{\pi}{45} \right)^{1/2} m_\chi M_{Pl} \left( \frac{g^* s}{\sqrt{g^*}} \right) \langle \sigma v \rangle(x)$$

and where $g^*$ and $g^*_s$ are the effective degrees of freedom.

The $Y_\chi(x_0)$ at the present epoch is,

$$\frac{1}{Y_\chi(x_0)} = \frac{1}{Y_\chi(x_f)} + \int_{x_f}^{x_s} \frac{dx}{x^2} \frac{\lambda(x)}{x^2}$$

where the freeze-out $x_f$ is obtained by solving $n_\chi(x_f) \langle \sigma v \rangle = H(x_f)$. We find that $x_f \sim 30$ and the relic density of $\chi$ is given by,

$$\Omega = \frac{m_\chi s_0 Y_\chi(x_0)}{\rho_c}$$

where $s_0 = 2890 \text{ cm}^{-3}$ is the present entropy density and $\rho_c = h^2 1.9 \times 10^{-29} \text{ gm/cm}^3$ is the critical density.
The differential annihilation rate,

\[ Q_{e^+}(E, \vec{r}) = \frac{\rho^2}{2m^2_{\chi}} \langle \sigma v \rangle_{\mu^+\mu^-} \frac{dN_{e^+}}{dE} \]

NFW profile,

\[ \rho_{\text{NFW}} = \rho_0 \frac{r_s}{r} \left(1 + \frac{r}{r_s}\right)^{-2}, \quad \rho_0 = 0.4 \text{ GeV/cm}^3, \quad r_s = 20 \text{ kpc}, \]

Muon magnetic moment,

\[ \Gamma_\mu = \frac{e g_H^2}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma^\beta (p' + k + m_{\mu'})}{(p' + k)^2 - m^2_{\mu'}} \frac{(p + k + m_{\mu'})}{(p + k)^2 - m^2_{\mu'}} \frac{\gamma^\alpha}{k^2 - M^2_{\theta^+}} \]

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The contribution of neutral higgs particles is,

$$[\Delta a_\mu]_{h,A} = \frac{m^2_\mu}{8\pi^2} \int_0^1 dx \frac{y_h^2 (x^2 - x^3 + \frac{m_{\mu'}^2}{m_\mu^2} x^2)}{m_\mu^2 x^2 + (m_{\mu'}^2 - m_\mu^2) x + m_h^2 (1 - x)}$$

$$+ \frac{m^2_\mu}{8\pi^2} \int_0^1 dx \frac{y_A^2 (x^2 - x^3 - \frac{m_{\mu'}^2}{m_\mu^2} x^2)}{m_\mu^2 x^2 + (m_{\mu'}^2 - m_\mu^2) x + m_A^2 (1 - x)}$$

The contribution of charged higgs is,

$$[\Delta a_\mu]_{H^\pm} = \frac{m^2_\mu}{8\pi^2} \int_0^1 dx \frac{y_{H^\pm}^2 \left( x^3 - x^2 + \frac{m_{\mu'}^2}{m_\mu^2} (x^2 - x) \right)}{m_\mu^2 x^2 + (m_{H^\pm}^2 - m_\mu^2) x + m_{\mu'}^2 (1 - x)}$$
The propagation eq. in Galprop is,

\[
\frac{\partial \psi}{\partial t} = Q(r, t) + \nabla \cdot (D_{xx} \nabla \psi - V \psi) + \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi \\
- \frac{\partial}{\partial p} \left[ \dot{p} \psi - \frac{p}{3} (\nabla \cdot V) \psi \right] - \frac{1}{\tau_f} \psi - \frac{1}{\tau_r} \psi
\]
Free parameters: $\tilde{p}$ masses and mixing, $\mu$ and $\tan\beta$. 
Neutralino and chargino contributions to $g - 2$ in MSSM.
MSSM contribution,

\[ \Delta a_\mu = 1.2 \times 10^{-9} \tan \beta \text{ sign}(\mu) \left( \frac{100 \text{ GeV}}{\tilde{M}_{\text{SUSY}}} \right)^2 \]

Potential enhancement \( \propto \tan \beta = 1, 2, \ldots, 50 \) and \( \propto \text{ sign}(\mu) \).

SUSY can explain 3\( \sigma \) deviation but large \( \tan \beta \) and small \( \tilde{M}_{\text{SUSY}} \) preferred.