Neutrino Masses, Mixings and Leptogenesis in a $A_4$ Symmetric Type-I See-saw.

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based on arXiv: 1407.5826

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**Introduction and Motivation**

- **Nonzero \( \theta_{13} \):**
  - Neutrino oscillation data indicates that the lepton mixing matrix can be described by tribimaximal mixing pattern at the zeroth order (before Double-Chooz results); \( A_4 \) is the simplest symmetry to realize TB pattern.  
    
    
    \[
    \begin{align*}
    &\text{Ma,Rajasekharan PRD64; Babu,Valle,Ma PLB512} \\
    &\text{Altarelli,Feruglio NPB741, Varzielas,King,Ross PLB648} \\
    
    &\text{...}
    \end{align*}
    \]

    After Double-Chooz and other experiments, \( \theta_{13} \neq 0 \) & hence deviation from TB is necessary.

- **The set-up:**
  - \( \star \) particle contents and symmetries
  - \( \star \) light neutrino mass generated by Type-I see-saw
  - \( \star \) parameters involved and their correlation with Majorana phases involved in \( U_{PMNS} \)

- **Leptogenesis:**
  - Baryon asymmetry can be realized with NLO contribution to the neutrino Yukawa matrix and dependence on Majorana phases and \( \theta_{13} \)
Neutrino Phenomenology and Parameters

Neutrino flavour eigenstates and mass eigenstates are related by

\[ |\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \]

Pontecorvo-Maki-Nakagawa-Sakata parametrization

\[
U_{PMNS} = \begin{pmatrix}
C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\
-S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & 0 \\
S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & C_{13}S_{23} & 0 \\
S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & C_{13}S_{23} & 0 \\
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
e^{i\alpha_{21}/2} & 0 & 0 \\
0 & e^{i\alpha_{31}/2} & 0
\end{pmatrix}
\]

(1)

here \( C_{ij} = \cos \theta_{ij} \) and \( S_{ij} = \sin \theta_{ij} \).

Neutrino Parameters

- Three mixing angles (\( \theta_{12}, \theta_{23}, \text{and} \theta_{13} \))
- Two mass squared differences, namely solar mass-squared difference (\( \Delta m^2_\odot \)) and atmospheric mass-squared difference (\( \Delta m^2_A \)). Atleast, two different neutrino mass spectrum is possible: Normal hierarchy (NH) for which conversion is \( m_1 < m_2 < m_3 \) and inverted hierarchy (IH) for which conversion \( m_3 < m_1 < m_2 \).
- CP violating Dirac phase (\( \delta \))
- Two Majorana phases (\( \alpha_{21} \text{ and } \alpha_{31} \))
### Neutrino Parameters

#### Neutrino Mixing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best Fit</th>
<th>$1\sigma$ range</th>
<th>$3\sigma$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2_{12} ; [\times 10^{-5}\text{eV}^2]$</td>
<td>7.60</td>
<td>$7.42 - 7.79$</td>
<td>$7.11 - 8.18$</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m^2_{21}</td>
<td>; [\times 10^{-3}\text{eV}^2]$</td>
<td>2.48 2.38</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.323</td>
<td>$0.307 - 0.339$</td>
<td>$0.278 - 0.375$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.567 0.573</td>
<td>$0.439 - 0.599$ $0.530 - 0.598$</td>
<td>$0.392 - 0.643$ $0.403 - 0.640$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.0234 0.0240</td>
<td>$0.0214 - 0.0254$ $0.0221 - 0.0259$</td>
<td>$0.0177 - 0.0294$ $0.0183 - 0.0297$</td>
</tr>
</tbody>
</table>

#### Bound on neutrino Mass and BAU

**PLANCK Collaboration, arXiv:1303:5076**

\[ \sum_i m_{\nu_i} < 0.230 \text{eV} \; (95\% \text{ CL}) \]

**WMAP, arXiv: 1212:5225**

\[ Y_B = \frac{n_B - n_{\bar{B}}}{s} = (8.77 \pm 0.24) \times 10^{-11} \]
Tribimaximal (TBM) mixing:

Global analysis of neutrino oscillation data suggests $\sin^2 \theta_{12} = 1/3$ and $\sin^2 \theta_{23} = 1/2$. With $\theta_{13} = 0$, neutrino mixing matrix takes the form

$$U_{TB} = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

(Talk by P. Roy and G. Rajasekharan)

Measurement of $\theta_{13}$

- Important discovery in 2012 apart from Higgs
  - **Daya Bay**: $\sin^2 2\theta_{13} = 0.089 \pm 0.010 (\text{stat.}) \pm 0.005 (\text{syst.})$, arXiv:1203:1669
  - **DOUBLE-CHOOZ**: $\sin^2 2\theta_{13} = 0.109 \pm 0.030 (\text{stat.}) \pm 0.025 (\text{syst.})$, :1112:6353
  - **RENO**: $\sin^2 2\theta_{13} = 0.113 \pm 0.013 (\text{stat.}) \pm 0.019 (\text{syst.})$, arXiv:1204:0626
  hence $\sin \theta_{13} \approx 0.155 \rightarrow \theta_{13} \approx 9^\circ$

- Non-Abelian discrete Groups:

  $A_4, A_5, S_3, S_4, \Delta(27)$ etc.

- $A_4$ has four object namely $1, 1', 1''$ and 3. (Talk by G. Rajasekharan)
Structure of the Model

**Modified Altarelli-Feruglio (AF) [arXiv:hep-ph/0512103] model with $A_4 \times Z_3$**

<table>
<thead>
<tr>
<th></th>
<th>$e^c$</th>
<th>$\mu^c$</th>
<th>$\tau^c$</th>
<th>$L$</th>
<th>$N^c$</th>
<th>$H_u$</th>
<th>$H_d$</th>
<th>$\phi_S$</th>
<th>$\phi_T$</th>
<th>$\xi$</th>
<th>$\xi'$</th>
<th>$\phi_0^S$</th>
<th>$\phi_0^T$</th>
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<td>1</td>
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</tr>
<tr>
<td>$A_4$</td>
<td>1</td>
<td>1&quot;</td>
<td>1'</td>
<td>3</td>
<td>3</td>
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<tr>
<td>$Z_3$</td>
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<td>$\omega$</td>
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<td>$\omega^2$</td>
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<tr>
<td>$U(1)_R$</td>
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<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
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Shimizu, Tanimoto, Watanabe PTP126; King, Lnhn JHEP1109

- Vacuum expectation values (vev) of the scalar fields
  $\langle \phi_T \rangle = (v_T, 0, 0)$, $\langle \phi_S \rangle = (v_S, v_S, v_S)$, $\langle H_{u,d} \rangle = v_{u,d}$, $\langle \xi \rangle = u$, $\langle \xi' \rangle = u_N$.

**Superpotential: without $\xi'$**

$$w = \left[ y_e e^c (\phi_T L) + y_\mu \mu^c (\phi_T L)' + y_\tau \tau^c (\phi_T L)'' \right] \frac{H_d}{\Lambda} + y L N^c H_u + x_A \xi (N^c N^c) + x_B \phi_S (N^c N^c)$$

- Dirac Mass matrix:

$$m_D = y v_u \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Charged lepton mass matrix: diagonal.
Deviation from TBM mixing

- To make $\theta_{13} \neq 0$, introduce new scalar singlet $\xi'$, which contributes in the heavy Majorana neutrino sector through $x_N \xi' (N^c N^c)$.
- Majorana mass matrix is then given by

$$M_{Rd} = \begin{bmatrix}
a + 2b/3 & -b/3 & -b/3 \\
-b/3 & 2b/3 & a - b/3 \\
-b/3 & a - b/3 & 2b/3
\end{bmatrix} + \begin{bmatrix}
0 & 0 & d \\
0 & d & 0 \\
d & 0 & 0
\end{bmatrix},$$

where $a = 2x_A u$, $b = 2x_B v_s$ and $d = 2x_N u_N$.

- Above structure of RH neutrino mass matrix is no longer diagonalizable by $U_{TB}$ only.
- Additional rotation is required. Let us consider a matrix $U_1$ (parametrized by $\theta$ and $\psi$), which do this job.
- Then RH neutrino mass matrix can be diagonalized by

$$\text{diag}(M_1 e^{i\varphi_1}, M_2 e^{i\varphi_2}, M_3 e^{i\varphi_3}) = (U_{TB} U_1)^T M_{Rd} U_{TB} U_1.$$ 

Here, $M_{1,2,3}$ all are real, positive eigenvalues of $M_{Rd}$. 

RH-Neutrino masses and phases associated

- Majorana neutrino mass eigenvalues and phases associated can be given by

\[
M_1 = \left| b + \sqrt{a^2 + d^2 - ad} \right| = \left| a \right| \lambda_2 e^{i\phi_{ba}} + \sqrt{1 + \lambda_1^2 e^{2i\phi_{da}} - \lambda_1 e^{i\phi_{da}}} \\
M_2 = \left| a + d \right| = \left| a \right| \left| 1 + \lambda_1 e^{i\phi_{da}} \right|, \\
M_3 = \left| b - \sqrt{a^2 + d^2 - ad} \right| = \left| a \right| \lambda_2 e^{i\phi_{ba}} - \sqrt{1 + \lambda_1^2 e^{2i\phi_{da}} - \lambda_1 e^{i\phi_{da}}} ,
\]

\[
\phi_1 = \arg(b + \sqrt{a^2 + d^2 - ad}) \\
\phi_2 = \arg(a + d) \\
\phi_3 = \arg(b - \sqrt{a^2 + d^2 - ad})
\]

where \( \lambda_1 = |d/a| \) and \( \lambda_2 = |b/a| \), also \( \phi_{da} = \phi_d - \phi_a \) and \( \phi_{ba} = \phi_b - \phi_a \) are phase difference between \((d, a)\) and \((b, a)\) respectively.

- Without loss of generality we will work with \( \phi_{da} = 0 \) and say \( K = \sqrt{1 - \lambda_1 + \lambda_1^2} \), then we have

\[
M_1 = \left| a \right| \lambda_2 e^{i\phi_{ba}} + K \quad \phi_1 = \arg(b + aK), \\
M_2 = \left| a \right| \left| 1 + \alpha_1 \right| \quad \phi_2 = \arg(a + d), \\
M_3 = \left| a \right| \lambda_2 e^{i\phi_{ba}} - K \quad \phi_3 = \arg(b - aK).
\]
Light neutrino masses and mixings

- Light neutrino masses obtained via \( m_\nu = m_D^T M_{Rd}^{-1} m_D = U_\nu^* m^\text{diag}_\nu U_\nu^{\dagger} \), here

\[
U_\nu = \frac{m_D^T}{y_{\nu u}} U_{TB} U_1^* \text{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, e^{i\varphi_3/2}).
\]

- Light neutrino masses given by

\[
m_i = \frac{(y_{\nu u})^2}{M_i},
\]

where \( m_i \)'s are real and positive.

- We can now remove one common phase by setting \( \varphi_1 = 0 \) and we find the Majorana phases as

\[
\varphi_2 = \alpha_{21}, \\
\varphi_3 = \alpha_{31}
\]

Majorana phases will be extremely important when we will study Leptogenesis.
Light neutrino masses and mixings

- Final form of unitary matrix that diagonalizes $m_\nu$ is given by

$$U_\nu = \frac{m_D^T}{y_{\nu u}} U_{TB} \begin{bmatrix} \cos \theta & 0 & \sin \theta e^{-i\psi} \\ 0 & 1 & 0 \\ -\sin \theta e^{+i\psi} & 0 & \cos \theta \end{bmatrix} \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$$

$$= \begin{bmatrix} \sqrt{\frac{2}{3}} \cos \theta & 1/\sqrt{3} & -\sqrt{\frac{2}{3}} \sin \theta e^{-i\psi} \\ -\frac{\cos \theta}{\sqrt{6}} + \frac{\sin \theta}{\sqrt{2}} e^{i\psi} & 1/\sqrt{3} & -\frac{\cos \theta}{\sqrt{6}} - \frac{\sin \theta}{\sqrt{2}} e^{-i\psi} \\ -\frac{\cos \theta}{\sqrt{6}} - \frac{\sin \theta}{\sqrt{2}} e^{i\psi} & 1/\sqrt{3} & -\frac{\cos \theta}{\sqrt{6}} + \frac{\sin \theta}{\sqrt{2}} e^{-i\psi} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}.$$  

here we have parametrized the extra $U_1$ matrix by $\theta$ and $\psi$ as mentioned earlier.

- We find a general sum rule for light neutrino masses as given by,

$$\frac{1}{m_1} - \frac{2K e^{i\alpha_{21}}}{m_2(1 + \lambda_1)} = \frac{e^{i\alpha_{31}}}{m_3}.$$  

- When, $K = \sqrt{1 - \lambda_1 + \lambda_1^2} \to 1$ (i.e. with $\lambda_1 = 0$), the sum rule is reduced to the one found in Altarelli,Meloni JGP36 and Hagedorn,Molinaro,Petcov JHEP0909.
Generation of nonzero $\theta_{13}$ and effect on other mixing angles

- Comparing $U_{PMNS}$ and $U_{\nu}$ we get

$$
\sin \theta_{13} = \sqrt{\frac{2}{3}} \sin \theta, \quad \delta_{CP} = \psi = \tan 2\theta = \frac{\sqrt{3}\lambda_1}{(2 - \lambda_1)},
$$

$$
\sin^2 \theta_{12} = \frac{1}{3(1 - \sin^2 \theta_{13})} \quad \text{and} \quad \sin^2 \theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}} \sin \theta_{13} \cos \psi.
$$

- $\sin^2 \theta_{13}$ vs $\lambda_1$

Here $\psi = 0$. Now, for $3\sigma$ (blue patch) and $1\sigma$ (red patch) range of $\sin^2 \theta_{13}$ we get $\lambda_1 = 0.328 - 0.413$ and $\lambda_1 = 0.357 - 0.386$ respectively. Best-fit value of $\sin^2 \theta_{13}$ makes $\lambda_1 = 0.378$ and $0.38$ for NH and IH respectively.
Non-zero $\theta_{13}$ and effect on other mixing angles

- Correlation plots: $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ vs $\lambda_1$

- Here, vertical blue patch indicates allowed value for $\alpha_1$ corresponding to $3\sigma$ range of $\sin^2 \theta_{13}$ and horizontal red dashed line represents $3\sigma$ allowed range for $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$

- Summary:

<table>
<thead>
<tr>
<th>Range of $\lambda_1$ obtained from Fig.1</th>
<th>$\sin^2 \theta_{12}$</th>
<th>$\sin^2 \theta_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.36 \lesssim \lambda_1 \lesssim 0.39$</td>
<td>0.341-0.342</td>
<td>0.604-0.614</td>
</tr>
<tr>
<td>$0.33 \lesssim \lambda_1 \lesssim 0.41$</td>
<td>0.339-0.343</td>
<td>0.595-0.620</td>
</tr>
</tbody>
</table>

Table: Allowed regions of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ obtained from Fig.2 for a restricted range of $\lambda_1$ (corresponding to Fig.1) in our set-up.
Correlation of parameters involved

- Parameters involved Majorana neutrino masses: $\lambda_1, \lambda_2, |a|, \phi_{ba}$.
- These can be constrained by low energy neutrino oscillation data through ratio of solar and atmospheric mass squared difference defined as

$$r = \frac{\Delta m^2_\odot}{|\Delta m^2_A|}, \quad \Delta m^2_\odot = \Delta m^2_{21} \equiv m^2_2 - m^2_1, \quad |\Delta m^2_A| = |\Delta m^2_{31}| \equiv |\Delta m^2_{32}|.$$

- Hence using above relations we get

$$r = \frac{[\lambda_2^2 + 2\lambda_2 K \cos \phi_{ba} + K^2 - (1 + \lambda_1)^2](\lambda_2^2 - 2\lambda_2 K \cos \phi_{ba} + K^2)}{4(1 + \lambda_1)^2 \lambda_2 K |\cos \phi_{ba}|}.$$

- Variation of $\cos \phi_{ba}$ with $\lambda_2$ for $r = 0.03$ (using best fit values)

- From the above plot for NH $\cos \phi_{ba} > 0$ ($\lambda_2 = 0.71 - 1.2$) and for IH $\cos \phi_{ba} < 0$ ($\lambda_2 = 1.1 - 2.3$).
Constraints on Light Neutrino Mass

- Using $r = \frac{\Delta m^2_{\odot}}{|\Delta m^2_A|}$ and neutrino mass eigenvalues we obtain

$$m_1^2 = |\Delta m^2_A| r \frac{(1 + \lambda_1)^2}{[\lambda_2^2 + 2\lambda_2K \cos \phi_{ba} + K^2 - (1 + \lambda_1)^2]} .$$

- $r = 0.03$ and $\lambda_1$ from best fit value of $\sin^2 \theta_{13}$, we can estimate $m_i$'s from the above relation for both the NH (and IH) as follows

**Figure**: Light neutrino masses $m_1$ (green dotted), $m_2$ (orange dashed), $m_3$ (blue dot-dashed) and $\sum m_i$ (red continuous) vs $\lambda_2$ for $\lambda_1$ fixed. Here in the right panel horizontal shaded region indicated disfavored region for $\sum m_i$ from Planck Collaboration.

$$0.07 \text{ eV} \leq \sum m_i \leq 0.10 \text{ eV} (\text{NH}, \lambda_2 = 0.73 - 1.20),$$

$$0.13 \text{ eV} \leq \sum m_i \leq 0.23 \text{ eV} (\text{IH}, \lambda_2 = 1.30 - 2.30).$$
Constraints on Majorana phases

- Majorana phases are insensitive to neutrino oscillation experiments.
- They can be constrained using parameters appearing in mass eigenvalues.
- With $\phi_{da}=0$, we found Majorana phases $\alpha_{21,31}$ as [modified form as obtained in Hagedorn, Molinaro, Petcov JHEP 0909, 115 (2009)]

\[
\tan \alpha_{21} = -\frac{\lambda_2 \sin \phi_{ba}}{K + \lambda_2 \cos \phi_{ba}}, \quad \tan \alpha_{31} = \frac{2K \lambda_2 \sin \phi_{ba}}{\lambda_2^2 - K^2}.
\]

**Figure:** Majorana phases $\alpha_{21,31}$ as function of $\lambda_2$ for NH (upper row with $\cos \phi_{ba} > 0$ and $\sin \phi_{ba} < 0$) and IH (lower row with $\cos \phi_{ba} < 0$ and $\sin \phi_{ba} < 0$) respectively.
Bound on neutrinoless double beta decay:

- **Effective neutrino mass parameter $|\langle m \rangle|$**

  $$|\langle m \rangle| = \left| \frac{2}{3} m_1 \cos^2 \theta + \frac{1}{3} m_2 e^{i\alpha_{21}} + \frac{2}{3} m_3 \sin^2 \theta e^{i\alpha_{31}} \right|.$$  

  with $\delta_{CP} = \psi$

- $|\langle m \rangle|$ vs $\lambda_2$ Plot:

  - **Summary**: $0.01 \text{ eV} \lesssim |\langle m \rangle| \lesssim 0.02 \text{ eV}$ (NH); $0.015 \text{ eV} \lesssim |\langle m \rangle| \lesssim 0.07 \text{ eV}$ (IH).

  - The current upper limit on $|\langle m \rangle|$ varies between 0.177 eV and 0.339 eV [arXiv:1407.4357].
Leptogenesis

1. Out of equilibrium decay of RH neutrinos can produce lepton asymmetry at a temperature $T \sim M_i \gtrsim (1 + \tan^2 \beta)10^{12}$ GeV

2. Lepton asymmetry parameter is defined as in the basis where RH neutrinos are diagonal

$$\epsilon_i = \frac{1}{8\pi} \sum_{j \neq i} \text{Im} \left[ \left( (\hat{\nu}_\nu \hat{\nu}_\nu^\dagger)_{ij} \right)^2 \right] f \left( \frac{m_i}{m_j} \right)$$

where $\hat{\nu}_\nu = \text{diag}(1, e^{-i\alpha_{21}/2}, e^{-i\alpha_{31}/2}) U_R^T Y_\nu$.

3. The loop factor $f(x)$ in above expression has been defined as follows

$$f(x) \equiv -x \left( \frac{2}{x^2 - 1} + \log \left( 1 + \frac{1}{x^2} \right) \right) \text{ with } x = m_i/m_j.$$

4. Leptogenesis can be linked to baryon asymmetry as

$$Y_B \approx \sum Y_{Bi} \text{ where } Y_{Bi} \simeq -1.48 \times 10^{-3} \epsilon_i \eta_{ii}.$$ 

$Y_{Bi}$'s are coming from decay of each Majorana neutrinos and $\eta_{ii}$ stands for efficiency factor

$$\frac{1}{\eta_{ii}} \approx 3.3 \times 10^{-3} \text{ eV} + \left( \frac{\tilde{m}_i}{0.55 \times 10^{-3} \text{ eV}} \right)^{1.16},$$

with washout mass parameter, $\tilde{m}_i = \frac{\langle \hat{\nu}_\nu \hat{\nu}_\nu^\dagger \rangle_{ii} \nu_{ii}^2}{M_i}$ for $M_i < 10^{14}$ GeV.
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where $\hat{Y}_\nu = \text{diag}(1, e^{-i\alpha_{21}/2}, e^{-i\alpha_{31}/2})U_R^T Y_\nu$.

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$$\frac{1}{\eta_{ii}} \approx \frac{3.3 \times 10^{-3}}{\tilde{m}_i} \text{ eV} + \left( \frac{\tilde{m}_i}{0.55 \times 10^{-3} \text{ eV}} \right)^{1.16},$$

with washout mass parameter, $\tilde{m}_i = \frac{(\hat{Y}_\nu \hat{Y}_\nu^\dagger)_{ii}}{M_i} \nu_{ii}^2$ for $M_i < 10^{14}$ GeV.
Leptogenesis

1. Out of equilibrium decay of RH neutrinos can produce lepton asymmetry at a temperature $T \sim M_i \gtrsim (1 + \tan^2 \beta)10^{12}$ GeV

2. Lepton asymmetry parameter is defined as in the basis where RH neutrinos are diagonal

$$\epsilon_i = \frac{1}{8\pi} \sum_{j \neq i} \frac{\text{Im} \left[ \left( \dot{Y}_\nu \dot{Y}_\nu^\dagger \right)_{ji} \right]^2}{\left( \dot{Y}_\nu \dot{Y}_\nu^\dagger \right)_{ii}} f \left( \frac{m_i}{m_j} \right)$$

where $\dot{Y}_\nu = \text{diag}(1, e^{-i\alpha_{21}/2}, e^{-i\alpha_{31}/2}) U_R^T Y_\nu$.

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$$Y_B \approx \sum Y_{Bi} \text{ where } Y_{Bi} \simeq -1.48 \times 10^{-3} \epsilon_i \eta_{ii}.$$  

$Y_{Bi}$’s are coming from decay of each Majorana neutrinos and $\eta_{ii}$ stands for efficiency factor

$$\frac{1}{\eta_{ii}} \approx 3.3 \times 10^{-3} \text{ eV} \left( \frac{\tilde{m}_i}{0.55 \times 10^{-3} \text{ eV}} \right)^{1.16},$$

with washout mass parameter, $\tilde{m}_i = \frac{(\dot{Y}_\nu \dot{Y}_\nu^\dagger)_{ii} \nu_{ii}^2}{M_i}$ for $M_i < 10^{14}$ GeV.
Leptogenesis

- At LO, $\hat{Y}_{\nu_0} \hat{Y}^\dagger_{\nu_0} \propto |y|^2 |1$
- Results, vanishing asymmetry parameter $\epsilon_i$ and zero matter-antimatter asymmetry.
- Possible remedy: NLO correction in Yukawa sector [arXiv:0807.4176].
- Relevant contribution Yukawa sector:

\[
y(LN^c)H_u + x_C N^c(L\phi_T)_{3S} H_u/\Lambda + x_D N^c(L\phi_T)_{3A} H_u/\Lambda
\]

- Yukawa matrix and $\hat{Y}_\nu \hat{Y}^\dagger_{\nu}$:

\[
Y_\nu = Y_{\nu_0} + \delta Y_{\nu}
\]

\[
= y \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \frac{x_C v_T}{\Lambda} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} + \frac{x_D v_T}{\Lambda} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}
\]

- Charged lepton mass-matrix remains diagonal

\[
\epsilon_1 = \frac{-1}{2\pi} \left( \frac{v_T}{\Lambda} \right)^2 \left[ \sin \alpha_{21} \left( 2\text{Re}(x_C)^2 \cos^2 \theta + \frac{2\text{Re}(x_D)^2}{3} \sin^2 \theta + \frac{2\text{Re}(x_C)\text{Re}(x_D)}{\sqrt{3}} \sin 2\theta \right) f \left( \frac{m_1}{m_2} \right) 
+ \sin \alpha_{31} \left( \text{Re}(x_C)^2 \sin^2 2\theta + \frac{\text{Re}(x_D)^2}{3} \cos^2 2\theta + \frac{\text{Re}(x_C)\text{Re}(x_D)}{\sqrt{3}} \sin 4\theta \right) f \left( \frac{m_1}{m_3} \right) \right]
\]

and similar expressions for $\epsilon_2$ and $\epsilon_3$.
- Here, $y \gg (\text{Re}(x_{C,D})v_T/\Lambda)$ since $\text{Re}(x_{C,D})$ are of same order of $y$ and $(v_T/\Lambda)$ is a suppression factor. Small value of $(v_T/\Lambda)$ is required to reproduce $|\epsilon_i| \gtrsim 10^{-6}$ for generating baryon asymmetry of proper order. With this consideration, the washout mass parameters becomes identical to light neutrino masses (i.e $\tilde{m}_i \approx m_i$).
Leptogenesis

Baryon Asymmetry

Figure: Green, orange and blue dashed lines stands for $Y_{B1}$, $Y_{B2}$ and $Y_{B3}$ respectively and red line for total baryon asymmetry $Y_B$.

Parameters involved and their contribution:

- $\frac{\nu_T}{\Lambda} \sim 10^{-2}$, and typical magnitude of lepton asymmetry $|\epsilon_i| \gtrsim 10^{-6}$
- $\lambda_1 = 0.37$ and $0.38$ for best fit value of $\sin^2 \theta_{13}$ for NH and IH respectively.
- $x_C = x_D = 0.2$ for NH and $x_C = x_D = 0.05$ for IH. Observed range: $Y_B = (8.77 \pm 0.24) \times 10^{-11}$. 
$Y_B$ vs. $\sin^2 \theta_{13}$: ($\lambda_2 = 1$ for NH and $\lambda_2 = 2.1$ for IH)

\begin{align*}
\theta_{13} &\neq 0 \text{ and Leptogenesis} \\
\end{align*}
Conclusion

- We have modified the original $A_4$ symmetry model of AF by extending the flavon sector with additional scalar singlet.

- The vacuum alignment of the flavon vevs are studied.

- Modified neutrino mass matrix (through Type-I seesaw) generates adequate $\theta_{13}$; other mixing angles are in desired range.

- A new sum rule for the model is obtained.

- Correlation among entries in mass parameters and Majorana phases are studied.

- Able to generate required matter-antimatter asymmetry of the universe, where Majorana phases play important role.
Questions and Comments....
**A₄ group multiplication** [arXiv:1002:0211]

- Even permutation group of four object.
- It has 4!/2 = 12 elements.
- It is a subgroup of S₄.
- Discrete group A₄ has three 1D representation 1, 1′, 1″ and a irreducible 3D representation 3. Product of the singlet and triplets are given by

\[
1 \otimes 1 = 1, \quad 1' \otimes 1' = 1'', \quad 1' \otimes 1'' = 1, \quad 1'' \otimes 1'' = 1' \&
\]

\[
3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_A \oplus 3_S
\]

where subscripts A and S stands for “asymmetric” and “symmetric” respectively. If we have two triplets \((a_1, a_2, a_3)\) and \((b_1, b_2, b_3)\), their products are given by

\[
\begin{align*}
1 & \sim a_1 b_1 + a_2 b_3 + a_3 b_2, \\
1' & \sim a_3 b_3 + a_1 b_2 + a_2 b_1, \\
1'' & \sim a_2 b_2 + a_3 b_1 + a_1 b_3, \\
3_S & \sim \begin{bmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{bmatrix}, \\
3_A & \sim \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{bmatrix}.
\end{align*}
\]
Vacuum Alignment

- Driving part of the LO superpotential, invariant under $A_4 \times Z_3$ with $R = 2$, is

$$w_d = M(\phi_0^T \phi_T) + g(\phi_0^T \phi_T \phi_T) + \phi_0^S(g_1 \phi_S \phi_S + g_2 \phi_S \xi + g_3 \phi_S \xi') + \xi_0(g_4 \phi_S \phi_S + g_5 \xi \xi).$$

Equations which give vacuum structure of $\phi_T$ are given by:

$$\frac{\partial w}{\partial \phi_{T01}} = M\phi_T1 + \frac{2g}{3} \left( \phi_T^2 - \phi_T^2 \phi_T^3 \right) = 0,$$

$$\frac{\partial w}{\partial \phi_{T02}} = M\phi_T1 + \frac{2g}{3} \left( \phi_T^2 - \phi_T^1 \phi_T^3 \right) = 0,$$

$$\frac{\partial w}{\partial \phi_{T03}} = M\phi_T1 + \frac{2g}{3} \left( \phi_T^3 - \phi_T^1 \phi_T^2 \right) = 0.$$

Solution of these equations can be given by: $\langle \phi_T \rangle = (v_T, 0, 0)$ where $v_T = -\frac{3M}{2g}$.

- Again, equations responsible for vacuum alignments of $\phi_S$, $\xi$ and $\xi'$ are:

$$\frac{\partial w}{\partial \phi_{S01}} = \frac{2g_1}{3} \left( \phi_{S1}^2 - \phi_{S2} \phi_{S3} \right) + g_2 \xi \phi_{S1} + g_3 \xi' \phi_{S3} = 0,$$

$$\frac{\partial w}{\partial \phi_{S02}} = \frac{2g_1}{3} \left( \phi_{S2}^2 - \phi_{S1} \phi_{S3} \right) + g_2 \xi \phi_{S2} + g_3 \xi' \phi_{S2} = 0,$$

$$\frac{\partial w}{\partial \phi_{S03}} = \frac{2g_1}{3} \left( \phi_{S3}^2 - \phi_{S1} \phi_{S2} \right) + g_2 \xi \phi_{S3} + g_3 \xi' \phi_{S1} = 0,$$

$$\frac{\partial w}{\partial \xi_0} = g_4(\phi_{S1}^2 + 2\phi_{S2} \phi_{S3}) + g_5 \xi \xi = 0$$

From these equations we obtain $\langle \phi_S \rangle = (v_S, v_S, v_S)$, $\langle \xi \rangle = u$ and $\langle \xi' \rangle = u' \neq 0$ with $v_S^2 = -\frac{g_5 u^2}{3g_4}$ and $u' = -\frac{g_2 u}{g_3}$. 

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$\theta_{13} \neq 0$ and Leptogenesis